A Study on Angular Stiffness and Damping Properties of Externally Pressurized Gas Thrust Bearing with Surface-restriction Compensation*

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An externally pressurized gas-lubricated thrust bearing with surface-restriction compensation is analyzed theoretically for an angular displacement scheme applying an equivalent clearance model, which considers an equivalent recessed thrust bearing neglecting the local flow components in the bearing clearance, yielding angular stiffness and damping coefficient. The experimental results coincide well with the theoretical ones, which may justify the theoretical flow model.

The design criterion of the bearing with surface-restriction compensation is also discussed from the results of theoretical calculations for angular stiffness and damping coefficient.

Key Words : Bearing, Lubrication, Gas Bearing, Hydrostatic Bearing, Surface-restriction Compensation, Angular Displacement Property, Flow Model.

1. Introduction

In the design of externally pressurized gas bearings, a restrictor is necessarily arranged in the gas supply line in order to produce a restoring stiffness of the bearing. Orifice restriction compensation or inherent compensation is commonly used as the restrictor, and step-restriction compensation is also used in some special applications. Surface-restriction compensation analyzed in this paper is a type of this step compensation. Figure 1 illustrates schematically an externally pressurized gas-lubricated thrust bearing with surface-restriction compensation. A number of radial shallow grooves are designed on the bearing surface from the central supply port, the outer edges of which act as restricting steps similarly to a common step compensation bearing.

Fig. 1 Externally pressurized gas thrust bearing with surface-restriction compensation

The performances of this type of externally pressurized gas bearing can be analyzed effectively by applying the so-called equivalent clearance model⁽¹⁾ to the gas flows in the bearing clearance. Bearing performances about parallel mode displacement of a thrust bearing with surface-restriction compensation were analyzed in the previous papers by using this model, obtaining static stiffness⁽¹⁾ and dynamic characteristics⁽²⁾ such as dynamic stiffness and damping coefficient of the bearing. Theoretical results show that good bearing performances can be obtained if we choose a comparatively small clearance and correspondingly shallow and narrow grooves extending close to outer periphery of the bearing. One of the advantages of the surface-restriction compensation mechanism lies thus in a possible reduction of bearing clearance without machining fine supply holes.

Another advantage is that the bearing with surface-restriction compensation is expected to have a larger restoring moment against angular displacement of the rotor. This is due to the fact that the restricting steps or outer ends of the grooves, which play the role of compensation mechanism in the bearing, are located near the outer periphery of the bearing unlike the radial position of supply holes in a common hole-admission type bearing. In this paper, then, the angular displacement properties of an externally pressurized gas-lubricated, circular thrust bearing with surface-restriction compensation are analyzed using an equivalent clearance model.

2. Nomenclature

B, B^*, C_θ : angular damping coefficients
 H, h : bearing clearances

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h_{g0}	: groove depth
h_0	: reference clearance
h_{en}	: equivalent clearance for contained gas volume
h_{eq}	: equivalent clearance for radial gas flow
h_θ	: equivalent clearance for circumferential gas flow
k_θ, M, M^*	: angular stiffnesses
P, p	: pressures
p_a	: ambient pressure
p_s	: supply pressure
R, r	: radial coordinates
r_s	: radius of supply port
r_0	: outer radius of bearing
r_1	: radial position of groove end
t, τ	: times
Γ	: feeding parameter
γ	: angular fraction of groove part
θ	: circumferential coordinate
θ_0	: half center angle of a pair of groove and land
λ	: squeeze number
μ	: viscosity coefficient
X	: amplitude of rolling motion of rotor
ω	: frequency of rolling motion of rotor
Subscripts	
g	: groove
l	: land
I, II	: regions I and II, respectively

3. Theoretical Analysis

Figure 2 shows a schematic diagram of the bearing analyzed in this paper. Some symbols are designated in the figure.

Compressed gas is fed from central supply port under supply pressure p_s , it flows radially through the grooved region (Region I: $r_s \leq r \leq r_1$) and the outer region (Region II: $r_1 \leq r \leq r_0$) and flows out into the atmosphere with pressure p_a . The angular fraction of groove is γ , which is assumed to be constant in the analysis. Manufacture of grooves may be rather easy if the groove width is constant, but, in this paper, the above mentioned type of groove configuration is dealt with because

Fig. 2 Schematic diagram of bearing

analytical formulations are simplified. The latter type can be analyzed similarly by taking into consideration the change of equivalent clearances in radial direction.

Now, consider the flow in the bearing clearance when the rotor surface rolls in an angular displacement scheme with a small amplitude keeping the center clearance h_0 constant. The maximum amplitude of rotor surface displacement at bearing periphery is denoted as Δh , from which circumferential coordinate θ is measured, then the gap heights h_l at land part and h_g at groove part can be expressed, respectively, as

$$h_l = h_0 + (r\Delta h/r_0)\cos\theta \sin\omega t \dots\dots\dots(1)$$

$$h_g = h_l + h_{g0} \dots\dots\dots(2)$$

where r_0 is bearing outer radius, h_{g0} is groove depth and ω is frequency of rotor motion.

We consider the gas flow in Region I. When there are designed numerous radial grooves with sufficiently small width, pressure distribution in Region I can be assumed to be smooth in circumferential direction regardless of groove part or land part neglecting the local flow components in the bearing clearance. This means that we assume the flow to be identical to the one in an equivalent recessed circular thrust bearing. The bearing clearance in the recess part of the latter bearing is named an equivalent clearance. The equivalent clearances for radial flow(1),(2) h_{eq} , for contained gas volume(1),(2) h_{en} , and for circumferential flow h_θ can be given by

$$h_{eq} = \{h_g^3\gamma + h_l^3(1-\gamma)\}^{1/3} \dots\dots\dots(3)$$

$$h_{en} = h_g\gamma + h_l(1-\gamma) \dots\dots\dots(4)$$

$$h_\theta = \{h_g^{-3}\gamma + h_l^{-3}(1-\gamma)\}^{-1/3} \dots\dots\dots(5)$$

respectively, where γ is angular fraction of groove for a pair of groove and land. The derivation of h_θ is described in Appendix I.

Reynolds equation can be derived from continuity condition and isothermal change of gas flow, and it can be expressed as follows in dimensionless form,

$$\begin{aligned} \frac{\partial}{\partial R} (H_{eq}^3 R \frac{\partial P_I^2}{\partial R}) + \frac{1}{R} \frac{\partial}{\partial \theta} (H_\theta^3 \frac{\partial P_I^2}{\partial \theta}) \\ = 2\lambda R \frac{\partial}{\partial \tau} (H_{en} P_I) \end{aligned} \quad \text{:Region I } (R_s \leq R \leq R_1) \dots\dots\dots(6)$$

$$\begin{aligned} \frac{\partial}{\partial R} (H_l^3 R \frac{\partial P_{II}^2}{\partial R}) + \frac{1}{R} \frac{\partial}{\partial \theta} (H_l^3 \frac{\partial P_{II}^2}{\partial \theta}) \\ = 2\lambda R \frac{\partial}{\partial \tau} (H_l P_{II}) \end{aligned} \quad \text{:Region II } (R_1 \leq R \leq 1) \dots\dots\dots(7)$$

in which

$$H = h/h_0, R = r/r_0, \tau = \omega t, P = p/p_a \dots\dots\dots(8)$$

where p_a is ambient pressure, and λ is squeeze number defined by

$$\lambda = (12\mu\omega/p_a) \cdot (r_0/h_0)^2 \dots\dots\dots(9)$$

μ being viscosity coefficient of gas.

Boundary conditions for Eqs. (6) and

(7) are as follows: at the inlet to bearing clearance ($R=R_s$), pressure loss occurs similarly to inherently compensated gas bearings, and it is evaluated by

$$R_s \frac{\partial P_I^2}{\partial R} \Big|_{R=R_s} = -\frac{\Gamma P_s \phi H_{eq}}{H_{eq}^3} \dots\dots\dots (10)$$

where Γ is feeding parameter defined by

$$\Gamma = (24\mu C_D r_s \sqrt{RT}) / (p_s h_0^2) \dots\dots\dots (11)$$

in which

$$\left. \begin{aligned} \phi &= \left(\frac{2\kappa}{\kappa-1}\right)^{1/2} \left(\frac{p_m}{p_s}\right)^{1/\kappa} \\ &\times \left\{ 1 - \left(\frac{p_m}{p_s}\right)^{(\kappa-1)/\kappa} \right\}^{1/2} \\ &\quad ; \frac{p_m}{p_s} \geq \left(\frac{2}{\kappa+1}\right)^{\kappa/(\kappa-1)} \\ \phi &= \left(\frac{2\kappa}{\kappa+1}\right)^{1/2} \left(\frac{2}{\kappa+1}\right)^{1/(\kappa-1)} \\ &\quad ; \frac{p_m}{p_s} < \left(\frac{2}{\kappa+1}\right)^{\kappa/(\kappa-1)} \end{aligned} \right\} \dots\dots (12)$$

and p_m is the pressure just after the supply port, κ is adiabatic index, C_D is discharge coefficient, R is gas constant and T is gas temperature. Their expressions are the same as used in the analysis of common gas bearings with hole-admission.

At the end of grooves ($R=R_1$), continuity conditions for pressure and gas flow rate yield

$$P_I = P_{II} \dots\dots\dots (13)$$

$$H_{eq}^3 \frac{\partial P_I^2}{\partial R} \Big|_{R=R_1} = H_{eq}^3 \frac{\partial P_{II}^2}{\partial R} \Big|_{R=R_1} \dots\dots (14)$$

At the outer periphery of the bearing ($R=1$),

$$P_{II} = 1 \dots\dots\dots (15)$$

Perturbation method with respect to the amplitude of rotor displacement is now applied. We assume the rotor displacement H_λ in the following form considering Eq. (1),

$$H_\lambda = 1 + R\chi \cos \theta \sin \tau \dots\dots\dots (16)$$

where $\chi = \Delta h / h_0$. Pressure distribution and equivalent clearances are also perturbed as

$$P_i = P_{i0} + P_{i1}\chi \cos \theta \sin \tau + P_{i2}\chi \cos \theta \cos \tau \quad (i = I, II) \dots\dots (17)$$

$$\left. \begin{aligned} H_{eq} &= H_{eq0} + H_{eq1}R\chi \cos \theta \sin \tau \\ H_{en} &= H_{en0} + H_{en1}R\chi \cos \theta \sin \tau \\ H_\theta &= H_{\theta0} + H_{\theta1}R\chi \cos \theta \sin \tau \end{aligned} \right\} \dots\dots (18)$$

Substituting these equations into Reynolds equations (6) and (7) and boundary conditions, and rearranging them, we can obtain the fundamental equations and boundary conditions for pressure P_{ij} ($j = 0, 1, 2$). The zeroth order solution P_{i0} is for steady state case ($\chi = 0$). First order solutions, P_{i1} and P_{i2} , give restoring moment and damping coefficient, respectively, for angular displacement scheme of the rotor. Dimensionless expressions of them are

$$\begin{aligned} M &= \frac{k_\theta}{\pi r_0^3 (p_s - p_a)} \\ &= \frac{1}{p_s - 1} \cdot \frac{r_0}{h_0} \left\{ \int_{R_s}^{R_1} P_{I1} R^2 dR + \int_{R_1}^1 P_{II1} R^2 dR \right\} \dots\dots\dots (19) \end{aligned}$$

$$\begin{aligned} B &= \frac{\omega C_\theta}{\pi r_0^3 (p_s - p_a)} \\ &= \frac{1}{p_s - 1} \cdot \frac{r_0}{h_0} \left\{ \int_{R_s}^{R_1} P_{I2} R^2 dR + \int_{R_1}^1 P_{II2} R^2 dR \right\} \dots\dots\dots (20) \end{aligned}$$

where k_θ and C_θ are dimensional restoring moment and dimensional damping coefficient, respectively.

The fundamental differential equations are integrated numerically by a computer except the zeroth order solution P_{i0} , which can be given analytically in a closed form.

4. Experimental Results and Discussions

Figure 3 illustrates a schematic diagram of experimental apparatus for angular displacement properties of a thrust bearing. The tested thrust bearing (1) is fixed on the carrier (2). The block (3), which is equivalent to the rotor, can roll around the stationary shaft (4), which in turn is fixed on the frame (5). The block (3) is supported on the shaft (4) by an externally pressurized gas journal bearing system; diameter of the shaft is 21 mm, radial clearance is 8 μ m, and gas is supplied from inside of the shaft at supply pressure higher than 0.59 MPa {6.0 kgf/cm²} through 6 $\times$ 2 inherently compensating supply holes with diameter of 0.5 mm. Externally pressurized gas thrust bearing (6) supports axially the rolling block (3). The carrier (2) can be moved vertically through the base (8) by a micrometer head (7). The bearing clearance is set by inserting a thickness gage between the tested bearing (1) and the rolling block (3).

Fig. 3 Experimental apparatus

Indicial response in rolling mode of the block to an impulse load was measured by a non-contact type transducer. Dynamic angular stiffness and angular damping coefficient were calculated from frequency and logarithmic damping ratio of the response wave form after a transient state. The displacement at the center of the block, that is, just above the supporting journal bearing part, was measured additionally to confirm that the motion of the block was only rotational (rolling) mode without parallel displacement in vertical direction.

It is noted that, in the analytical model, angular displacement around a center line of just the rotor surface is assumed, while the experimental block rolls around the axis of the shaft (4), which is 16 mm apart from the former analytical model case. The effect of this difference on experimental results, however, can be concluded to be negligibly small by geometrical consideration.

Fig. 4 Experimental results of angular stiffness

Fig. 5 Experimental results of angular damping coefficient

Examples of experimental results are shown in Figs. 4 and 5 together with theoretical ones for the case of $r_0 = 2.0$ cm, $R_s = r_s/r_0 = 0.2$, $R_l = r_l/r_0 = 0.8$, $\gamma = 0.5$, number of grooves = 15 ($\theta_0 = 12^\circ$), $h_{g0} = 19$, 26 and 58 μm , and $p_s - p_a = p_{sg} = 0.059$ MPa $\{0.6 \text{ kgf/cm}^2\}$ ($p_{sg} = p_{sg}/p_a$). Both of experimental results for angular stiffness and damping coefficient coincide well with theoretical ones in the region of comparatively small bearing clearance, thus it may be concluded that the equivalent clearance model is effective for the analysis of bearing performances. Experimental results exceed theoretical ones when the bearing clearance increases. This may be due to inertia effect of the gas flow in the bearing clearance(3). It should be noted, however, that the surface-restriction compensation bearing may be designed to have a rather small bearing clearance so the inertia effect may be small enough in a practical bearing.

In Appendix II, the solution by an equivalent clearance model is compared with that by divergence formulation method(4), which may also verify the former analytical model.

5. Discussions on Theoretical Bearing Characteristics

Some theoretical bearing performances using an equivalent clearance model will be discussed in this section. For an example, we consider a typical thrust bearing with $R_s = 0.2$, $R_l = 0.8$ and $\gamma = 0.2$. A dimensionless parameter with respect to groove depth h_{g0} is introduced as

$$\Gamma_{g0} = (24\mu C_D r_s \sqrt{IRT}) / (p_a h_{g0}^2) \dots\dots (21)$$

Theoretical bearing characteristics for angular displacement mode are shown in Figs. 6 and 7. The angular stiffness and damping coefficient are nondimensionalized in the forms of M^* and B^* , respectively, as

$$M^* = M \cdot \left(\frac{h_0}{r_0}\right) = \frac{k_\theta h_0}{\pi r_0^4 (p_s - p_a)} \dots\dots\dots (22)$$

$$B^* = B \cdot \left(\frac{h_0}{r_0}\right) = \frac{C_\theta \omega h_0}{\pi r_0^4 (p_s - p_a)} \dots\dots\dots (23)$$

These dimensionless forms are introduced by considering similarity to the case of parallel mode displacement, and they are somewhat different from definitions by Eqs. (19) and (20). The abscissa of the figures is Γ defined by Eq. (11), which corresponds to reference clearance h_0 .

The angular stiffness shown in Fig. 6 becomes maximum at a certain value of bearing clearance when groove depth is given. If groove depth is comparatively small, which corresponds to a large Γ_{g0} value, this optimum value of bearing clearance is nearly equal to that for the maximum stiffness in parallel mode displacement. It can be seen in the figure that the optimum groove depth for maximum angular stiffness is several times as large as the bearing clearance. The an-

gular damping coefficient, on the other hand, becomes larger as groove depth decreases, which means that a design with deep grooves should be avoided from stability characteristics of the bearing operation.

It should be noted that the angular stiffness and damping coefficient are normalized in Eqs. (22) and (23) by multiplying with reference clearance h_0 , hence dimensional values of them become inversely large if the clearance h_0 is as small as is the case of surface-restriction compensation bearing.

In order to compare very roughly with performances of a conventional thrust bearing with hole-admission supplies, we consider a circular thrust bearing with inherently compensating supply holes locating just at the same radial position of groove ends in surface-restriction compensation bearing. Theoretical bearing performances of the former bearing analyzed under line-source assumption are indicated by broken curves in Figs. 6 and 7. The feeding parameter is defined by

$$\Gamma = (24\mu C_{Dn} r_s \sqrt{IT}) / (p_a h_0^2) \quad (24)$$

where n is number of supply holes. The definition of Γ is somewhat different from Eq. (11) for surface-restriction compensation bearing, so direct comparison of two bearings is not possible. It can be seen, however, from qualitative comparison that surface-restriction compensation bearing becomes effective if we choose a small bearing clearance and a correspondingly small value of groove depth.

Figures 8 and 9 show theoretical bearing characteristics with angular fraction of groove, γ , as parameter. As can be seen in Fig. 8, for a small bearing clearance case, the maximum angular stiffness increases as γ value decreases, and the optimum Γ value increases at the same time. If, on the other hand, the bearing clearance value is fixed, there exists optimum γ value which gives maximum angular stiffness. The angular damping coefficient increases as γ decreases. This is easily understood from the fact that pneumatic hammer instability largely depends on the contained volume of gas in the bearing clearance.

As to the effect of radial position of groove end R_1 , though diagrams of theoretical results are omitted in this paper, it can be concluded that a large R_1 causes a large angular stiffness. This is due to the fact that the sensitivity of pressure distribution to gap height change in rolling mode is large for a large R_1 value, and also that supplied gas pressure can be

Fig. 6 Theoretical results of angular stiffness

Fig. 7 Theoretical results of angular damping coefficient

Fig. 8 Effect of γ on theoretical angular stiffness

Fig. 9 Effect of γ on theoretical angular damping coefficient

distributed effectively over bearing surface at the same time. The angular damping coefficient decreases with an increase of R_1 .

These results are qualitatively the same as those obtained for the parallel displacement scheme. Hence, design criterion for surface-restriction compensation bearing that we should choose a small bearing clearance and a correspondingly shallow grooves extending close to outer periphery of the bearing, which has been deduced from bearing characteristics for parallel mode displacement, is quite applicable for angular bearing performances.

6. Concluding Remarks

In this paper, an externally pressurized gas thrust bearing with surface-restriction compensation is analyzed for angular bearing performances by applying an equivalent clearance model. Theoretical results coincide well with experimental ones, which may justify the flow model.

Theoretical discussion reveals that angular bearing performances are qualitatively the same as those for parallel mode displacement, leading to a conclusion that for the former scheme we can apply well the design criterion obtained for the latter one.

Appendix I. Equivalent clearance for the flow across grooves

We consider a flow passage as shown in Fig. 10, in which one of parallel walls has grooves with rectangular cross-section. The pressure gradient, hence the fluid

Fig. 10 Flow passage across grooves

Fig. 11 Calculated results of equivalent clearances

flow, exists only in the direction across grooves. Pressure drop for unit pitch b , that is a pair of a groove with width b_g and a land, is denoted as Δp . Pressure drops for groove and land parts are denoted as Δp_1 and Δp_2 , respectively, then we have

$$\Delta p = \Delta p_1 + \Delta p_2 \quad \dots\dots\dots (25)$$

Volume rate of incompressible viscous flow for unit length is

$$Q = \frac{(h_0 + h_{g0})^3}{12\mu} \cdot \frac{\Delta p_1}{b_g} = \frac{h_0^3}{12\mu} \cdot \frac{\Delta p_2}{b - b_g} \quad \dots\dots\dots (26)$$

The equivalent clearance for the flow across groove, h_θ , has the following relation with Q ,

$$Q = \frac{h_\theta^3}{12\mu} \cdot \frac{\Delta p}{b} \quad \dots\dots\dots (27)$$

Eq. (5), or h_θ , can be obtained from Eqs. (25)~(27) and the definition that $\gamma = b_g/b$.

Calculated results for three kinds of equivalent clearances, H_{eq} , H_{en} and H_θ , which are defined by Eqs. (3)~(5), are

Fig. 12 Results by equivalent clearance model and by divergence formulation method (angular stiffness)

Fig. 13 Results by equivalent clearance model and by divergence formulation method (angular damping coefficient)

shown in Fig. 11. H_{eq} increases, or in other words, the resistance of passage to the flow along grooves decreases monotonously with an increase of groove depth H_{g0} , while H_{θ} saturates to an asymptotic value, $(1-\gamma)^{-1/3}$, which means that circumferential flow may not be affected so much by existence of grooves. This fact is an important feature when we are concerned with angular performances of a thrust bearing. In this respect, the nature of surface-restriction compensation bearing may be essentially different from that of a shallow-recessed circular thrust bearing with step-restriction compensation.

It is worth noting that the magnitude of the flow across grooves may also play an essential role in performances of an externally pressurized gas journal bearing with surface-restriction compensation.

Appendix II. An investigation of equivalent clearance model

In order to verify the equivalent clearance model, bearing performances are analyzed by divergence formulation method(4). Fundamental equations by this method are based on the expressions of continuity condition of flow in a discretized area of bearing surface, so the difficulty in numerical calculation due to discontinuity of gap height can be overcome.

Figures 12 and 13 are examples of comparison of the results by equivalent clearance model and by divergence formulation method. In the figures, N denotes number of grooves. Theoretical results by these two methods coincide well with each other, though the equivalent clearance model gives a little smaller values for both of M and B .

From these as well as from the discussions on comparison with experimental results in the body, we can conclude that the equivalent clearance model is a simple but effective method to analyze performances of a surface-restriction compensation bearing.

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