Laser Doppler Velocimetry ———

Effect of Refractive Index Change at Flame Front*

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In application of laser doppler velocimetry (LDV) to the flow field including flames, there are interactions of the laser beams with flames. Those interactions are schlieren effect and phase effect. In this study, such interactions were investigated experimentally and theoretically.

The main results of the investigation are as follows :

- (1) The effect of interaction between laser beams and flame front on LDV was measured actually.
- (2) The effect is serious only for near-tangential incidence of laser beams to flame front, when large pseudo-velocities are produced in very short time interval.
- (3) The value of pseudo-velocity is proportional to the flame speed, and it increases with the flame size.
- (4) The pseudo-velocity maybe become serious when the relative incidence angle of laser beam to flame front fluctuates by such a high frequency as in strong turbulent flames.

Key Words : Flow Measurement, Combustion, Laser Doppler Velocimetry

1. Introduction

A pseudo-velocity appears in a system of laser doppler velocimetry (LDV) applied to the measurements of gas flow in turbulent combustion system. Because of some difference in refractive indices between burned and unburned gases, the directions and the phases of laser beams which traverse flame fronts change and the fringe grid at the intersecting point of beams moves, and then a pseudo-velocity is detected. These are called the "schlieren effect" and the "phase effect", as pointed out by Hong, et al.⁽¹⁾ And its velocity has been recorded as an error in the measurement of gas flow velocity by the LDV.

In this study, the pseudo-velocity could be measured experimentally by the LDV, in the case of a premixed spherical propagating laminar flame traversing laser beams. The theoretical calculation results of schlieren and phase effects were comparable with experimental ones. Namely, it was confirmed that an error of the LDV due to changing refractive indices could be calculated. Therefore it was proved that this pseudo-velocity is proportional to the flame speed, increases with the flame size, and maybe become serious when the relative incidence angles of laser beams to flame front fluctuate by such a high

frequency as in strong turbulent flames.

2. Pseudo-velocity due to Refractive Indices Change

2.1 Experimental apparatus and procedures

The system of the LDV applied to experimentation is forward scattering fringe mode, as illustrated in Fig.1. The optical and electronic processing instruments are composed of a 15mW He-Ne laser, a beam splitter, a frequency shifter, a photomultiplier and a frequency tracker. Twin laser beams intersect at the outside of a combustion chamber, and at the center of this intersection (measuring point), scattering particles (JIS Z 8901 grade 11) are fixed on a thin glass plate. The combustion chamber of 80mm cube has two observation windows (transparent acrylic resin) as shown in Fig.1. Homogeneous mixture of propane and air (equivalence ratio $\phi = 1.0$ and 1.4, pressure before ignition = 0.101 MPa) is ignited by electric spark at the center of combustion chamber, and then a laminar flame develops nearly spherically and interferes with laser beams.

Figures 2 and 3 show examples of the frequency tracker outputs with lapse of time after ignition (t). The intersection angle of twin laser beams in these experiments is 0.2023rad (11.59°) and the wave length is 632.8nm, and then a doppler frequency (f_D)

Fig.1 Experimental system

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of 1MHz corresponds to about 3.13m/s as flow velocity. It is verified by high speed motion pictures of flame that the time interval of two peaks a and b, as shown in Fig.2 or 3, is equal to the time of flame travel between beam 1 and beam 2. In spite of stationary scattering particles, a pseudo-velocity is detected at the instant of interference between flame front and laser beams. In these experiments, the frequency of beam 1 is shifted 40MHz by the frequency shifter, and the output of the photomultiplier as a detector is added a 39.95MHz alternating current, then an output signal from the detector is taken as a beat of these frequencies. This is the same as the frequency of beam 1 being shifted 0.05MHz. Figure 4 shows an example of beam 2 being shifted 0.05MHz, that is an inverted shape to Fig.2.

2.2 Calculation method of pseudo-velocity

The interference between flame front and laser beams is shown schematically in Fig.5. A rectangular coordinate system (X,Y) is adopted. A left side line (X=0,Y-axis) in this figure indicates the inner surface of observation window of laser source side. The width of this combustion chamber is 2Xc. The thickness of an observation window is (Xa-2Xc). The center of spherical flame of radius R comes at (Xc,0). Beam 1 enters at the origin (0,0), beam 2 does at the point of (0,H), and the incidence angle of each beam from X-axis is θ_0 . Laser beams go straight through each medium of unburned gas, burned gas, observation windows and air; refract at each boundary; and then intersect at the point of (X₀,Y₀) which is out of the chamber. Coordinates of refraction points are given as (X_{j,k}, Y_{j,k}) (j=1 for beam 1, and 2 for beam 2, k=1,2,3,4), a beam angle to X-axis at each point is denoted as $\theta_{j,k}$, each incidence and refractive angles of laser beams to the flame front are denoted as $i_{j,k}$ and $r_{j,k}$, and angles from X-axis to the connecting lines between the flame center (Xc,0) and each refraction point on the flame front are denoted as α_1 , β_1 , α_2 and β_2 as shown in Fig.5. Refractive

indices of air, unburned gas, burned gas and observation windows are given as n_0, n_1, n_2 and n_3 , respectively. It is assumed for calculation that the flame propagates spherically, the thickness of a flame front and the diameter of laser beams being ignored.

A two-dimensional system is considered for simplicity as shown in Fig.5. From the situation of Fig.5, the following equation is obtained about the optical path of beam 1.

$$R \sin \alpha_1 = (X_c - R \cos \alpha_1) \tan \theta_0.$$

Then α_1 can be expressed as follows :

$$\alpha_1 = \sin^{-1}(X_c \sin \theta_0 / R) - \theta_0. \quad (1)$$

The incidence angle (i_{11}) of beam 1 to the flame front,

$$i_{11} = \alpha_1 + \theta_0. \quad (2)$$

And the refractive angle (r_{11}) is given from Snell's law, as

$$r_{11} = \sin^{-1}(n_1 \sin i_{11} / n_2). \quad (3)$$

Following relations are obtained from the geometry of laser beams shown in Fig.5.

$$i_{12} = r_{11}, \quad (4)$$

$$r_{12} = i_{11}, \quad (5)$$

$$\beta_1 = (\pi - 2r_{11}) + \alpha_1, \quad (6)$$

$$\theta_{11} = r_{11} - \alpha_1, \quad (7)$$

$$\theta_{12} = \pi - (\beta_1 + i_{11}). \quad (8)$$

The refractive angles at the observation window are as follows :

$$\theta_{13} = \sin^{-1}(n_1 \sin \theta_{12} / n_3), \quad (9)$$

$$\theta_{14} = \sin^{-1}(n_3 \sin \theta_{13} / n_0). \quad (10)$$

From these equations, the coordinates of each refraction point about beam 1 are given as follows :

$$X_{11} = X_c - R \cos \alpha_1, \quad Y_{11} = R \sin \alpha_1, \quad (11)$$

$$X_{12} = X_c - R \cos \beta_1, \quad Y_{12} = R \sin \beta_1, \quad (12)$$

$$Y_{13} = Y_{12} + (2X_c - X_{12}) \tan \theta_{12}, \quad (13)$$

$$Y_{14} = Y_{13} + (X_a - 2X_c) \tan \theta_{13}. \quad (14)$$

The optical path of beam 2 can be obtained by replacing the values for beam 1 (X₁₁, θ_{11} , etc.) with the ones for beam 2 (X₂₁, θ_{21} , etc., see Fig.5). Therefore the intersection coordinates (X₀,Y₀) of laser beams are expressed as follows :

$$X_0 = X_a + (Y_{24} - Y_{14}) \cos \theta_{14} \cos \theta_{24} / \sin(\theta_{14} + \theta_{24}), \quad (15)$$

$$Y_0 = Y_{14} + (X_0 - X_a) \tan \theta_{14}. \quad (16)$$

The moving speed of the intersection is expressed as follows :

Fig.2 Pseudo-velocity

Fig.3 Pseudo-velocity Fig.4 Pseudo-velocity

Fig.5 Optical path and flame front

$$\begin{aligned}\dot{X}_0 &= [\cos \theta_{14} \cos \theta_{24} (\dot{Y}_{24} - \dot{Y}_{14}) - (X_0 - X_a) \cos(\theta_{14} + \theta_{24}) \\ &\quad + (Y_{24} - Y_{14}) \sin \theta_{14} \cos \theta_{24}] \dot{\theta}_{14} - [(X_0 - X_a) \cos(\theta_{14} + \theta_{24}) \\ &\quad + (Y_{24} - Y_{14}) \cos \theta_{14} \sin \theta_{24}] \dot{\theta}_{24} / \sin(\theta_{14} + \theta_{24}), \dots\dots\dots (17)\end{aligned}$$

$$\dot{Y}_0 = \dot{Y}_{14} + (X_0 - X_a) \dot{\theta}_{14} / \cos^2 \theta_{14} + \dot{X}_0 \tan \theta_{14}, \dots\dots\dots (18)$$

where $\dot{X}_0 = dX_0/dt$, $\dot{Y}_0 = dY_0/dt$. Furthermore the optical path length of beam 1 (l_1) and the rate of change of $l_1 (= dl_1/dt)$ are given as follows :

$$\begin{aligned}l_1 &= n_1 X_{11} / \cos \theta_0 + n_2 (X_{12} - X_{11}) / \cos \theta_{11} + n_1 (2X_c - X_{12}) / \cos \theta_{12} \\ &\quad + n_3 (X_a - 2X_c) / \cos \theta_{13} + n_0 (X_0 - X_a) / \cos \theta_{14}, \dots\dots\dots (19)\end{aligned}$$

$$\begin{aligned}\dot{l}_1 &= \left(\frac{n_1}{\cos \theta_0} - \frac{n_2}{\cos \theta_{11}} \right) \dot{X}_{11} + \left(\frac{n_2}{\cos \theta_{11}} - \frac{n_1}{\cos \theta_{12}} \right) \dot{X}_{12} + \frac{n_0}{\cos \theta_{14}} \dot{X}_0 + n_2 (X_{12} - X_{11}) \frac{\tan \theta_{11}}{\cos \theta_{11}} \dot{\theta}_{11} \\ &\quad + n_1 (2X_c - X_{12}) \frac{\tan \theta_{12}}{\cos \theta_{12}} \dot{\theta}_{12} + n_3 (X_a - 2X_c) \frac{\tan \theta_{13}}{\cos \theta_{13}} \dot{\theta}_{13} + n_0 (X_0 - X_a) \frac{\tan \theta_{14}}{\cos \theta_{14}} \dot{\theta}_{14}, \dots\dots\dots (20)\end{aligned}$$

Similarly the optical path length of beam 2 (l_2) and its rate of change ($\dot{l}_2$) can be obtained by replacing the values of beam 1 (X_{11} , θ_{11} , etc.) with the ones of beam 2 (X_{21} , θ_{21} , etc.).

By changing the difference in two optical path lengths, i.e. $\Delta l = l_2 - l_1$, the fringe grid in the intersection (X_0, Y_0) changes by a frequency of $\Delta f = (\dot{l}_2 - \dot{l}_1) / \lambda$.

As the fringe spacing is $\lambda / \left\{ 2 \sin \left(\frac{\theta_{14} + \theta_{24}}{2} \right) \right\}$, one component of the pseudo-velocity, V_p , due to the phase effect,

$$V_p = (\dot{l}_2 - \dot{l}_1) / \left\{ 2 \sin \left(\frac{\theta_{14} + \theta_{24}}{2} \right) \right\}, \dots\dots\dots (21)$$

and the other component, V_s , due to the schlieren effect,

$$V_s = \dot{Y}_0 \cos \left(\frac{\theta_{14} - \theta_{24}}{2} \right) \dots\dots\dots (22)$$

In case of shifting the frequency of beam 1 in the LDV system applied in these experiments, positive velocity can be detected when the measuring point changes in the negative direction of Y-axis. Then ($\dot{l}_2 - \dot{l}_1$) > 0 yields $V_p > 0$, because the fringe grid changes in the negative direction of Y-axis due to the phase of beam 2 being in advance of that of beam 1. And $\dot{Y}_0 > 0$ yields $V_s > 0$. Therefore the pseudo-velocity, V_a , can be obtained as a sum of V_p and V_s , considering the plus or minus sign.

From a mole fraction, v_i , and a refractive index⁽²⁾, n_i , of each component in mixture, the refractive index of gas, n_N (0°C, 0.101 MPa) is given as,

$$n_N - 1 = \sum (n_i - 1) v_i, \dots\dots\dots (23)$$

and from Gladstone-Dale's equation,

$$(n - 1) / \rho = (n_N - 1) / \rho_N = K,$$

where ρ is the density and K is a constant depending only on the wave length of light and nature of the substance (suffix N indicates the value for 0°C and 0.101 MPa), so the effect of temperature and pressure on the refractive index is expressed by

$$n - 1 = (n_N - 1) \rho / \rho_N \dots\dots\dots (24)$$

As measurements were made in the early stage of flame propagation, the changes of pressure and unburned gas temperature in the chamber were very little. The composition and temperature of burned gas were calculated in adiabatic and equilibrium condition, and the refractive index of mixture could be calculated by using the value given in the literature (2). So refractive indices could be obtained as follows; n_0 of air (288K, 0.101

MPa) 1.000277; n_1 of unburned gas ($\phi=1.0$, 288K, 0.101MPa) 1.000304; n_2 of burned gas ($\phi=1.0$, 2260K, 0.101MPa) 1.000037; and n_3 of observation window (acrylic resin⁽³⁾) 1.476. In Fig.5, $H=20\text{mm}$ and $\theta_0=0.10115\text{rad}$ (5.795°). The flame propagating speed, U , was 2.58m/s ($\phi=1.0$) which was obtained by analyzing the high speed motion pictures of flame. The following calculations were carried out by using above values.

2.3 Calculation results and their comparison with measured values

Calculations were made in the case of C_3H_8 -air homogeneous mixture ($\phi=1.0$). First, the changing of the measuring point is investigated. Denoting the flame radii as R_{p1} and R_{p2} when the flame front comes in contact with laser beams 1 and 2, respectively, and as R_{c1} and R_{c2} when the incidence angles (i_{11} and i_{21}) reach the critical value of 1.5478 rad in $\phi=1.0$, in this experimental system, the following relations were obtained :

$$R_{c1} = R_{p1} + 0.001 \text{ mm} \quad (\text{Beam 1}),$$

$$R_{c2} = R_{p2} + 0.004 \text{ mm} \quad (\text{Beam 2}).$$

The measuring point changes extremely with refraction of laser beams, while the flame radius does from R_{p1} or R_{p2} to a little larger than R_{c1} or R_{c2} . Figure 6 shows the calculated value of ΔX_0 which changes the deflection of measuring point in X-direction.

The calculated measuring volume in this experimental system was approximately 1.2mm in X-direction, 0.12mm in Y and 0.12mm in Z. These values were verified by investigating the detection range of the doppler frequencies caused by traveling a string of 7 μm diameter.

In the case of an extreme change in the deflection of measuring point, the scattering particles get out of the measuring range because of being fixed on a thin glass plate,

Fig.6 Deflection of measuring point in X-direction, ΔX_0 .

and then the measurement becomes impossible, i.e., the measuring limit of these experiments is 0.6mm of ΔX_0 .

Figures 7 and 8 show the calculated results of pseudo-velocity components, V_p and V_s , under the phase and the schlieren effects. U in figures denotes the flame propagating velocity. The phase and the schlieren effects work opposite to each other.

A pseudo-velocity of V_a can be obtained by summation of V_p and V_s . Figure 9 shows the experimental and calculated values of V_a . The extent of $\Delta X_0 > 0.6$ mm is excluded for calculation. Both values are in good agreement with each other except immediately after the flame front intersects each beam. The disagreement during very short time is likely to come from the fact that beams have finite diameter, the change of refractive index on the flame front is not discontinui-

Fig. 7 V_p , V_s Fig. 8 V_p , V_s

ty, and the frequency tracker cannot follow very rapid changes so well.

2.4 Effect of flame propagating velocity

Here, the case is considered that the spherical flame develops beyond the laser beams and the beams intersect at the outside of flame (detector side) in the combustion chamber of Fig. 5.

Although flame propagating velocities may be different, the optical paths of laser beams will be the same under condition of the same flame radius. So $\dot{Y}_0/U$, $\dot{i}_1/U$ and $\dot{i}_2/U$ are given as follows:

$$\dot{Y}_0/U = 2C, \dots\dots\dots (25)$$

$$\dot{i}_1/U = D_1, \dots\dots\dots (26)$$

$$\dot{i}_2/U = D_2, \dots\dots\dots (27)$$

where $C = (1 - N_{12})\{A_1 + M_1(X_0 - X_a)N_{10}/\cos^2 \theta_{14}\} + B \tan \theta_{14}/\sin(\theta_{14} + \theta_{24})$,

$$D_1 = n_1/\cos i_{11} - n_2 \cos \theta_0/(\cos i_{11} \cos \theta_{11}) + 2n_0 B/\cos \theta_{14}$$

$$- (n_2/\cos \theta_{11} - n_1/\cos \theta_{12})\{\cos \beta_1 + (1 - 2N_{12})\sin \beta_1 \tan i_{11}\}$$

$$+ M_1(1 - N_{12})\{n_2(X_{12} - X_{11})\tan \theta_{11}/\cos \theta_{11} + 2n_1(2X_c - X_{12})\tan \theta_{12}/\cos \theta_{12}$$

$$+ 2(X_a - 2X_c)\tan \theta_{13} \cos \theta_{12}/\cos^2 \theta_{13} + 2(X_0 - X_a)\tan \theta_{14} \cos \theta_{12}/\cos^2 \theta_{14}\},$$

$$M_1 = \tan i_{11}/R, N_{12} = (n_1/n_2)(\cos i_{11}/\cos r_{11}),$$

$$N_{13} = (n_1/n_3)(\cos \theta_{12}/\cos \theta_{13}), N_{10} = (n_1/n_0)(\cos \theta_{12}/\cos \theta_{14}),$$

$$A_1 = (1 - N_{12})\{(\sin i_{11}/\cos \theta_{12})\{1 + (X_c/R + \cos \beta_1)/(\cos i_{11} \cos \theta_{12})\} + M_1(X_a - 2X_c)N_{13}/\cos^2 \theta_{13}\},$$

$$B = [\cos \theta_{14} \cos \theta_{24}\{(1 - N_{22})A_2 - (1 - N_{12})A_1\} + \cos(\theta_{14} + \theta_{24})(X_0 - X_a)\{N_{10}M_1(1 - N_{12})$$

$$+ N_{20}M_2(1 - N_{22})\} + (Y_{24} - Y_{14})\{N_{10} \sin \theta_{14} \cos \theta_{24}M_1(1 - N_{12})$$

$$+ N_{20} \cos \theta_{14} \sin \theta_{24}M_2(1 - N_{22})\}]/\sin(\theta_{14} + \theta_{24}).$$

(Regarding D_2 , M_2 , N_{22} , N_{20} and A_2 , the values of θ_{24} , i_{24} and r_{24} from beam 2 are applied.)

Therefore the components of a pseudo-velocity due to the phase and the schlieren effects, V_p and V_s , are expressed as follows:

$$V_p/U = (D_2 - D_1)/\left\{2 \sin\left(\frac{\theta_{14} + \theta_{24}}{2}\right)\right\}, \dots\dots\dots (28)$$

$$V_s/U = 2C \cos\left(\frac{\theta_{14} - \theta_{24}}{2}\right), \dots\dots\dots (29)$$

Accordingly, a pseudo-velocity is in proportion to a flame propagating velocity, if the flame shape and its position are the same.

As T means a time interval of the flame propagation from contact with beam 1 to contact with beam 2, the lapse time (t) from the flame coming in contact with beam 1 can be made dimensionless by T . Therefore V_a/U ($V_a = V_p + V_s$) can be given as one curved line for t/T , having no relation with flame propagating velocity, as shown in Fig. 10.

3. Pseudo-velocity due to Fluctuating Flame Front

Fig. 9 Pseudo-velocity (comparison between experimental and calculated results)

Fig. 10 Pseudo-velocity

Fig.11 Optical paths and flame front

As mentioned above, it was recognized that the pseudo-velocity can be predicted by means of the calculating method mentioned in the section 2.2. In practical measurement of gas flow velocity in a combustion system, the measuring point (intersection of laser beams) exists in a combustion chamber which is described in section 2.4, then there is no influence of an observation window. A few examples of calculation under such condition are given as follows.

3.1 Spherically propagating flame

In the case of a spherically propagating laminar flame as shown in Fig.11, the pseudo-velocity was investigated. In this figure, R_p is the flame radius of the flame front coming in contact with beam 1. When the flame radius grows a little larger than R_p , beam 1 enters the burned gas zone from (X_1, Y_1) , and enters the unburned gas zone from (X_2, Y_2) , but the point of (X_2, Y_2) is not indicated in this figure. During a short time interval after the flame front comes in contact with beam 1, we have $Y_2 < Y_0$, and the intersection of laser beams (X_0, Y_0) is expressed as follows:

$$\left. \begin{aligned} X_0 &= \{X_2 \sin \theta_2 + (H - Y_2) \cos \theta_2\} \cos \theta_0 \\ & / \sin(\theta_0 + \theta_2), \\ Y_0 &= H - X_0 \tan \theta_0 \end{aligned} \right\} \dots (30)$$

The optical path length of laser beams, l_1 and l_2 ,

$$\left. \begin{aligned} l_1 &= n_1 \{X_1 / \cos \theta_0 + (X_0 - X_1) / \cos \theta_2\} \\ & + n_2 (X_2 - X_1) / \cos \theta_1, \\ l_2 &= n_1 X_0 / \cos \theta_0 \end{aligned} \right\} \dots (31)$$

In case of $Y_2 = Y_0$, coordinates of the intersection (X_0, Y_0) are as follows:

$$\left. \begin{aligned} X_0 &= A - R \cos(i_1 + \theta_0) \\ Y_0 &= H - X_0 \tan \theta_0 \end{aligned} \right\} \dots (32)$$

where R is a flame radius and the coordinates of flame center are given as (A, B) . And the optical path lengths,

$$\left. \begin{aligned} l_1 &= n_1 X_1 / \cos \theta_0 + n_2 (X_0 - X_1) / \sin i_1 \\ l_2 &= n_1 X_0 / \cos \theta_0 \end{aligned} \right\} \dots (33)$$

Then if the intersection of laser beams occurs in the flame due to a growing flame, the coordinates of beam intersection will be as follows:

$$\left. \begin{aligned} X_0 &= \{H + X_1 (\tan \theta_1 - \tan \theta_0) + X_3 (\tan \theta_3 \\ & - \tan \theta_0) / (\tan \theta_1 + \tan \theta_3) \\ Y_0 &= Y_1 + (X_0 - X_1) \tan \theta_1 \end{aligned} \right\} \dots (34)$$

The optical path lengths,

$$\left. \begin{aligned} l_1 &= n_1 X_1 / \cos \theta_0 + n_2 (X_0 - X_1) / \cos \theta_1 \\ l_2 &= n_1 X_3 / \cos \theta_0 + n_2 (X_0 - X_3) / \cos \theta_3 \end{aligned} \right\} \dots (35)$$

Figure 12 shows the range of movement of intersection (measuring point) calculated by equations (30), (32) and (34), where ΔX_0 or ΔY_0 is a deflection from origin which is the intersection of laser beams before the flame coming in contact with beam 1. This deflection is made dimensionless by the value of R_p in this figure. In a short time after the flame coming in contact with beam 1, the incidence angle of i_1 becomes critical, and the intersection reaches the maximum deflection point of $(\Delta X_{0\max}, \Delta Y_{0\max})$. Then the deflection decreases to a fixed value.

The pseudo-velocity in the case described above is investigated in this paragraph. In the case of $Y_2 < Y_0$,

$$\left. \begin{aligned} \dot{Y}_0 / U &= -\sin \theta_0 \tan i_1 \\ & \times \sin(\theta_1 - \theta_0) / (\cos^2 \theta_1 \cos r_1) \\ (\dot{i}_2 - \dot{i}_1) / U &= n_1 \{ \tan r_1 \sin(\theta_2 - \theta_1) \\ & + 2 \cos \theta_0 \cos \theta_2 \} / (\cos^2 \theta_1 \cos i_1) \\ & - 2n_2 / \cos r_1 \end{aligned} \right\} \dots (36)$$

and in the case of $Y_2 = Y_0$,

$$\left. \begin{aligned} \dot{Y}_0 / U &= -\sin(2i_1 + \theta_0) / \cos i_1 \\ (\dot{i}_2 - \dot{i}_1) / U &= 2 \{ n_1 \sin(i_1 + \theta_0) - n_2 \} / \sin \theta_0 \end{aligned} \right\} \dots (37)$$

furthermore, when the beam intersection occurs in the burned gas zone,

$$\left. \begin{aligned} \dot{Y}_0 / U &= \sin r_1 \sin^2(\theta_1 - \theta_0) \\ & / (\cos \theta_0 \cos r_1 \cos i_1), (\dot{i}_2 - \dot{i}_1) / U = 0 \end{aligned} \right\} \dots (38)$$

A pseudo-velocity can be calculated from equations (36), (37) and (38). So each angle is same for same flame radius, and the pseudo-velocity is in proportion to the flame propagating velocity. In the system as shown in Fig.11, the beam intersection exists in the burned gas zone, and then a pseudo-velocity can be only due to the schlieren effect, as shown clearly in the equation (38). As T denotes the time from the flame coming in contact with beam 1 to the

Fig.12 Range of movement of measuring point

Fig.13 Pseudo-velocity

state of $Y_2 = Y_0$, the change of V_a/U with dimensionless time t/T is shown in Fig.13.

3.2 Effect of fluctuation of relative angles of laser beams to the flame front

As shown in Fig.14, the laser beams 1 and 2 refract at the boundary (flame front) between unburned and burned gases, and intersect at a point of (X_0, Y_0) in burned gas. The pseudo-velocities are investigated in the cases that angles (i_1 and i_2) change in the same or opposite direction.

In the same ; $di_1/dt = di_2/dt = 1 \text{ rad/s}$,

in the opposite ; $di_1/dt = -di_2/dt = 1 \text{ rad/s}$, when $t = 0$, $i_1 = -\pi/2 \text{ rad}$.

The intersection of laser beams, (X_0, Y_0) , is shown as follows :

$$\begin{aligned} X_0 &= X_1 + (Y_2 - Y_1) / (\tan \theta_1 + \tan \theta_2) \\ Y_0 &= Y_1 + (X_0 - X_1) \tan \theta_1 \end{aligned} \quad \dots (39)$$

and the rate of change,

$$\begin{aligned} \dot{X}_0 &= (Y_1 - Y_2) \{ \cos^2 \theta_2 \dot{\theta}_1 \\ &\quad + \cos^2 \theta_1 \dot{\theta}_2 \} / \sin^2 (\theta_1 + \theta_2) \\ \dot{Y}_0 &= \tan \theta_1 \dot{X}_0 + (X_0 - X_1) \dot{\theta}_1 / \cos^2 \theta_1 \end{aligned} \quad \dots (40)$$

The difference in optical path length between the two laser beams,

$$\Delta l = n_2 (1 / \cos \theta_2 - 1 / \cos \theta_1) (X_0 - X_1), \dots (41)$$

and the rate of change,

$$\begin{aligned} \dot{l} &= n_2 (1 / \cos \theta_2 - 1 / \cos \theta_1) \dot{X}_0 \\ &\quad + n_2 (X_0 - X_1) \{ \dot{\theta}_2 \tan \theta_2 / \cos \theta_2 \\ &\quad - \dot{\theta}_1 \tan \theta_1 / \cos \theta_1 \} \dots (42) \end{aligned}$$

By using equations (40) and (42), when the incidence angles (i_1 and i_2) change from $-\pi/2 \text{ rad}$ to $\pi/2 \text{ rad}$, the pseudo-velocities are calculated as shown in Fig.15, where $L=H=20 \text{ mm}$ and $\theta_0=0.10115 \text{ rad}$. This figure shows pseudo-velocities in two cases of i_1 and i_2 changing in the same and in the opposite direction. In the case of same direction change, the value of V_a becomes large when i_1 or i_2 is within the range of critical angles, but V_a decreases rapidly with deviation from the range. On the other hand, in the case of opposite direction change, V_a fluctuates

Fig.14 Optical paths and flame front

Fig.15 Pseudo-velocity

hard from plus to minus when both angles i_1 and i_2 come within the range, but V_a can be ignored as it becomes less than 10^7 m/s when i_1 and i_2 are out of the range.

Furthermore, Fig.16 shows the value of V_a calculated in the case of the incidence angle of beam 1 (i_1) changing from $-\pi/2 \text{ rad}$ to $\pi/2 \text{ rad}$, but i_2 is fixed at θ_0 . Also, this figure is shown the effect of flame size (L) on the pseudo-velocity of V_a . The value of L is defined in Fig.14.

The effect of L on V_a is expressed by equation (43), when the absolute value of incidence angle, $|i_1|$, changes nearly from 0.25 rad to 1.52 rad .

$$\frac{\log_{10} V_a}{\log_{10} V_a(H)} = 6.22 L^{1.803} \times 10^{-3}, \dots (43)$$

where $V_a(H)$ is the value of V_a in the case of $L = H$ (see Fig.14), unit of V_a being m/s and those of L and H mm .

Fig.16 Pseudo-velocity

4. Conclusions

The pseudo-velocity as detected in the LDV due to refractive index change, was calculated in a simplified system, and the result turned out almost equivalent to the experimental ones.

According to the calculated results in various cases, the pseudo-velocity V_a has large value like a pulse shortly after the propagating flame front comes in contact with laser beams, but afterward, it becomes rapidly so small that it can be almost ignored, and V_a is in proportion to the flame propagating velocity and increases with the flame size. Moreover, when the relative angles between laser beams and flame front fluctuate with high frequency, it is likely that the value of V_a as an error in the velocity measurement cannot be ignored.

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