Curvature Effects on Two-dimensional Turbulent Wall Jets along Concave Surfaces *

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The purpose of this paper is to study experimentally the curvature effects on the temporal mean and turbulence characteristics of two-dimensional turbulent wall jets along concavely curved surfaces with constant radii of curvature. The surface curvature adopted ranges over $b/R=0\sim 0.1$, where b is the half width of the wall jet and R is the radius of surface curvature. The turbulent energy balance in the wall jets is obtained by measuring advection, production and dissipation of the turbulent energy. It shows that their nondimensional magnitudes of the turbulent energy decrease as the surface curvature increases. The effects of curvature on the time- and space-correlations of fluctuating velocities are also considered.

Key Words: Turbulence, Boundary Layer, Wall Jet,
Concave Surface, Curvature, Energy Balance

1. Introduction

The turbulent wall jet is applied to boundary-layer controls, thermal controls of solid surfaces and many other engineering purposes. The flow regime of the wall jet can be divided into two regions; the one is the region between the solid surface and the location of the maximum velocity, behaving like a boundary layer (called wall region), and the other is the rest of the wall jet, behaving like a free jet (called jet region). It is, however, clear that the structure of turbulence in the wall jet is different from those of the boundary layers or of the free jets, especially in a region between the wall region and the jet region, where they are not simply connected but influence each other.

The first theoretical study was made by Glauert⁽¹⁾ for a two-dimensional turbulent wall jet along a plane surface using an eddy-viscosity hypothesis, and the predicted temporal mean characteristics (velocity profile, jet width etc.) are in good agreement with the experiment⁽²⁾⁻⁽⁴⁾. Further studies have been carried out experimentally about the mean flow and turbulence characteristics of the wall jet along plane surfaces⁽⁵⁾⁻⁽¹²⁾.

In many cases, the wall jet is applied to curved surfaces. It is, there-

fore, important to study the effects of surface curvature not only on the mean flow characteristics but also on the structure of turbulence in the wall jet. Recently, the mean flow and turbulence characteristics of wall jets along convex surfaces have been measured in cases of constant radii⁽¹¹⁾⁽¹³⁾ and logarithmic spirals⁽¹⁴⁾⁻⁽¹⁶⁾. For concave surfaces, on the other hand, since the measurements of the mean flow characteristics by Guitton⁽¹⁷⁾, the turbulence characteristics were measured only by Spettel et al.⁽¹⁸⁾. A general feature for the curvature effects on turbulent wall jets along concave surfaces is not yet obtained. Previously, the authors measured in detail the mean flow and turbulence characteristics of the plane wall jet⁽¹²⁾ and discussed its structure of turbulence compared with two cases of free jet and boundary layer.

The purpose of this paper is to study experimentally the curvature effects on the mean flow and turbulence characteristics of two-dimensional turbulent wall jets along concave surfaces of constant radii. The radii of curvature are varied in series in a moderate range of curvatures $b/R=0\sim 0.1$.

Nomenclature

b : half width of wall jet, where $U=U_m/2$ in jet region
 c_f : local skin friction coefficient = $\tau_w / (1/2)\rho U_m^2$
 d : height of jet at nozzle exit
 l : length of leading flat plate
 R : radius of curvature of test plate
 Re : Reynolds number = $U_m b / \nu$
 Re_m : Reynolds number = $U_m y_m / \nu$
 Re_o : Reynolds number = $U_o d / \nu$
 R_x, R_y, R_z : correlation coefficients
 U, V : temporal mean velocity components in x, y directions, respectively

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U_0 : velocity at exit of contraction nozzle
 U_m : maximum velocity in wall jet
 U_τ : friction velocity at solid surface
 u', v', w' : fluctuating velocity components in x, y, z directions
 x, y, z : coordinate system (Fig. 1)
 y_m : location of maximum velocity
 y_0 : location where Reynolds stress becomes zero
 $\Lambda_x, \Lambda_y, \Lambda_z$: integral scales of turbulence
 λ_x : dissipation scale of turbulence
 ν : kinetic viscosity of fluid
 ρ : density of fluid
 τ_0 : shear stress at wall

2. Experimental Apparatus and Procedures

A schematic diagram of the test section is shown in Fig. 1. A contraction nozzle A (15mm x 588mm at the exit) of a wind tunnel is tangentially joined to an aluminum test plate B having a rounded leading edge. The test plate has dimensions of 588mm wide, 2800mm long and 4mm thick. Two end plates with sufficiently large sizes are attached to both ends of the test plate. Experiments were made for four test concave plates of constant radii of $R = 5m, 2.5m, 1.5m, 1.0m$ and also for a plane surface in downstream direction (x direction). As shown in Fig. 1 and Table 1, all the test concave plates have leading plane surfaces with length l in order to obtain fully developed plane wall jets at the beginning of the concave surfaces. Temporal mean flow velocities and fluctuating velocities were measured using hot-wire anemometers with a tungsten wire of $5\mu m$ diameter. A distance from the wall can be read to an accuracy of 0.01mm near the wall by a traversing mechanism with a dial gauge. The velocity

Table 1 Test plates

Test plate No.	Radius R m	Leading length		Curvature b/R
		l m	l/d	
1	Plane	---	---	0
2	5	0.56	70	0.012 ~ 0.021
3	2.5	0.80	100	0.030 ~ 0.042
4	1.5	0.80	100	0.050 ~ 0.064
5	1.0	0.80	100	0.074 ~ 0.095

Fig. 1 Schematic diagram of test section
A: contraction exit of wind tunnel, B: test plate, $d=8mm$

U_0 at the nozzle exit was set at 32m/s, 40m/s and 48m/s, which correspond to the Reynolds number Re_0 (based on the velocity U_0 and the height d of the jet at the nozzle exit) of $(1.7 \sim 2.6) \times 10^4$. The turbulence intensity in x direction was 0.9 to 1.1 percent at the nozzle exit. Two-dimensionality of the flow field was checked by measuring the streamwise velocity profile in z direction. The velocity profile at $x=50d$ was uniform within 1.5 percent of the average value in a range of 400mm in z direction. In the concave section, the two-dimensionality of the wall jet was rather improved. The shear stress at the wall was measured by Stanton tubes of razor-blade type, as illustrated in Fig. 2. The Stanton tubes were calibrated using the calibration equipment⁽¹²⁾ with a straight circular pipe of 70mm inside diameter.

3. Experimental Results and Discussions

3.1 Mean flow characteristics

Figure 3 shows temporal mean velocity profiles of the turbulent wall jets along the concave surfaces of $R=1.0m$ measured at $x=50d$ in the leading plane section and at various measuring stations 30d (shown as c 30d in Fig.3), 55d, 80d and 105d downstream from the inlet of the concave surface. They reduce to a single curve when the velocity U and the distance y measured from the surface are expressed in dimensionless forms using the local maximum velocity U_m and the half width b , respectively. The half width b is defined as the y value of $U=U_m/2$ in the jet region. The results of all other cases tested are given by the same curve, and are shown by a solid line which agrees with the measurement by Wilson and Goldstein⁽¹¹⁾ for a plane wall jet. It is noted from Fig.3 that the velocity profile of the plane wall jet is fully developed already at $x=50d$ and that the velocity profile is not affected by the surface curvature in the

Fig. 2 Stanton tube

Fig. 3 Mean velocity profiles
(c 30d means the measuring station downstream 30d from the inlet of the concave section.)

curvature range measured. The velocity profile near the wall is given in the expression of the law of the wall in Fig.8.

Figures 4 and 5 show streamwise variations of the half width b/d and the location y_m/d of the maximum velocity U_m , respectively. The gradients db/dx and dy_m/dx , obtained from these figures, are given in Table 2.

For the plane wall jet, the present result of b can be formulated as follows:

$$b/d = 0.066(x + x_1)/d \quad (1)$$

where the virtual origin is $x_1 = 10d$. The gradient $db/dx = 0.066$ and the virtual origin $x_1 = 10d$ agree with average values⁽⁹⁾ of previous experiments: $db/dx = 0.068$ and $x_1 = 10d$.

In the case of concave wall jet, the values of db/dx and dy_m/dx decrease as the radius R of curvature is reduced, but should be noticed that the ratio y_m/b remains unchanged irrespective of the curvature, as shown in Fig.5, and agrees with the value 0.16 for the plane case.

Figure 6 shows a decay of the local maximum velocity U_m in x direction, where the ordinate of this figure is a squared reciprocal of U_m/U_0 . It is evident that the decay rate of the maximum velocity U_m downstream decreases as the surface curvature is increased. These relations are formulated as follows:

Fig.4 Streamwise variations of half width b of wall jet

Fig.5 Streamwise variations of location y_m of maximum velocity U_m

Fig.6 Streamwise variations of maximum velocity U_m

$$(U_0/U_m)^2 = a(x + x_2)/d \quad (2)$$

where the value of a is given in Table 2. The virtual origin was $x_2 = -8d$ for the plane wall jet.

Figure 7 gives the local skin friction coefficient $c_f (=2\tau_0/\rho U_m^2)$, obtained from the wall shear stress τ_0 measured by using the Stanton tube, in relation to the Reynolds number $Re_m (=U_m y_m/\nu)$. Logarithmic scales are taken for the abscissa and the ordinate of this figure. No difference is found between the concave and the plane cases. These measured results can be formulated as Eq.(3):

$$c_f = 0.117 Re_m^{-0.31} \quad (3)$$

Figure 8 shows the mean velocity profiles in the wall region for various radii R of curvature, rendered dimensionless in terms of the friction velocity $U_\tau (= \tau_0/\rho)$, which was obtained from the present measurement of τ_0 . All the results of the law of the wall are in agreement in both cases of the concave and the plane wall jets, which are expressed by the following logarithmic law:

$$U/U_\tau = 5.0 \log_{10}(U_\tau y/\nu) + 4.9 \quad (4)$$

Table 2 Mean flow characteristics

Test plate No.	Radius R m	$\frac{db}{dx}$	$\frac{dy_m}{dx}$	$\frac{y_m}{b}$	a
1	Plane	0.066	0.0106	0.16	0.067
2	5	0.052	0.0083	0.16	0.064
3	2.5	0.041	0.0066	0.16	0.049
4	1.5	0.035	0.0056	0.16	0.043
5	1.0	0.029	0.0046	0.16	0.041

Fig.7 Local skin friction coefficient c_f in relation to Reynolds number Re_m

Fig.8 Mean velocity profiles in wall region
a: viscous sublayer, b: authors; Eq. (4), c: Myers et al.⁽⁷⁾; plane wall jet, d: Tailland-Mathieu⁽⁸⁾; plane wall jet, e: Spettel et al.⁽¹⁸⁾; concave wall jet ($R=1.2m$)

3.2 Turbulence characteristics

Figures 9 and 10 show the distributions of the three components ($\sqrt{u'^2}/U_m$, $\sqrt{v'^2}/U_m$, $\sqrt{w'^2}/U_m$) of the turbulence intensities and the Reynolds stress ($\overline{u'v'}/U_m^2$) across the concave wall jet for $R=1.0m$, where $Re=Umb/\nu$. The measurement by Wilson and Goldstein⁽¹¹⁾ for the plane wall jet is shown with a broken line for comparison. It is evident from these figures that the magnitudes of the turbulence intensities and the Reynolds stress for the concave case are reduced rapidly from the plane case and then decrease slowly downstream, although their main features of the distributions are unchanged. The results of Figs. 9 and 10 are qualitatively in agreement with the measurement for a concave wall jet in a case of $R=1.2m$ by Spettel et al⁽¹⁸⁾.

Figures 11 and 12 show the turbulence intensities and the Reynolds stress for various dimensionless curvatures b/R as a

(a) $\sqrt{u'^2}/U_m$

(b) $\sqrt{v'^2}/U_m$

(c) $\sqrt{w'^2}/U_m$

Fig.9 Distributions of turbulence intensities
(a) $\sqrt{u'^2}/U_m$, (b) $\sqrt{v'^2}/U_m$, (c) $\sqrt{w'^2}/U_m$

Fig.10 Distributions of Reynolds stresses

parameter. It can be said from these results that the magnitudes of the turbulence intensities and the Reynolds stress decrease monotonically as the curvature b/R is increased.

As is well known as one of the significant features for turbulent plane wall jets, the location (y_0/b) where the Reynolds stress becomes zero does not agree with the location (y_m/b) of the maximum velocity, which is in contrast to other shear flows such as pipe flows and free jets, where y_0 is equal to y_m . Figure 13 shows that the ratio y_0/y_m appears to increase and the location of y_0 approaches y_m when the curvature b/R is increased beyond 0.07. Figure 14 shows the maximum value of the Reynolds stress ($\overline{u'v'}/U_m^2$)_{max} in relation to the curvature. The relation is expressed by the following formula for the curvature range of $b/R=0\sim0.1$:

(a) $\sqrt{u'^2}/U_m$

(b) $\sqrt{v'^2}/U_m$

(c) $\sqrt{w'^2}/U_m$

Fig.11 Distributions of turbulence intensities for various curvature b/R
(a) $\sqrt{u'^2}/U_m$, (b) $\sqrt{v'^2}/U_m$, (c) $\sqrt{w'^2}/U_m$

Fig.12 Distributions of Reynolds stresses for various curvatures b/R

$$(\overline{u'v'})_{\max}/U_m^2 = 0.012(1 - 5.8b/R) \quad (5)$$

3.3 Turbulent energy balance

The equation governing the balance of turbulent energy in the wall jet along a curved surface can be written as follows:

$$\begin{aligned} & \frac{bU}{2U_m^3} \frac{\partial \overline{q^2}}{\partial x} + \left(1 - \frac{y}{R}\right) \frac{bV}{2U_m^3} \frac{\partial \overline{q^2}}{\partial y} \\ & + \frac{(\overline{u'^2} - \overline{v'^2})b}{U_m^3} \frac{\partial U}{\partial x} + \left\{ \left(1 - \frac{y}{R}\right) \frac{\partial U}{\partial y} + \frac{U}{R} \right\} \\ & \times \frac{\overline{u'v'}b}{U_m^3} + \left(1 - \frac{y}{R}\right) \frac{b}{U_m^3} \frac{\partial}{\partial y} \left\{ \nu \left(\frac{q^2}{2} + \frac{p'}{\rho} \right) \right\} \\ & + \frac{b}{U_m^3} \epsilon = 0 \quad (6) \end{aligned}$$

where $\overline{q^2} = \overline{u'^2} + \overline{v'^2} + \overline{w'^2}$, p' is the fluctuating pressure and ϵ is the dissipation of the turbulent kinetic energy. The first and the second terms of Eq.(6) represent advection, the third and the fourth terms production, the fifth term diffusion and the sixth (last) term dissipation. The dissipation of the turbulent kinetic energy ϵ was estimated from the following Eq.(7), assuming Taylor's hypothesis and local isotropy of turbulence:

$$\epsilon = 15 \frac{\nu}{U^2} \left(\frac{\partial u'}{\partial t} \right)^2 \quad (7)$$

Figure 15 shows the distributions of the advection, the production, the diffusion and the dissipation in Eq.(6) for $R=1.0m$, which are compared with the plane case. The distributions of the advection, the production and the dissipation were obtained from direct measurements, but because it is difficult to measure the fluctuating pressure p' directly, the diffusion term was obtained as the residue of Eq.(6). In this case, the

Fig.13 Relative position y_0/y_m of zero Reynolds stress in relation to curvature b/R

Fig.14 Maximum Reynolds stress $(\overline{u'v'})_{\max}/U_m^2$ in relation to curvature b/R
 • Authors, ○ Tailland-Mathieu⁽⁸⁾,
 ■ Wilson-Goldstein⁽¹¹⁾, ○ Spettel et al⁽¹⁸⁾.

dissipation term was corrected by a constant ratio so that the integral of the diffusion term across the wall jet becomes zero. It is noticed in Fig.15 that all the terms of the turbulent energy for the concave wall jet are smaller than those of the plane wall jet. More detailed examination indicated that the location where the advection becomes zero in the jet region of the concave wall jet moves outward compared with the plane case. The minimum value of the production is found at around the location $y_m/b (=0.16)$ of the maximum velocity. For the concave case, the minimum value is smaller and its location moves outward as compared with the plane case. This result coincides with the deviation of the location y_0 where the Reynolds stress becomes zero. The magnitude of the dissipation is decreased with its main feature unchanged. Figure 16 shows the turbulent energy balance in the wall region, where the scale of the abscissa is magnified. In the wall region, the magnitudes of the advection and the dissipation are smaller than in the plane case. The production term was not affected by the curvature, because the wall shear stress τ_w (Fig.7) and the velocity profile near the wall (Fig.8) for the concave case are not changed by the curvature.

The result also shows that the turbulent energy in the jet region which is gained mainly by the production (the fourth term of Eq.(6)) and the advection (the first term), is partly dissipated there, but partly diffused either to the wall region or to the outer area of the jet region. In the wall region ($y/b < 0.16$), on the other hand, the production around the location y_m of the maximum velocity U_m is much smaller than the dissipation. A part of the turbulent energy is gained from the upstream by the advection (the first term) and the other part from the jet region by the diffusion.

Fig.15 Turbulent energy balance for concave wall jet
 [Gain in Eq.(6) is shown here as positive.]

3.4 Velocity-correlation

coefficients and scales of turbulence
Figure 17 shows a typical result of the auto-correlation coefficient R_x ($=u'(0)u'(t)/\sqrt{u'(0)^2}\sqrt{u'(t)^2}$) at the fixed location ($y_1/b=0.16$) of the maximum velocity for $R=1.0m$, where t is the delay time. As seen from Fig.17, the auto-correlation coefficient R_x for the concave wall jet is almost in agreement with the plane case. Figure 18 shows the measured cross-correlation coefficient R_y ($=u'(y_1)u'(y)/\sqrt{u'(y_1)^2}\sqrt{u'(y)^2}$) between two locations of y_1 (fixed at $y_1/b=0.16$) and y (variable). The cross-correlation coefficient R_y for the concave wall jet is smaller than the plane case. Figure 19 shows the cross-correlation coefficient R_z ($=u'(0)u'(z)/\sqrt{u'(0)^2}\sqrt{u'(z)^2}$) between two locations of $z=0$ (fixed) and z (variable) at $y_1/b=0.16$. The cross-correlation coefficient R_z is larger than the plane case. Similar results for the correlation coefficients R_x , R_y and R_z were obtained for other fixed locations of y_1 .

Figure 20 represents the distributions of the following four integral scales of turbulence $\Lambda_x, \Lambda_{y_i}, \Lambda_{y_o}$ and Λ_z :

$$\left. \begin{aligned} \Lambda_x &= U \int_0^\infty R_x dt, \quad \Lambda_{y_i} = \int_0^{y_1} R_y dy \\ \Lambda_{y_o} &= \int_{y_1}^\infty R_y dy, \quad \Lambda_z = \frac{1}{2} \int_{-\infty}^\infty R_z dz \end{aligned} \right\} \quad \dots\dots(8)$$

which can be obtained from the integration of the measured correlation coefficients R_x, R_y, R_z at various fixed locations of y_1 . Λ_x is the integral scale of turbulence in x direction, Λ_{y_i} in y direction from y_1 toward the solid surface, Λ_{y_o} in y direction

Fig.16 Turbulent energy balance in wall region
[Gain in Eq.(6) is shown here as positive.]

Fig.17 Auto-correlation coefficient R_x

from y_1 toward the outside of the wall jet and Λ_z in z direction. The region of the negative correlations is not involved in the integration. In this figure, the concave case of $R=1.0m$ whose data are independent of x is compared with the plane case. As shown in Fig.20, the integral scales Λ_{y_i}/b and Λ_{y_o}/b decrease and Λ_z/b increase (except near the surface), as the surface curves become concave. The integral scale Λ_x/b is larger than $\Lambda_{y_i}/b, \Lambda_{y_o}/b$ and Λ_z/b , and agrees with the plane case.

The dissipation scale of turbulence λ_x defined by Eq.(9) is also shown in Fig.20.

$$\lambda_x = \sqrt{2}U\sqrt{u'^2} / \sqrt{\left(\frac{\partial u'}{\partial t}\right)^2} \quad \dots\dots\dots(9)$$

The distribution of $\lambda_x(y_1)/b$ across the plane wall jet becomes approximately similar in shape downstream in a region $x>150d$. The fact is not affected by the curvature. The dissipation scale λ_x/b is approximately constant in the jet region.

Fig.18 Cross-correlation coefficient R_y

Fig.19 Cross-correlation coefficient R_z

Fig.20 Scales of turbulence for wall jets
(plane: $x=150d$, concave: c 80d ($x=180d$))

4. Conclusions

The effects of surface curvature on the mean flow and turbulence characteristics of wall jets along concave surfaces with constant radii were examined by experiment. The results are summarized as follows:

(1) The velocity profile $U/U_m = f(y/b)$ and the local skin friction coefficient c_f in relation to the Reynolds number Re_m are in agreement with the case of plane wall jet in the curvature range $b/R = 0 \sim 0.1$.

(2) The turbulence intensities and the Reynolds stress for the turbulent wall jets along the concave surfaces decrease as the curvature b/R increases. The maximum values of the Reynolds stress $(\overline{u'v'}/U_m^2)_{max}$ in relation to the curvature b/R are given as follows in the curvature range $b/R = 0 \sim 0.1$:

$$(\overline{u'v'}/U_m^2)_{max} = 0.012(1 - 5.8b/R)$$

(3) The turbulent energy balance for the concave wall jet is considered. The magnitudes of the advection, the production and the dissipation of turbulent energy are small, compared with the case of plane wall jet.

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