Both Surface Removal-Both Surface X-Ray Diffraction Method for Residual Stress Measurement in a Hollow Cylinder*

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The authors proposed in this paper a new method of X-ray diffraction to measure three principal residual stresses in a hollow cylinder. With thin layers of equal thickness removed from both surfaces of a hollow cylinder, surface stresses are measured by X-ray on both surfaces, and then the residual stress distributions can be calculated from both surfaces at the same time. As an application of the new method, the authors measured the residual stresses of a heat treated hollow cylinder of STPT 38.

Key Words: Experimental Stress Analysis, Strength, X-ray Diffraction Measurement of Residual Stress, Cylinder

1. Introduction

The theory of residual stress measurement with X-ray diffraction of a cylinder has been proposed, in which thin layers are removed successively from either outer surface or inner one, and then the surface stresses are measured by means of X-ray on the outer surface or the inner one^{(1)~(5)}.

In the same manner as the both surface diffraction method proposed by the authors on a plate⁽⁶⁾, a new method is to be introduced in this paper on a hollow cylinder.

The method is applied to a steel pipe and the residual stress distributions from both surfaces are calculated at the same time.

2. Analytical Formulas

Assumptions are as follows: (1) Residual stresses are symmetric with respect to the axis. (2) Residual stresses are functions of radius only. (3) Analytical formulas are derived from the theory of thick-walled cylinder.

As shown in Fig.1, r_1 , r_2 : radii of the internal and the external surface; r , ρ : arbitrary radii ($r_1 \leq \rho \leq r \leq r_2$); $\sigma_i(r)$, $\sigma_t(r)$, $\sigma_r(r)$: axial, tangential and radial residual stresses at an arbitrary radius r ; After thin layers of equal removal from both surfaces, $\sigma_L(r)$, $\sigma_T(r)$: axial and tangential surface stresses measured by X-ray on an inner surface; $\sigma_L(r_1+r_2-r)$, $\sigma_T(r_1+r_2-r)$: axial and tangential surface stresses measured by X-ray on an outer surface; By thin layer removal from an inner surface; $\sigma'_{i1}(r)$, $\sigma'_{t1}(r)$: axial and tangential stress changes on an inner surface; $\sigma'_{i2}(r)$, $\sigma'_{t2}(r)$: axial and

tangential stress changes on an outer surface; By thin layer removal from an outer surface; $\bar{\sigma}'_{i1}(r)$, $\bar{\sigma}'_{t1}(r)$: axial and tangential stress changes on an inner surface; $\bar{\sigma}'_{i2}(r)$, $\bar{\sigma}'_{t2}(r)$: axial and tangential stress changes on an outer surface.

Fig.1 Coordinates of a hollow cylinder

From Fig.1, we have

$$\sigma_i(r) + \sigma'_{i1}(r) + \bar{\sigma}'_{i1}(r) = \sigma_L(r) \quad (1)$$

$$\sigma_i(r_1+r_2-r) + \sigma'_{i2}(r) + \bar{\sigma}'_{i2}(r) = \sigma_L(r_1+r_2-r) \quad (2)$$

$$\sigma_t(r) + \sigma'_{t1}(r) + \bar{\sigma}'_{t1}(r) = \sigma_T(r) \quad (3)$$

$$\sigma_t(r_1+r_2-r) + \sigma'_{t2}(r) + \bar{\sigma}'_{t2}(r) = \sigma_T(r_1+r_2-r) \quad (4)$$

Where

$$\sigma'_{i1}(r) = \sigma'_{i2}(r) = \frac{2}{(r_1+r_2-r)^2 - r^2} \int_{r_1}^r \rho \sigma_i(\rho) d\rho \quad (5)$$

$$\bar{\sigma}'_{i1}(r) = \bar{\sigma}'_{i2}(r) = \frac{2}{(r_1+r_2-r)^2 - r^2} \times \int_{r_1+r_2-r}^{r_2} \rho \sigma_i(\rho) d\rho \quad (6)$$

$$\sigma'_{i1}(r) = \frac{(r_1+r_2-r)^2 + r^2}{(r_1+r_2-r)^2 - r^2} \sigma_r(r) \quad (7)$$

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$$\sigma'_{12}(r) = \frac{2r^2}{(r_1+r_2-r)^2-r^2} \sigma_r(r) \dots\dots\dots (8)$$

$$\bar{\sigma}'_{11}(r) = \frac{-2(r_1+r_2-r)^2}{(r_1+r_2-r)^2-r^2} \sigma_r(r_1+r_2-r) \dots\dots\dots (9)$$

$$\bar{\sigma}'_{12}(r) = -\frac{(r_1+r_2-r)^2+r^2}{(r_1+r_2-r)^2-r^2} \sigma_r(r_1+r_2-r) \dots\dots\dots (10)$$

$$\sigma_r(r) = \frac{1}{r} \int_{r_1}^r \sigma_i(\rho) d\rho \dots\dots\dots (11)$$

$$\sigma_r(r_1+r_2-r) = \frac{1}{(r_1+r_2-r)} \int_{r_2}^{r_1+r_2-r} \sigma_i(\rho) d\rho \dots\dots\dots (12)$$

From eqs.(1), (2) and (5), $\sigma_L(r)$ and $\sigma_L(r_1+r_2-r)$ are obtained as follows:

$$\sigma_i(r) + \frac{2}{(r_1+r_2)(r_1+r_2-2r)} \left\{ \int_{r_1}^r \rho \sigma_i(\rho) d\rho + \int_{r_1+r_2-r}^{r_2} \rho \sigma_i(\rho) d\rho \right\} = \sigma_L(r) \dots\dots\dots (13)$$

$$\sigma_i(r_1+r_2-r) + \frac{2}{(r_1+r_2)(r_1+r_2-2r)} \left\{ \int_{r_1}^r \rho \sigma_i(\rho) d\rho + \int_{r_1+r_2-r}^{r_2} \rho \sigma_i(\rho) d\rho \right\} = \sigma_L(r_1+r_2-r) \dots\dots\dots (14)$$

The integral equations (13) and (14) can be solved as follow:

$$\begin{aligned} \sigma_i(r) = & \sigma_L(r) - \frac{(r_1+r_2-2r)}{r} \left[(r_1+r_2) \int_{r_1}^r \frac{\sigma_L(\rho)}{(r_1+r_2-2\rho)^2} d\rho \right. \\ & \left. - \frac{2}{(r_1+r_2)} \int_{r_1}^r \frac{\rho(r_1+r_2-\rho)}{(r_1+r_2-2\rho)^2} [\sigma_L(\rho) - \sigma_L(r_1+r_2-\rho)] d\rho \right] \dots\dots\dots (15) \end{aligned}$$

$$\begin{aligned} \sigma_i(r_1+r_2-r) = & \sigma_L(r_1+r_2-r) - \frac{(r_1+r_2-2r)}{(r_1+r_2-r)} \left[(r_1+r_2) \int_{r_1}^r \frac{\sigma_L(r_1+r_2-\rho)}{(r_1+r_2-2\rho)^2} d\rho \right. \\ & \left. + \frac{2}{(r_1+r_2)} \int_{r_1}^r \frac{\rho(r_1+r_2-\rho)}{(r_1+r_2-2\rho)^2} [\sigma_L(r_1+r_2-\rho) - \sigma_L(\rho)] d\rho \right] \dots\dots\dots (16) \end{aligned}$$

From eqs.(13) and (14), we have

$$\sigma_i(r) - \sigma_i(r_1+r_2-r) = \sigma_L(r) - \sigma_L(r_1+r_2-r) \dots\dots\dots (17)$$

From eqs.(16) and (17), we have

$$\begin{aligned} \sigma_i(r) = & \sigma_L(r) - \frac{(r_1+r_2-2r)}{(r_1+r_2-r)} \left[(r_1+r_2) \int_{r_1}^r \frac{\sigma_L(r_1+r_2-\rho)}{(r_1+r_2-2\rho)^2} d\rho \right. \\ & \left. + \frac{2}{(r_1+r_2)} \int_{r_1}^r \frac{\rho(r_1+r_2-\rho)}{(r_1+r_2-2\rho)^2} [\sigma_L(r_1+r_2-\rho) - \sigma_L(\rho)] d\rho \right] \dots\dots\dots (18) \end{aligned}$$

From eqs.(15) and (17), we have

$$\begin{aligned} \sigma_i(r_1+r_2-r) = & \sigma_L(r_1+r_2-r) - \frac{(r_1+r_2-2r)}{r} \left[(r_1+r_2) \int_{r_1}^r \frac{\sigma_L(\rho)}{(r_1+r_2-2\rho)^2} d\rho \right. \\ & \left. - \frac{2}{(r_1+r_2)} \int_{r_1}^r \frac{\rho(r_1+r_2-\rho)}{(r_1+r_2-2\rho)^2} [\sigma_L(\rho) - \sigma_L(r_1+r_2-\rho)] d\rho \right] \dots\dots\dots (19) \end{aligned}$$

Axial residual stress can be calculated by combining eqs.(15) and (19) or eqs.(16) and (18). From eqs.(3), (4), (7), (8), (9) and (10), $\sigma_T(r)$ and $\sigma_T(r_1+r_2-r)$ are obtained:

$$\sigma_i(r) + \frac{\{(r_1+r_2-r)^2+r^2\} \sigma_r(r) - 2(r_1+r_2-r)^2 \sigma_r(r_1+r_2-r)}{(r_1+r_2)(r_1+r_2-2r)} = \sigma_T(r) \dots\dots\dots (20)$$

$$\sigma_i(r_1+r_2-r) - \frac{\{(r_1+r_2-r)^2+r^2\} \sigma_r(r_1+r_2-r) - 2r^2 \sigma_r(r)}{(r_1+r_2)(r_1+r_2-2r)} = \sigma_T(r_1+r_2-r) \dots\dots\dots (21)$$

The integral equations (20) and (21) can be solved.

$$\begin{aligned} \sigma_i(r) = & \sigma_T(r) + \int_{r_1}^r \frac{1}{\rho} \sigma_T(\rho) d\rho + \frac{1}{(r_1+r_2)(r_1+r_2-r)^2} \left[\int_{r_1}^r \frac{(r_1+r_2-\rho)^2 \{(r_1+r_2-\rho)^2 + \rho^2\}}{(r_1+r_2-2\rho)^2} \sigma_T(\rho) d\rho \right. \\ & + 2 \int_{r_1}^r \frac{\rho(r_1+r_2-\rho)^3}{(r_1+r_2-2\rho)^2} \sigma_T(r_1+r_2-\rho) d\rho \left. - \frac{1}{(r_1+r_2)} \left[\int_{r_1}^r \frac{(r_1+r_1-\rho) \{(r_1+r_2-\rho)^2 + \rho^2\}}{\rho(r_1+r_2-2\rho)^2} \sigma_T(\rho) d\rho \right. \right. \\ & \left. \left. + 2 \int_{r_1}^r \frac{(r_1+r_2-\rho)^2}{(r_1+r_2-2\rho)^2} \sigma_T(r_1+r_2-\rho) d\rho \right] \right] \dots\dots\dots (22) \end{aligned}$$

$$\begin{aligned} \sigma_i(r_1+r_2-r) = & \sigma_T(r_1+r_2-r) - \int_{r_1}^r \frac{1}{(r_1+r_2-\rho)} \sigma_T(r_1+r_2-\rho) d\rho \\ & - \frac{1}{r^2(r_1+r_2)} \left[\int_{r_1}^r \frac{\rho^2 \{(r_1+r_2-\rho)^2 + \rho^2\}}{(r_1+r_2-\rho)^2} \sigma_T(r_1+r_2-\rho) d\rho + 2 \int_{r_1}^r \frac{\rho^3 (r_1+r_2-\rho)}{(r_1+r_2-2\rho)^2} \sigma_T(\rho) d\rho \right] \\ & + \frac{1}{(r_1+r_2)} \left[\int_{r_1}^r \frac{\rho \{(r_1+r_2-\rho)^2 + \rho^2\}}{(r_1+r_2-\rho)(r_1+r_2-2\rho)^2} \sigma_T(r_1+r_2-\rho) d\rho + 2 \int_{r_1}^r \frac{\rho^2}{(r_1+r_2-\rho)^2} \sigma_T(\rho) d\rho \right] \dots\dots\dots (23) \end{aligned}$$

From eqs.(11),(12),(22) and (23), $\sigma_r(r)$ and $\sigma_r(r_1+r_2-r)$ are obtained.

$$\begin{aligned} \sigma_r(r) = & \int_{r_1}^r \frac{1}{\rho} \sigma_T(\rho) d\rho + \frac{1}{r(r_1+r_2)(r_1+r_2-r)} \left[\int_{r_1}^r \frac{(r_1+r_2-\rho)^2((r_1+r_2-\rho)^2+\rho^2)}{(r_1+r_2-2\rho)^2} \sigma_T(\rho) d\rho \right. \\ & + 2 \int_{r_1}^r \frac{\rho(r_1+r_2-\rho)^3}{(r_1+r_2-2\rho)^2} \sigma_T(r_1+r_2-\rho) d\rho \left. \right] - \frac{1}{(r_1+r_2)} \left[\int_{r_1}^r \frac{(r_1+r_2-\rho)((r_1+r_2-\rho)^2+\rho^2)}{\rho(r_1+r_2-2\rho)^2} \sigma_T(\rho) d\rho \right. \\ & \left. + 2 \frac{(r_1+r_2-\rho)^2}{(r_1+r_2-2\rho)^2} \sigma_T(r_1+r_2-\rho) d\rho \right] \dots\dots\dots (24) \end{aligned}$$

$$\begin{aligned} \sigma_r(r_1+r_2-r) = & - \int_{r_1}^r \frac{1}{(r_1+r_2-\rho)} \sigma_T(r_1+r_2-\rho) d\rho - \frac{1}{r(r_1+r_2)(r_1+r_2-r)} \\ & \times \left[\int_{r_1}^r \frac{\rho^2((r_1+r_2-\rho)^2+\rho^2)}{(r_1+r_2-2\rho)^2} \sigma_T(r_1+r_2-\rho) d\rho + 2 \int_{r_1}^r \frac{\rho^3(r_1+r_2-\rho)}{(r_1+r_2-2\rho)^2} \sigma_T(\rho) d\rho \right] \\ & + \frac{1}{(r_1+r_2)} \left[\int_{r_1}^r \frac{\rho((r_1+r_2-\rho)^2+\rho^2)}{(r_1+r_2-\rho)(r_1+r_2-2\rho)^2} \sigma_T(r_1+r_2-\rho) d\rho + 2 \int_{r_1}^r \frac{\rho^2}{(r_1+r_2-2\rho)^2} \sigma_T(\rho) d\rho \right] \\ & \dots\dots\dots (25) \end{aligned}$$

3. Example of Application

As an application of the method, the authors measured residual stresses in a headed steel pipe STPT 38.

The condition of heat treatment and size of test piece are shown in Table 1 and Fig.2. The removal of thin layers from the inner surface and the outer surface of a hollow cylinder is done by electropolishing. The experimental apparatus is shown in Fig. 3 and condition of electropolishing is shown in Table 2. The removal of the equal thickness from both surfaces was confirmed experimentally by keeping the current density on each surface at the respective values determined from the calibration curves shown in Fig. 4.

The X-ray measurement is done under the condition shown in Table 3. The inner surface stress is measured through a slit as shown in Fig.2. The slit is cut in axial or tangential direction with an end-mill after the heat treatment of test pipe.

The measured stresses are shown in Fig.5 and the residual stress distributions calculated from the formulas of the new method are shown in Fig.6. The measured stress values at the point A in Fig.2 are shown by white marks in Fig.5.

Fig.2 Test piece

Table 1 Condition of heat treatment

Heating temperature	850°C
Holding time	45 min
Coolant & Temperature	Water, 19°C

Table 2 Condition of electropolishing

Chemical components %	HCl 1 H ₂ O 99
Temperature °C	20 - 22
Current density	As in Fig.4

Table 3 Condition of X-ray stress measurement

Characteristic X-ray	Cr-Kα
Tube voltage KV	30
Tube current mA	10
Time constant s	16
Slit divergent angle	1.02°
Irradiated area mm ²	4 x 2
Determination of peak point	Half width method

Fig.3 Electropolishing apparatus

Fig.4 Amount of electropolishing

a) Axial-direction

b) Tangential-direction

Fig.5 Measured stresses by X-ray

a) Axial-direction

b) Tangential and radial direction

Fig.6 Residual stress distributions

4. Conclusions

- (1) The authors proposed a new method of both surface removal and both surface X-ray diffraction to measure the three principal residual stresses in a hollow cylinder.
- (2) The advantage of this method is that the residual stress distributions from both surfaces can be calculated at the same time in a hollow cylinder.
- (3) The present method is applied to measurement of the residual stresses in the hardened steel pipe STPT 38.

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