Flow characteristics of a Jet Adjacent to a Plane Wall*

(2nd Report, Jet Impinging on Inclined Adjacent Wall)

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Flow characteristics of a two-dimensional air jet issuing from a nozzle of 5mm in height and impinging against an inclined wall, which has the vertical distance L between its leading edge and the jet axis plane and an angle θ of 15° – 45° against the plane, have been experimentally and analytically investigated.

The maximum velocity profile in the flow direction was predicted semi-theoretically in the upper stream area and analyzed in the wall jet area by the same numerical method as taken in our previous report.

The momentum flux decreases along the wall owing to the impingement and so on, the coefficient of the momentum decrease η_m at the cross section of $s/t=140$ (s : distance along the main stream line, t : nozzle height) is in the range of 0.85 to 0.55.

1. Introduction

Flow characteristics of an impinging jet in which the jet axis is parallel to an adjacent wall have been presented in the 1st report⁽¹⁾. In the present report, the flow characteristics of an impinging jet whose axis is inclined against an adjacent wall are presented.

The influences of the inclined angles on the velocity and the static pressure distribution of the jets are investigated.

A number of studies on a jet impinging normally on a plate have been carried out^{(2), (3), (4)} experimentally and theoretically. However, regarding an impinging jet with an inclined angle, only three investigations have been reported. Schach⁽⁵⁾ studied a stream line and the difference between the geometrical impinging point and the real stagnation point on the assumption of a potential flow. Schauer⁽⁶⁾ and Kamoi et al.⁽⁷⁾ made an evaluation of shear stress mainly in the impinging region.

However, the characteristics of the main stream line and pressure rise in the vicinity of the impinging point and the flow characteristics in the downstream region have not been clarified not only theoretically but also experimentally.

Therefore, in this report the curvature of the locus of the maximum velocities (the main stream line), the dis-

tributions of maximum velocities along the jet axis and the variation of the boundary layer are investigated on the basis of the measurements of the velocity and the pressure distribution in impinging jets with various inclined angles. And also, empirical generalized expressions for the velocity and pressure distribution and the variation of the thickness of the boundary layers are obtained, and a method to predict the characteristic values of the wall jet is proposed.

2. Experimental Apparatus and Procedure

Fig. 1 shows a schematic diagram of the experimental apparatus. A two-dimensional air jet is discharged horizontally in the still air from a nozzle of the pressure chamber (1) which is 5mm in height, and it impinges on the inclined smooth flat wall (3) which has the angle θ against the jet axis.

The width of the nozzle is 300mm and two end plates (2) are installed on both sides of the nozzle in order to get a two dimensional jet. There are no cover plates between the end plates except the adjacent wall. The velocity distribution in the jet boundary at sections perpendicular to the wall are measured by traversing an I-type hot wire, an L-type hot wire, and a pitot tube of 3mm diameter in the y -direction.

Fig.1 Experimental apparatus

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The pressure distributions in the jet boundary are measured with a disc-type static tube, and the pressure on the wall is measured with static pressure taps of a diameter 0.8mm. Total pressure tube of 2mm outer diameter with a hole of diameter 0.4mm is used to determine the flow direction by rotating the tube around the axis in the x-y plane. The minimum distance between the tube axis and the wall surface in the measurements is 6mm.

The experiment was carried out under the following conditions:

The distance between the jet axis and the leading edge of the wall (point A in Fig. 2), L , was in the range of 20 to 40mm ($L/t=4-8$), and the Reynolds number at the leading edge of the wall ($=V_0 L/\nu$) was in the range of 3.4×10^4 to 5.3×10^4 (V_0 : the maximum velocity at the section of the leading edge of the wall, ν : kinematic viscosity). The angle between the inclined wall and the jet axis θ ranged from 15° to 90° .

By preliminary experiments the wall was set for each run with a different angle of θ as follows. The leading edge of the wall was placed at the point A (Fig. 2) which was a point of outer edge of a free jet which is determined by the following formula of the velocity profile, and the length of l_a was $5L$.

$$V/V_m = \{1 - (\eta/\eta_{max})^{1.5}\}^2$$

The length $5L$ was selected so as to get a fully developed turbulent flow before the impinging of a jet on the leading edge of the wall.

Furthermore, since the lower space between the exit plane of nozzle and the leading edge of the wall was not restricted, the jet flowed straight up to the leading edge of the wall without bending the jet axis by the Coanda effect.

3. Flow Pattern

The flow characteristics of a turbulent jet impinging against the inclined wall are very complicated, because in the jet boundary the distribution of velocities and pressures significantly changes before and after impingement. Therefore, it is very difficult to discuss the characteristics of the whole

region as a unified form. Then, in the present report, the flow characteristics are analyzed by dividing the region into three areas as follows (see Fig. 2):

(1) Free jet area: from the nozzle to the section (AEB) at the leading edge of the wall

(2) Upper stream area: from the section AEB to the section A'E'B'. This area is such divided into US-1 region and US-2 region as follows (see Fig. 2):

US-1: from the section AEB to DFC at the downstream end (pressure reaches P_{wm} at DFC)

US-2: from the section DFC to A'E'B' at the downstream end (pressure reaches approximately atmospheric pressure at A'E'B')

(3) Wall jet area: downstream of the section A'E'B' in which the flow is parallel to the wall.

In the free jet area the jet flows straight and its behavior is the same as that of the ordinary free jet.

In the US-1 region the kinetic energy is at first partially converted into pressure energy owing to the impingement against the wall and the velocity decreases, and then in the US-2 region the pressure energy is converted into kinetic energy again. As the result the main stream line is curved like the curve EFE'F' as shown in Fig. 2.

The main stream line EFE'F' is expressed approximately by a circular arc of radius r , whose centre is located on an extension of the line A'E'B' and the circular arc is tangent to the jet axis OJE at the point E. The magnitude of dimensionless radius r/L obtained by this manner is plotted against θ in Fig. 3.

Since the inner jet layer between the main stream line and the wall surface is in the transient region from the diffusion layer of the free jet to the boundary layer in the vicinity of the wall, and the pressure rises in the jet layer owing to a stagnation of the flow, the velocity distribution in this layer is very complicated.

The thickness of the inner jet layer, δ_m , is equal to $L/\cos\theta$ at the leading edge of the wall ($x=0$), and the thickness decreases remarkably along x after the impingement, and reaches the minimum value, δ_{m0} , at the downstream

Fig. 2 Flow pattern

Fig. 3 Dimensionless radius r/L for main stream line and location of the maximum pressure on the wall

end of the upper stream area x_0 .

The thickness of the outer jet layer, δ , between the main stream line and the outer edge of the jet increases approximately in proportion to the distance x .

4. Velocity Distribution

4.1 Variation of velocity along the main stream line

The variation of the maximum velocity V_m with the distance s from the nozzle along the main stream line is shown in a dimensionless form in Figs. 4(a), (b) and (c), where V_j is the discharge velocity at the nozzle exit.

As shown in this figure, the distance from the nozzle to the point, O_j' , that is, the length of the potential core is nearly equal to $6.5t$ in this experiment.

The rate of maximum velocity decay is equal to that of the free jet, that is, the slope of the decay curve is $-1/2$ between the point O_j' and the leading edge of the wall, E_1 .

The rate of velocity decay increases owing to impingement from the point E_1 to the section F_1 where the pressure on the wall reaches the maximum value, P_{wm} .

Next, the pressure energy is converted into kinetic energy again and the velocity increases from the section F_1 to the section E_1' where the velocity is u_{m0} . Furthermore, the flow downstream of the section E_1' becomes a wall jet and the rate of velocity decay is similar to that of an ordinary wall jet in the previous report⁽¹⁾.

The relation between the dimensionless velocity $U (=V_m/V_c)$ and the dimensionless distance $X (=x/L)$ is shown in Fig. 5. The values of U at the same values of X are constant independently of L in the present experimental conditions. Namely, the distribution of U is independent of L , and the similarity of the velocity characteristics is held in the present flow configurations.

4.2 Velocity Profile in y Direction

The velocity profile of the inner jet layer in the wall jet area, $X > X_0$, is approximately expressed by a formula,⁽⁸⁾ $u/u_m = (y/\delta_m)^{1/5}$ as shown in Fig. 6. But, since the inner jet layer in the upper stream area, $X < X_0$, is in the transient region from the free diffusion layer to the boundary layer of the wall jet, the velocity profile in this layer has non-similarity. Therefore, in this report a discussion about this region is omitted.

The experimental results of velocity profiles of the outer jet layer in each region are shown in Fig. 7 and they are expressed by the following empirical formulas which coincide with those in free jet and the ordinary wall jet respectively:

Fig. 5 Comparison of experiment with calculated result

Fig. 6 Dimensionless velocity profile in the inner jet layer

Fig. 7 Dimensionless velocity profile in the outer jet layer

The upper stream area: $V/V_m = (1 - ((y - \delta_m)/\delta) 1.5)^2$, and the wall jet area: $u/u_m = (1 - ((y - \delta_m)/\delta) 1.5)^2$, where u is the velocity in the x direction and the width of the outer layer δ is set at $2.27\delta_{0.5}$ where $\delta_{0.5}$ is the width at $u=0.5u_m$ (or $V=0.5V_m$) which is obtained by the experiment.

4.3 Empirical expressions for V_c , V_{ms} , u_{mo}

4.3.1 Maximum velocity V_c at the point E

Assuming the jet as a straight free jet, one can express the decay of the velocity along O_jE by Eq.(1). Then, the measured values of V_c are plotted against the values obtained from Eq.(1) in Fig.8 (a).

The agreement is good, therefore V_c can be expressed by Eq.(1).

$$\frac{V_c}{V_j} = \sqrt{\frac{\text{length of potential core}}{O_jE}} = \sqrt{\frac{6.5t}{L(5-\tan\theta)}} \quad \dots(1)$$

4.3.2 Maximum velocity V_{ms} at the end of the US-1 region

The maximum velocity V_{ms} at a cross section where the pressure on the wall has the maximum value is important for discussing the velocity distribution characteristics in this jet. Therefore the experimental values are expressed in a generalized form as follows.

The maximum velocity of the ordinary free jet at the cross section of the distance $(O_jE + ED)$ from the nozzle is given by

$$V_j \sqrt{\frac{6.5t}{O_jE + ED}} = V_j \sqrt{\frac{6.5t}{L(5-\tan\theta + \frac{1}{\cos\theta \cos\varphi})}}$$

where $\varphi = \angle DEA = \tan^{-1}(\frac{x_s}{L} \cos\theta)$.

Then the value of V_{ms} in this jet is compared with the above value. As the result, the ratio of these values, ξ , is obtained to as 0.835, and V_{ms} is expressed in a dimensionless form by Eq.(2)

$$\frac{V_{ms}}{V_j} = 0.835 \sqrt{\frac{6.5t}{L(5-\tan\theta + \frac{1}{\cos\theta \cos\varphi})}} \quad \dots(2)$$

Fig.8 (a),(b),(c) Maximum velocities in the upper stream area

Comparison of the experimental values and Eq.(2) is made in Fig.8(b). From the figure it is apparent that the velocity can be predicted approximately from Eq.(2).

4.3.3 Maximum velocities at the section of the downstream end of US-2 region, u_{mo}

The maximum velocity at the cross section of the downstream end of the upper stream area, u_{mo} , is compared with the empirical formula Eq.(3) as shown in Fig.8(c). The form of the formula is postulated in the following manner.

As the pressure energy is converted into kinetic energy before the flow reaches this cross section, the dynamic pressure $(1/2)\rho u_{mo}^2$ may be proportional to the maximum pressure, P_{wm} , thus the relation can be expressed by $((1/2)\rho u_{mo}^2)/P_{wm} = K_p$. The measured values of K_p are shown in Fig.9. From this figure it can be seen that the ratio K_p depends on only θ and does not depend on L . Moreover, P_{wm} is given by Eq.(6), then the value of u_{mo} can be expressed in a dimensionless form by Eq.(3).

$$\frac{u_{mo}}{V_j} = \sqrt{K_p \sin\theta} \cdot \sqrt{\frac{6.5t}{L(5+1/\tan\theta)}} \quad \dots(3)$$

Comparison of this empirical formula with the experimental result shows good agreement as shown in Fig.8(c).

4.4 Velocities along the main stream line in the upper stream area

The relation between V_{ms} and V_c and that between u_{mo} and V_c can be expressed by the following equations respectively.

$$\frac{V_{ms}}{V_c} = \frac{Eq.(2)}{Eq.(1)} = f_1(\xi, \theta, \varphi), \quad \frac{u_{mo}}{V_c} = \frac{Eq.(3)}{Eq.(1)} = f_2(K_p, \theta)$$

These two dimensionless velocities are also independent of L , because ξ and K_p are independent of L as shown above, therefore the velocity profile has a similarity in the case of the same value of θ . Furthermore, the velocity distribution on the main stream line in US-1 and US-2 regions can be expressed by straight lines as shown in Figs.4(a), (b) and (c), then the linear relations can be expressed by Eqs.(4) and (5) respectively.

$$\frac{V_m}{V_j} = \frac{V_c}{V_j} \left(\frac{s}{L(5-\tan\theta)} \right)^m, \quad L(5-\tan\theta) \leq s < s_s, (m < 0) \quad \dots(4)$$

$$\frac{V_m}{V_j} = \frac{V_{ms}}{V_j} \left(\frac{s}{s_s} \right)^n, \quad s_s \leq s \leq s_o, (n > 0) \quad \dots(5)$$

Fig.9 K_p, m, n , $(1/2)C_{fo}$, γ_m and δ_{mo}/L

where s_s and s_0 are the distances along the main stream line from the nozzle to cross sections at V_{ms} and u_{mo} respectively. The values of m and n can be determined from the slopes of the lines in Fig. 4 in US-1 and US-2 regions respectively. The values determined were $-1.36, -1.45, -1.53$ and $0.04, 0.241, 0.496$ in the case of $\theta = 15^\circ, 30^\circ$ and 45° as shown in Fig. 9. The calculated values of Eqs. (4) and (5) are shown by solid lines in Figs. 4 (a), (b) and (c) and by curves U in Fig. 5. The agreement between the calculated values and the experiments is good.

5. Pressure Distribution

An example of the measured static pressure profiles at the cross sections perpendicular to the wall is shown in Fig. 10. As shown in this figure, the pressure in the inner jet layer at the leading edge cross section AEB is higher than the atmospheric pressure due to the existence of the wall. Owing to the impingement of jet in US-1 region, the pressure in the boundary layer increases with the distance x and reaches the maximum pressure $P_{max} (=P_{wm})$ at the cross section DFC at a distance $AD=x_s$ from the leading edge of the wall. In this cross section the velocity on the main stream line takes the minimum value, V_{ms} , as shown in Fig. 5, where the ratio of the conversion of kinetic energy to pressure energy becomes maximum.

Next, an empirical expression of the maximum pressure on the wall P_{wm} will be described. As shown in Fig. 2 the momentum flux in the axial direction (the direction of O_jEP) is given by $J = 2\rho \int_0^{\eta_{max}} v^2 d\eta$, on the assumption that the jet flows

Fig. 10 An example of static pressure distributions ($\theta = 15^\circ, L/t=4$)

Fig. 11 Half width of pressure profile

Fig. 12 The maximum pressures on the wall

straight along the axis O_jEP similarly to the ordinary two-dimensional free jet. The component J perpendicular to the wall is $J \cdot \sin \theta (=0.6312 \cdot (1/5) \cdot \rho V_{mp}^2 (5+1/\tan \theta) \cdot L \cdot \sin \theta)$, where ρ is the density, V_{mp} is a hypothetical maximum velocity at the point P in the ordinary free jet and $\eta_{max} = (1/5) \cdot (5+1/\tan \theta) \cdot L$ in the present case that the jet spread angle is $\tan^{-1}(1/5)$. On the other hand, as to the pressure force, an integration of P_w in the upstream and downstream area of the reference point D is made as follows

$$F_p = \int_0^{x_{pu}} P_w dx' + \int_{x_s}^{x_{pd}} P_w dx' = 0.3156 P_{wm} m p L$$

where x' is the distance measured upstreamwise from the point D, x_{pu} and x_{pd} are the widths of the pressure distribution in the upstream and downstream sides of the point D respectively, and m_p is the whole width in a dimensionless form $(= (x_{pu} + x_{pd})/L)$. The location of D (values of x_s) is correlated by the relation between $x_s/(L/\sin \theta)$ and θ as shown in Fig. 3.

The comparison of the pressure force, F_p , with the momentum, $J \sin \theta$, lead to an empirical relation, that is, $P_{wm} \propto (1/2) \cdot K_m \cdot \rho \cdot V_{mp}^2 \sin \theta$. Here, the proportional constant $K_m = (4/5) \cdot (5+1/\tan \theta)/m_p$ calculated from the experimental values is approximately equal to unity as shown in Fig. 11. Consequently, the dimensionless pressure P_{wm} based on the pressure p_j in the pressure chamber can be expressed by the following empirical formula (6), and this agrees with the experimental values as shown in Fig. 12.

$$\frac{P_{wm}}{P_j} = \frac{1}{2} \frac{\rho}{P_j} \left\{ V_j \sqrt{\frac{6.5t}{L(5+1/\tan \theta)}} \right\}^2 \sin \theta \dots (6)$$

Next, the pressure distribution on the wall which is shown by a double-dotted line in Fig. 2 will be discussed as follows.

The pressure distributions in the regions US-1 and US-2 which are expressed in a nondimensional form are shown in Fig. 13 (a) and Fig. 13(b) respectively. As can be seen from the figures, the experimental points fall on a curve independently of L and θ .

Therefore, it is shown that the profile of the pressure distributions has a similarity and the distributions in both regions are expressed by the following equations (3).

Fig. 13 (a), (b) Pressure profiles on the wall

$$\frac{p_w}{p_{wm}} = \left\{1 - \left(\frac{x}{x_{pu}}\right)^{1.5}\right\}^4, \quad \frac{p_w}{p_{wm}} = \left\{1 - \left\{\frac{(x-x_s)}{x_{pd}}\right\}^{1.5}\right\}^4$$

Here, $x_{pd}0.5$ is defined as a location, where the value of p_w/p_{wm} is equal to 0.5, from the measured pressure distribution in the region US-2. The location of the downstream end of US-2 (point A' in Fig. 2) is defined from the relation of $x_{pd}/L = 3.41(x_{pd}0.5/L)$. Also, in the region US-1, the values of $x_{pu}0.5$ and x_{pu} are defined in a similar manner.

The calculated results of $X_0 = x_0/L = (x_s + x_{pd})/L$ (the non-dimensional length between A and A' in Fig. 2) are shown in Table 1.

6. Thickness of Jet Boundary Layer

6.1 The Thickness of the inner Jet layer

In this section, the variation of the thickness of the inner jet layer δ_m along x will be described. The thickness decreases with x from the value of $\delta_m = AE = L/\cos \theta$ at the leading edge of the wall to the minimum value of δ_m at the cross section x_0 .

As described in Section 6.3 of the previous report (1) on the locus of the main stream line, the wall shear stress τ_{wo} is equal to the turbulent shear stress τ_t in the outer diffusion layer at the cross section where δ_m is minimum.

In the present experiment, τ_{wo} is given by $\tau_{wo} = (1/2) \cdot C_{fo} \cdot u_{mo}^2$ on the assumption described in Section 7.1, where, $(1/2)C_{fo} = 0.0225(\nu/u_{mo} \delta_{mo})^{1/8}$ and τ_t is given by $\rho \kappa u_{mo}^2$ (1), and κ is a proportional constant for the turbulent shear stress. Then, when the two shear stresses are equal to each other, the relation $(1/2) \cdot C_{fo} = 0.0225(\nu/u_{mo} \delta_{mo})^{1/8} = \kappa$ should hold.

The calculated values of δ_{mo}/L and $(1/2) \cdot C_{fo}$ obtained by substituting the value of κ into the above relation are shown in Fig. 9. Thus the main stream line can be drawn as a circular arc of the dimensionless radius r/L passing through the points E' and E at the leading edge of the wall. This arc is represented by the curve S_m in Fig. 5 and it agrees approximately with the experimental result.

In addition, in the wall jet area, the experimental value S_m increases with the distance like the wall jet described in the previous report (1) and the calculated result described in section 7.2 shown by the solid line in Fig. 5 agrees with the experimental result.

6.2 Calculation of the outer jet layer

The momentum flux at X_0 in each experimental condition is assumed to be expressed by an equation $J_0 = \eta_m J_j$. Here, J_j is the momentum flux at the nozzle exit. The values of the coefficient of momentum decrease, η_m , which were obtained from the experimental values of velocity profiles are shown in

Fig. 9 and Table 1. As can be seen from them, η_m depends on only θ and not on L . On the other hand, J_0 is expressed by $J_0 = \rho \int_0^b u^2 dy$ ($J_j = \rho V_j^2 b$; the whole width of jet $b = \delta_m + \delta$).

Then substitution of the velocity profile in the inner jet layer $u = u_{mo}(y/\delta_{mo})^{1/5}$ (see Fig. 6) and the one in the outer jet layer $u/u_{mo} = (1 - \{(y - \delta_{mo})/\delta_o\}^{1.5})^2$ into the expression of J_0 yields the following relation

$$\eta_m = \frac{t}{L} = 0.8824 \left(\frac{u_{mo}}{V_j}\right)^2 \frac{\delta_{mo}}{L} + 0.3156 \left(\frac{u_{mo}}{V_j}\right)^2 \frac{\delta_o}{L} \dots \quad (7)$$

By substituting δ_{mo}/L shown in Fig. 9 (see Section 6.1 and 7.1) and u_{mo}/V_j obtained from Eq. (3) into Eq. (7) one can obtain the value of δ_o/L . In addition, δ_c at the leading edge of the wall is given as $\delta_c = BE = bE \sin(\angle BBE)/\sin(\angle bBE)$ according to the geometrical relation obtained from the lower figure in Fig. 2.

The value of δ/L in the region between δ_c/L and δ_o/L is expressed approximately by an equation $\delta/L = ((\delta_o - \delta_c)/x_0) \cdot x/L + \delta_c/L$ which is obtained by interpolation. Thickness of the outer jet layer obtained in this manner is shown by the curve BCB' in Fig. 2 and this agrees with the experimental result as shown in Fig. 5.

Here, the experimental value of the outer jet layer thickness, δ , is set at $2.27 \delta_{0.5}$, and $\delta_{0.5}$ is defined as half the jet width from the velocity profile of V (which is denoted as u in the wall jet area and as V in the upper stream area). Thus each dimensionless layer width, the velocity, and the coefficient at X_0 are shown in Table 1.

7. Calculated Results of Flow Characteristics in Wall Jet Area and Comparison with Experiments

7.1 Assumption and basic equations

As shown in Fig. 5, similarity holds in each characteristics in the dimensionless width of the inner and outer jet layer, S_m and S , and the dimensionless velocity U is independent of L and depends on only θ . These values along the axis were calculated numerically by using values at X_0 given in Table 1 as the initial conditions in the wall jet area in the manner described in the previous report (1).

Here, the wall shear stress is expressed by the equation $\tau_w = C_0 \rho u_m^2 (\nu/u_m \delta_m)^k$, where C_0 is a constant

Table 1 Characteristic values and the coefficient at section X_0

θ°	15	30	45
$X_0 = X_0/L$	6.02	4.40	3.52
$U = u_{mo}/V_c$	0.741	0.827	0.881
$S_m = \delta_{mo}/L$	0.225	0.136	0.097
$S = \delta_o/L$	2.97	1.98	1.38
η_m	0.85	0.75	0.65

value of 0.0225 which is deduced from Blasius' relation in a uniform flow, and u_m is the velocity u at the location of δ_m . The value of index k can be estimated at $1/8$ by substitution of $n=1/15$, which was obtained by the present experiment, into the equation⁽⁹⁾ in a uniform flow, that is, $k=2n/(n+1)$.

The momentum balance equations on control volumes E'B'C'F' and A'B'C'D', and the energy balance equation on E'B'C'F', which are basic equations for the present numerical calculation, are the same as Eqs.(8),(9) and (10) in the previous report (1) respectively.

7.2 Calculated results and discussions

The initial condition for the numerical calculation is determined in the following manner: The rate of velocity decay in the wall jet area is affected significantly by the value of the empirical constant κ . Therefore, in order to determine the value of κ , the calculation was repeated using the tentative initial values instead of the values of δ_{m0}/L and δ_0/L which are shown in Table 1 till the velocity gradient by the calculation agrees with that in experiment.

By substituting the value of κ determined above into the relation $\kappa = 0.0225 (\nu/(u_m \delta_m))^{1/8}$ we obtain the values of δ_{m0}/L and then δ_0/L from Eq.(7). In this way, κ and the initial values of δ_{m0}/L and δ_0/L have been decided as shown in Table 1. The values of κ are 0.0076, 0.0078, 0.0080 for the case of $\theta = 15^\circ, 30^\circ, 45^\circ$ respectively at Reynolds number 3.4×10^4 . As shown in Fig.5, the calculated values of U, S_m and S agree well with the experimental ones in the wall jet area.

Then, the value of κ in the present experiment represents the strength of the diffusion into the surrounding still air and the value in this study comes between 0.0053 in the wall jet and 0.016 in the free jet. Thus the value is considered reasonable for an impinging jet.

In the case of the same angle θ , the experimental points of U, S_m and S for different values of L fall approximately on a single curve (see Fig.5), so that the influence of Reynolds number ($Re = V_c L / \nu$) on the characteristics of the present jet is considered very small.

Furthermore, the influence of Reynolds number on the distributions of U, S_m and S was examined by the numerical calculations in the range of $Re: 3.4 \times 10^4 - 5.3 \times 10^4$ and the results showed that the influence was of order of a few percent.

8. Distribution of Momentum and Energy Flux

An example of the variation of the dimensionless momentum flux J/J_j of the adjacent wall jet ($L/t=4, L=20\text{mm}, \theta = 15^\circ - 45^\circ$) along the flow path is shown in Fig.14. The one-, two-, and three-

dotted chain lines in this figure represent the calculated results and the keys of small circles identify the experiments. In addition, J represents the momentum flux which is obtained from the experiment as

$$J = \rho \int_0^{\delta_m + \delta} u^2 dy + \int_0^{\delta_m + \delta} p dy \text{ and suffix } j \text{ represents the value at the nozzle exit.}$$

Experimental value of J at the leading edge of the wall agrees approximately with the value $J = J_j \cos \theta$ which Chigier et al.⁽¹⁰⁾ obtained for a jet impinging against the inclined wall with angle θ . It can be seen from the figure that the decrease of the momentum flux in the region between the nozzle exit and the leading edge of the wall is heavy and is affected greatly by the angle θ .

Also, in the region between the leading edge of the wall and the cross section at X_0 (the downstream end of the upper stream area) the momentum flux decreases by about 10% from the value at the leading edge for all cases of θ . In the area, kinetic energy is converted partially into pressure energy and then pressure energy is converted partially into kinetic energy, consequently the momentum flux decreases.

Furthermore, the rate of decay of J in the wall jet area in each case of θ is approximately the same. As shown in this figure the calculated values agree well with the experimental ones.

The decay of the momentum flux in the present adjacent wall jet is larger than that in the ordinary wall jet as shown by the solid line which was calculated from the relation

$$\tau_w = 0.0225 \rho u_m^2 (\nu/(u_m \delta_m))^{1/4}$$

presented by Myers and others' work⁽¹¹⁾. The reason can be explained by the fact that the coefficient of skin friction $\tau_w/(1/2 \rho u_m^2)$ in the present experiment is about 3 times as large as the theoretical one in the ordinary wall jet.

Examples of the variation of the normalized energy flux E/E_j along the main stream line in the adjacent wall jet ($L/t=4, L=20\text{mm}, \theta = 15^\circ - 45^\circ$) are shown in Fig.14. The one-, two-, and three-dotted chain line and the keys of small circles represent the calculated and experimental values respectively. These experimental values of E were obtained using the measured values of u and p by the integration

Fig.14 Distribution of momentum and energy

$$E = (1/2) \cdot \rho \int_0^{\delta_m + \delta_s} u^2 dy + \int_0^{\delta_m + \delta_s} p |u| dy \quad (\text{in the wall jet area}).$$

As shown in this figure the normalized energy E/E_j at the leading edge of the wall is about 0.5 and amounts to 0.3 at the end of the upper stream area in each case of θ . That is, approximately 20% of the energy is lost in the region between the leading edge of the wall and the end of the upper stream area. In the downstream area from X_0 to $s/t=50$ the larger θ is, the larger the loss of energy is, and in the downstream area which belongs to the wall jet area the rate of decay of energy is approximately constant independently of θ .

By comparison of the energy distribution in the present adjacent wall jet with that in the free jet shown in Fig. 14, the energy distribution of the adjacent wall jet at $\theta=15^\circ$ is found approximately equal to that in the free jet.⁽¹²⁾ The reason lies in the fact that an increase of the energy loss by the skin friction and a decrease of strength of diffusion due to the presence of the wall in the adjacent wall jet result in the same order of magnitude in the energy loss in the free jet. Furthermore, the reason why the energy flux of the ordinary wall jet based on Myers and others⁽¹¹⁾ theoretical work is larger than those in the adjacent wall jet and the free jet is due to the inhibition effect of presence of a wall on the diffusion of the jet in the region between the exit of the nozzle and the wall.

Next, the experimental momentum flux and energy flux of the adjacent wall jet ($L/t=4$, $\theta=0^\circ-45^\circ$) at $s/t=140$ are shown in Fig.15 to compare the characteristics of the present jet with those of the wall jet and the free jet. The momentum flux at this section, decreases 90% to 55% with an increasing θ , and the energy flux decreases 20% to 12% with θ .

9. Conclusions

The flow characteristics of a two-dimensional adjacent jet against the inclined wall such as distributions of velocities, pressures, momentum fluxes and energy fluxes have been studied experimentally.

Since generalized empirical formulas for each parameter in the flow characteristics are obtained, using these

Fig.15 Conservation of momentum and energy at the section $s/t=140$

formulas the flow characteristics in adjacent wall jets with arbitrary geometry of the inclined wall can be estimated in the following manner.

(1) The location of the maximum wall pressure, x_s/L , is determined from Fig.3, and the location of the upper stream area (US-2) is determined as $x_{pd}/L=3.41(x_{pd}/0.5/L)$ from Fig.11.

(2) The velocities at the leading edge of the wall, at the maximum wall pressure, and at the end of the upper stream area, V_c , V_{ms} , u_{m0} are obtained by Eq.(1), Eq.(2) and Eq.(3) and Fig.9 respectively.

(3) The main stream line in the upper stream area EFE' is approximated by a circular arc which passes the points E and E' shown in Fig.2. The distance E'A', that is, the thickness of the inner layer δ_{m0} is determined from Fig.9, and the radius r is determined from Fig.3. The thickness of the outer jet layer δ_c at the leading edge of the wall and that at the end of the upper stream area, δ_0 , are obtained from the geometrical relation in Fig.2 and Eq.(7) respectively.

(4) The velocity distribution along the main stream line in the upper stream area is expressed by Eqs.(4) and (5).

(5) The thicknesses of the inner jet layer and the outer jet layer and the velocity distribution in the wall jet area can be numerically calculated by a method similar to that reported in the previous report (1).

(6) The distributions of momentum fluxes and energy fluxes along the flow axis were measured, and the cause of decay was discussed.

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