Simulation on Rotational Vibration of Spur Gears*

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This paper introduces a developed simulator on a rotational vibration and a strain at a root of a tooth, on a power transmission spur gear. The simulator solves a differential equation with one degree of freedom in consideration of the behaviour of the stiffness around tooth tip meshing. The stiffness function is discussed and determined with static and dynamic meshing tests under load.

The developed simulator has one more distinctive feature that the errors of the gears are put into the simulator with a newly developed automatic gear accuracy measuring instrument.

The simulated outputs depict precisely the experimental behaviours.

Through the development of this simulator, the the discussion becomes easier on the effectiveness of a spur gear for the decrease of vibration.

Key Words: Spur Gear, Rotational Vibration, Simulator, Overall Errors of Gears, Stiffness of Tooth Tip Meshing

1. Introduction

Many researches on the rotational vibration of spur gears have been reported(1)(2)(3). Utagawa(1) studied theoretically the vibration using the differential equations with one degree of freedom, in which the gear blank is mass and gear tooth is spring, and verified experimentally the theoretical values are reasonable on the behaviour of the strain at the root of teeth. On the power transmitting spur gears, when the tip of driven gear starts to mesh, an interference between the tip and the tooth surface of the driving gear appears with a deflection of a pair of teeth meshing before one pitch (hereinafter called the "interference of the tip"). He applied the interference of the tip in the differential equation as an equivalent error at the tip.

Aida studied theoretically the vibrations with the acceleration and compared them with the experimental results. But he could not show the influence of the interference of the tip on the vibration, because the experimented load was not sufficiently heavy.

On the influence of gear errors on the rotational vibration, Utagawa(4)(5), Kubo(6)(7), Opitz(8) discussed under the assumption that these are typical errors on the gear. So the vibrations have not been studied sufficiently under the condition that there are actually many kinds of errors on the gear.

Taking account of the stiffness of axis, Rettig(9), Kiyono(10) solved the differential equation. But their solutions could not depict sufficiently the actual acceleration.

In this study, a newly developed simulator is discussed. The simulator can depict accurately the rotational acceleration and the strain at the root on the given actual pairs of spur gears under a wide range of transmitting loads and speeds. This has two main characteristics. One is that errors of the given gear pair are measured with the Automatic Gear Accuracy Measuring Instrument (hereinafter called "AGAMI") which has been developed by one of the authors(11)(18), and the measured errors are accounted for through the differential equation. The second one is that the behaviours of the stiffness around the region where the tip of tooth of a driven gear starts and moves on to mesh with the flank of tooth of driven gear, are studied experimentally (hereinafter called "stiffness function"). The defined stiffness function contains the behaviours of stiffness with the interference of the tip.

2. Simulator

2.1 Differential equation

To calculate the gear rotational acceleration and dynamic strain at the tooth root, the simulator solves a differential equation with one degree of freedom (1)(2). In the equation, the system of gear pair is modelled such that gears are connected by a spring representing the stiffness of the meshing teeth. The equation is written as follows:

$$M\ddot{x} + D\dot{x} + K(t,x)x = W + F(t,x)$$

M: equivalent inertia on line of action,
D: damping coefficient,
K(t,x): meshing stiffness of a pair,
W: statical normal force,
F(t,x): total applied tooth force caused

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by gear errors,
t: time, and x: displacement.

This equation is calculated by the Runge-Kutta Gill method using the computer. The number of the calculated points is about 320 on a normal pitch of gear. The relations between the accuracy of the calculated points are studied. When the number of the calculated points is over 200, it has been made sure that the solutions saturate and become stable.

The actual inertia M is calculated on the blank and the teeth. Damping coefficient D is calculated from the relation $D=2\zeta\sqrt{MK(t,x)}$ on the free vibration. Determined as a behaviour of experimental acceleration, the damping factor ζ is 0.07. The experimental condition is from 400rpm to 600rpm in the speed and from 49Nm (5kgfm) to 196Nm (20kgfm) in the torque. The stiffness function and the method how to sample the gear errors are detailed in the following.

The dynamic strain at the root is calculated from the deflection using Aida and Terauchi's equation (12).

2.2 Mesh stiffness of a pair of gears

A number of studies have been performed on the tooth deflection which gives mesh stiffness.

Ishikawa presented an equation for tooth deflection which was based on the analysis of a tooth modelled as three trapezoidal beams (13). In this simulator, Ishikawa's equation (13)(14) is used to obtain the bending deflection, the shear deflection and the stamp effect of base of a tooth. And Weber's equation (15) is used to obtain the Hertzian or contact deflection. The value obtained from Lundberg's equation(16) was too large in this study.

2.3 Stiffness function

Tooth deflection produces naturally the tooth tip meshing on power transmission gear. How this tooth tip meshing appears becomes an important factor for the simulator as the transmitted force increases.

Based on the behaviour of transmission error under static load(17), the discussions are performed on the stiffness function of tooth tip meshing. That is, when a pair of teeth start to get out of the line of contact, the stiffness is zero. And the stiffness increases linearly to the stiffness at normal meshing condition, being the width of Hertzian contact inner from the tip, where the width of complete Hertzian contact is guaranteed.

The following seven kinds of stiffness functions are discussed to see which function can depict the behaviours of transmission error under static load and dynamic meshing test.

G_0 : The stiffness function which has been discussed up to this time, that is, at the start of the interference of the tip, a pair of gears transmit the load with the ideal flanks of pinion and gear.

G_{L0} : At the start of the interference of the tip, the stiffness is zero. And the stiffness increases linearly to the geometrical meshing point where the meshing starts on the line of action.

G_{LH} : Near the tooth tip meshing, the meshing is performed with the flanks, but the width of Hertzian contact a_H is not sufficiently built up by the tip of tooth (as shown in Fig.2). So at the start of interference of the tip, the stiffness is zero. And the stiffness increases linearly to the inner meshing point where the width of Hertzian contact a_H is built up. This stiffness function has the characteristic

Fig.1 Stiffness functions and the behaviours in meshing test under static load

that the start point of the stiffness function near the tip is varied depending on the transmitting load. This stiffness function is recommended in this study.

G_s , G_{SR} , G_c , G_{CR} : Actually the linearity of the stiffness function G_{LH} at the entry region must be discussed elastically, but in this study the behaviours of the stiffness at the entry region are assumed to be squared, square root, cubic and cube root functions instead of the linearity, and discussed by simulation to see which function can depict actually the experimental results.

Fig.1(a) shows the experimental and the theoretical behaviours with each stiffness function on the meshing test under static load. The abscissa is the length of action. The ordinate is the stiffness of one pair and of two pairs. In the figure, L_e means the length of the meshing with a tip and L_{aH} means the width of Hertzian contact from the tip. All values are converted into ones on the line of action. The behaviours of meshing error change according to the stiffness functions. Checking the agreement between the theoretical and experimental results, the meshing error h and the width of the behaviours W , are shown in Fig.1(b). It is observed that the theoretical results for G_{LH} agree well with the experimental results.

2.4 Overall gear errors measured by AGAMI

One of the authors developed AGAMI which provides simultaneous overall gear errors including tooth profile error, pitch error, run out and so on, as the digital output data to be punched on a paper tape. The measured interval P on the line of action is shown by the following formula, where r_g is radius of base circle,

$$P = \frac{r_g \times \pi}{6 \times 180}$$

In this study, P is 0.27mm for the test gear and the number of the measured points on the line of action is 81.

On the other hand, designed profile, estimated pitch error and so on are used in the differential equation.

Fig.2 shows a flow diagram of the simulator. The differential equation is simulated and solved on the digital computer using Runge-Kutta Gill method, with the data given above. The output of this analysis is then digitally sampled and time-domain response data are plotted with the use of the X-Y plotter. Finally gear rotational acceleration and dynamic strain are obtained for a revolution.

3 Dynamic Meshing Test under Loading

3.1 The test gears

For the clarification of the recommended stiffness function and of the relation between error and rotational vibration, a dynamic meshing test under loading is performed on the test gear as shown in Table 1. The profile errors measured are shown in Fig.3. There are about 6 μ m errors at the root of driving gear and the tip of driven gear. One of the overall gear errors measured by AGAMI is shown in Fig.4.

Table 1 Specifications of tested gear

Pressure angle (°)	14.5
Module	4
Number of teeth	48
Addendum modification coefficient	0
Reference diameter (mm)	192
Tip circle diameter (mm)	198.62
Center distance (mm)	192.0
Face width (mm)	10
Material	SCM 3
Finish	MAAG grinding

Fig.2 Simulator for the rotational vibration of spur gears

3.2 Gear test rig

The test gear is driven by a 55kW variable speed motor, and an eddy current type dynamometer (max. load capacity 210Nm (21.4kgfm)) is connected to the driven gear as a loading device (19).

For lubrication the #180 turbine oil is supplied at a rate of 1.3 liter per minute to meshing side.

3.3 The vibration characteristic of the test rig

3.3.1 The stiffness of axis

The stiffness of shaft is designed to be as small as about 1/90, unlike the stiffness of teeth. So the tested gear system is simulated by a differential equation of one freedom.

Fig.3 Profile errors of the tested gears

Fig.4 One of the overall gear errors by AGAMI

3.3.2 The vibration of bearing and housing.

The bearing and housing of the test rig are shown in Fig.5. The following vibration characteristics are measured on the bearing and housing. Loading the shaft by a lever, the characteristic in the vertical direction is measured as spectra of acceleration as shown in Fig.6. In this measurement the teeth are separated to avoid the influence of rotational vibration of gears.

Fig.5 Tested gears, bearing and housing

Fig.6 Vibration characteristic of bearing and housing in the vertical direction

The resonance frequency increases as shown by a dotted line owing to an increase of load on the axes F1, F2. When the load is 0 N, the resonance frequencies on both sides are coincident. But when the load is increased to 2000 N that of driving side is 2400 Hz and that of driven side is 2150 Hz. That is, there is some difference.

When the pair of gears is loaded statically by 196 Nm, the rotational accelerations measured turn out as shown in Fig.7 owing to an increase of the power of exciter. Thereby 2630 Hz is the resonance frequency with the meshing of two pair of teeth. 2180 Hz and 2400 Hz are the resonance frequencies as shown in Fig.6. These measurement data show that the acceleration increases in proportion to the force and is remarkably large at the resonance frequency of housing.

3.4 Measuring

Two accelerometers are set facing each other as shown in Fig.8. The signal of acceleration is recorded through a differential amplifier, which means only rotational acceleration is shown.

The strain at root is measured by the strain gauges located at the center of this facewidth.

4 The Experimental Results and the Simulated Outputs

4.1 The stiffness functions

When the transmitting load is heavy, the difference of the stiffness functions appears remarkably in the behaviour of the rotational acceleration.

The experimental results are measured under the maximum torque 196 Nm in this test rig. Under the same conditions the simulated outputs are calculated with each stiffness function, as shown in Fig.9. When the test gears transmit 196 Nm (20 kgf·m, 208 kgf/cm on the pitch circle), in the simulation the interference of the tip is equivalent to 0.1te on the line of action, and has 32 points where the differential equations are calculated. The width of

Fig.7 Measured rotational accelerations of one pair of tested gears

Hertzian contact is $0.04te$ and 12 points. So this torque is sufficient to discuss the difference of the stiffness functions. In the figure, abscissa means time. And the domain where a pair of teeth mesh geometrically is shown in the figure. Under the condition that test gears have no error, only simulator results cannot faithfully depict actually the experimental results, because each behaviour of the strain at root is spiky at the start of meshing of the tooth.

On the other hand, when the measured errors with AGAMI are counted, the simulated outputs are shown as "AGAMI error" in the figure. The simulated outputs with G_0 and G_{L0} show a steep increase at the one pair area, but this characteristic cannot be found in the experimental results. The simulated output with G_s shows good agreement with the experimental results in figure, but it is too large in the amplitude.

Finally the output with G_{LN} shows good agreement with the experimental result. From the above discussion and Fig.1(b), the stiffness function G_{LN} is recommended in this study.

Still more, it is observed in Fig.9 that the simulated outputs on the strain at root with every stiffness function cannot show the same difference. That is, behaviours of the strain at root are insensible to the vibration of gears unlike acceleration.

4.2 Transmitting torque and behaviours of acceleration and strain at root

When the transmitting torque is changed, the experimental results and the simulated output with the recommended

Fig.8 Schematic diagram of setup for measuring the acceleration

stiffness function G_{LH} and conventional G_o are shown in Fig.10 under the condition that the speed is constantly 1200rpm.

It is observed in the experimental results that a peak of acceleration at the geometrical one pair meshing appears under

heavy torque 196Nm. So the number of peaks changes from three to two in a meshing interval. The simulator outputs depict remarkably well the above characteristic of the experimental results.

Still more it is observed that both simulator outputs with G_{LH} and G_o coincide under light torque, because the area of meshing with tip becomes small relatively.

4.3 Rotational speed and behaviours of acceleration and stress of root

When the rotational speed is changed experimental results and simulated outputs with G_{LH} are shown in Fig.11 under the condition that transmitting torque is constantly 196Nm.

At the low speed 400rpm, the acceleration and stress of root of simulated output depict faithfully the behaviours of the experimental results from one tooth to one revolution of gear.

At the high speed 1700rpm, the simulator output depicts almost perfectly the experimented results. Under more precise observation, every amplitude of acceleration coincides in experimental result, but that of simulator output shows some difference.

This phenomenon comes from the fact that the simulator calculates the errors on only one transverse section of cylinder and does not count the lead error.

On the other hand, the actual experimental results are affected by the profile of lead, and are smoothed in the behaviours of acceleration. The experimental results of strain at root show the plus and minus in this measuring system, but actually they are in the same direction.

The differences in the value and the resonance frequency and tooth separation are discussed with the rotational vibration from 500rpm to 3000rpm in the experiment and simulator. In Fig.12 the lines indicate the experimental results and points do the simulator outputs. The two

Fig.9 Stiffness functions and behaviours of acceleration and strain at root

Fig.10 Transmitting load, behaviours of acceleration and strain at root

Fig.11 Rotational speed, behaviours of acceleration and strain at root

coincide actually all over the speed and torque ranges.

Many high harmonic frequencies (shown as $1/2f_n$, $1/3f_n$) of the meshing resonance frequency f_n appear in the experimental results under the torque 196Nm. It is observed that there are some differences between the measured resonance frequency f_n and the estimated frequency from these harmonics. Since at high speed, tooth separation occurs in meshing, the peak frequency deviates a little from the theoretical value. The same phenomena are observed at the resonance frequency of the housing f_b (2180Hz) and $1/2f_b$ (1090Hz) as shown in Fig.7.

When the effect of the tooth separation at the resonance frequency f_n is avoided and comparison is made with the

Fig.13 Transmitting load and frequency at which tooth separation occurs

Fig.12 Rotational speed and acceleration

second resonance frequency $1/2f_n$, the difference between the two is under 80rpm in speed and 64Hz in frequency.

And under torque 98Nm and 49Nm, it is observed that the acceleration becomes suddenly large at some frequency due to the tooth separation. The experimental and simulator frequencies where the tooth separation occurs, are shown in Fig.13. There is about 300rpm difference between the two values, because the simulator considers only the stiffness of teeth and errors. On the other hand, the experimental results are affected by other influences, for instance, the housing and shafts, and the tooth separation occurs at some lower speed.

5 Conclusions

A simulator which takes account of actual errors of gear, is developed for the rotational vibration and the strain at root in the power transmitting spur gear. And the dynamic meshing tests of the simulator under loading are performed for the discussion of it. The following results have been obtained:

- 1) The developed simulator can depict precisely the phenomena under wide ranges of transmitting loads and speeds.
- 2) With use of the recommended stiffness function G_{LH} , the simulated outputs can be made more precise to depict the experimental behaviours.
- 3) Through the development of this simulator, the discussion is made easier on the profile of a spur gear which decreases the vibration.

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