An Investigation into Laser Diagnostic

Technique of Liquid Droplets*

(Part 2 : Measurement of Evaporation Rate Constant
In Hot Spray)**
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A measurement method for spray droplet evaporation rate constant was shown. In this study a novel laser diffraction technique was applied to the investigation of spray vaporizations. Measurements were made of local spatial size and distribution in volume element within a quasi two dimensional spray generated from Fan Spray nozzle.

Detected diffracted total light energy and its relative diffracted light energy were defined. To predict the evaporation rate constant, the detected relative light energy was compared with the total light energy in vaporizing spray in hot gas stream.

For ethanol cloud spray, evaporation rate constant was given as $0.04-0.1 \text{ mm}^2/\text{s}$ (373K-573K). It is found that this method is available to estimate the evaporating rate constant of liquid spray

Key Words : Measurement, Laser Diagnostics, Spray Characteristics
Evaporation Coefficient

1. Introduction

Evaporation rate constant of a single liquid droplet and/or droplets has been studied intensively since about 1950, and the evaporation rate constant for various liquids have been made clear from theoretical or experimental studies[1][2][3]. Most of these studies are done with a large single droplet at order $1000 \mu\text{m}$ and a few results are published for the spray droplets generated by the atomizer[10].

In our previous work[4], the local spray structures in a hot gas flow (in hot spray) are measured, and their changes which depend on the local temperature and the spatial position are discussed using Fraunhofer diffraction theory. But, the evaporation rate constant can not be estimated.

The present paper intends to predict the evaporation rate constant of spray droplets. The relative total energy of the diffracted light power from particles is defined by reevaluating the detected signal from the photodetector of the particle analyser, and the change of the weight fraction and the weight of spray droplets which depend on time are discussed. And, a technique is proposed for comparing the detected relative total light energy with the total light energy in vaporizing spray flow which is calculated by the diffraction theory. Droplets vaporization model is produced by applying the single droplet vaporization model(d^2 -Law). In this paper the evaporation rate constant for ethanol

cloud spray is measured by this method and compared with the published results.

2. Nomenclature

C', C''	: constant numbers
$C_e, C_{e,h}$	: evaporation rate constants
C_{eo}	: limit evaporation rate constant
d	: droplet diameter
f	: focal length
J_n	: n-th order Bessel function
L	: diffracted light energy
L_R	: relative total light energy
L_h	: latent heat
N, n	: particle number
N_c	: spatial number density
N_u	: Nusselt number
P_r	: Prandtl number
R	: cumulative weight distribution

$$R = \exp\left\{-\left(\frac{X}{\bar{X}_R}\right)^w\right\}$$

R_e	: Reynolds number
S	: diffracted light energy (arbitrary unit)
s	: radius of detector
T_a	: air temperature
T_g	: ambient gas temperature
T_w	: liquid temperature
t	: time
t_a	: arrival time
V_d	: particle velocity
W	: index of Rosin-Rammler function
w	: weight or relative weight
X	: droplet diameter
$\bar{X}_R$	: characteristic diameter
$\bar{X}_{10}$	$= \left\{ \int_0^\infty X dn / \int_0^\infty dn \right\} \times 10^{-6}$
$\bar{X}_{30}$	$= \left\{ \int_0^\infty X^3 dn / \int_0^\infty dn \right\}^{1/3} \times 10^{-4}$
Z	: distance from nozzle tip
ρ_i	: density
λ	: wave length
λ_g	: gas conductivity

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Subscripts

0 : initial condition
i : classified number
 max : maximum value
 min : minimum value

3. Definition of Total Energy and Relative Total Energy

As shown in previous paper, the angular distribution of the diffraction light energy from particles is measured by a photodetector which consists of 30 concentric annulus. The diffracted light energy within any ring in focal plane, bounded by radii s_1 and s_2 due to droplets, is given:

$$L_{s_1, s_2} = C' \sum_{i=1}^M N_i d_i^2 [(J_0^2 + J_1^2)_{s_1} - (J_0^2 + J_1^2)_{s_2}] \quad \dots\dots\dots (1)$$

and minimum and maximum radii of the detector ring are s_{min} and s_{max} respectively.

$$s_1 = s_{min}, \quad s_2 = s_{max} \quad \dots\dots\dots (2)$$

Eq.(1) thus becomes

$$L_{s_{min}, s_{max}} = C' \sum_{i=1}^M N_i d_i^2 [(J_0^2 + J_1^2)_{s_{min}} - (J_0^2 + J_1^2)_{s_{max}}] \quad \dots\dots\dots (3\cdot a)$$

$$= C'' \sum_{i=1}^M \frac{w_i}{d_i} [(J_0^2 + J_1^2)_{s_{min}} - (J_0^2 + J_1^2)_{s_{max}}] \quad \dots\dots\dots (3\cdot b)$$

$L_{s_{min}, s_{max}}$ is the summation of the output signal from each detector ring. $L_{s_{min}, s_{max}}$ is called "total energy".

If there are particles whose number, diameter and weight are initially N_i , d_i and w_i respectively, the total diffraction energy is given by Eq.(3-b). After Δt time, all particles with diameter d_i change to d'_i and their weight are w'_i . Then, at $t_0 + \Delta t$, using Eq.(3-b), the total energy is:

$$L_{s_{min}, s_{max}}(t_0 + \Delta t) = C'' \sum_{i=1}^M \frac{w'_i}{d'_i} [(J_0^2 + J_1^2)_{s_{min}} - (J_0^2 + J_1^2)_{s_{max}}] \quad \dots\dots\dots (4)$$

where the distribution of w_i and w'_i is defined as relative weight distribution.

The profile of diffracted light energy is detected, and from this, the relative distribution of weight fractions can be estimated, but the absolute value of the weight fraction can not be estimated. L_R (relative total energy) is normalized by the total energy which is detected at time t_0 . Thus, the change in the relative weight distribution and diameter of spray droplets is estimated by L_R . Using Eq.(3-b), the relative total energy is:

$$L_R(\Delta t) = \frac{L_{s_{min}, s_{max}}(t_0 + \Delta t)}{L_{s_{min}, s_{max}}(t_0)} \quad \dots\dots\dots (5)$$

Therefore, the relative total energy L_R can be calculated by Eq.(5), if the change in the relative total energy and diameter of spray droplets is known. And, in the range where the total energy is calculated by Eq.(3-a) or (3-b), if the change of L_R can be measured, then the change in the relative distribution of weight fraction and diameter, which depend

Fig.1 Total diffracted light energy and measured characteristic particle size $\bar{X}_R$

Fig.2 Diffracted light energy profile from particles

on time, can be estimated.

As described above, the change of the spray structures, which depends on time, can be explained by the change of L_R . It is necessary that the total energy has to satisfy Eq.(3-a) or (3-b). Therefore, from Eq.(3-b), the total energy is a function of the relative distribution of weight fraction and diameter. If the spatial number density is not high, the total energy is proportional to the spatial number density and the projected cross section of the particles. Therefore, the range of lower or upper number density where Eq.(3-a) and (3-b) are applicable is measured using the known size standard glass beads of diameter from 177 μm to 255 μm , and the applicable range is verified. The results are shown in Fig.1. The abscissa is the product of the spatial number density and the projected cross section of particles; $(N_c \pi \bar{X}_{30}^2 / 4)$, is given by

$$\bar{X}_{30} = \left\{ \frac{\int_0^\infty X^3 dn}{\int_0^\infty dn} \right\}^{1/3} \times 10^{-4} \quad \dots\dots\dots (6)$$

where dn is written as

$$dn = \frac{w}{\rho_i (\pi/6)} \left(\frac{1}{\bar{X}_R} \right)^w X^{w-4} \exp \left\{ - \left(\frac{X}{\bar{X}_R} \right)^w \right\} dX \quad \dots\dots\dots (7)$$

The total energy is obtained for the spatial number density range 100-300 1/cm and it is proportional to the product of the spatial number density and the projected cross section of particles below $N_c = 1000$. $\bar{X}_R$ is estimated exactly at con-

siderably high density $N_c=3000$ and $\bar{X}_R$ is kept constant.

Fig.2 shows the relation between the number density N_c and the signal S . The condition a-d in Fig.2 corresponds to a-d in Fig.1. In this figure the detected diffraction energy distribution does not fall below $N_c=1000$, and the signal S is linearly proportional to N_c . Thus, it is considered that the results obtained by the particle analyser have good reliability and the change of the total energy corresponds to that of the number of the particle and diameter.

4. Detected Signal and Total Energy in Hot Spray

The quasi-two dimensional spray pattern, generated by Fan Spray nozzle (spray angle 25° , equivalent diameter 0.33mm), is used for experiments. This spray has an elliptic pattern and the laser path is set up to the major axis of the spray. In order to obtain a local spray structure, the window of the slit is used and the nozzle can be offset to angle of 0° or 12° (see Fig.3). Thus, the spray structures of both inner part (offset 0°) and neighborhood of the rim (offset 12°) of the spray are measured. The injection pressure is kept at 0.68 MPa and the test liquid is ethanol. T_a is air temperature at the upper point from the spray nozzle.

The spatial number density and droplets velocity are measured by two-beam laser correlation method. The results in this experiment show that N_c is not more than 1000 and total energy is also less than 600. Therefore, it is considered that the measured value of total energy corresponds to a change in the weight and diameter of the droplets. The detected signal profile is shown in Figs.4 and 5 for offset angle of 0° . The results shown in Figs. 4 and 6 are obtained for $T_a=473$ K and the results shown in Figs.5 and 7 are obtained for $T_a=573$ K.

As mentioned above, it is considered that present experimental results are obtained in the range where L_R satisfies Eqs.(3-a) and (3-b). Therefore, with a vaporizing model of spray droplets it seems that the vaporizing process of spray droplets can be explained by making a comparison of the experimental result with the calculated L_R .

Fig.3 Nozzle set up angle

Fig.4 Diffraction energy profile from particles

Fig.5 Diffraction energy profile from particles

Fig.6 Diffraction energy profile from particles

Fig.7 Diffraction energy profile from particles

5. Discussion of Evaporating Model in Hot Spray

In vaporization process of a single droplet, it is made clear theoretically and experimentally that the change of the droplet diameter can be explained by "d²-Law". Assuming that the single droplet vaporization Law "d²-Law" applies to the vaporization process of spray droplets, it is considered that the vaporization model of spray droplets is based on the following assumptions.

(1) The change of each droplet diameter, which depends on time, follows d²-Law. That is,

$$d^2 = d_0^2 - C_e t \times 10^3 \quad \text{.....(8)}$$

where C_e is given as follows;

$$C_e = 8\lambda_g(T_g - T_w) / (L_g \rho_l) \times 10^6 \quad \text{.....(9)}$$

(2) The evaporation rate constant of an individual droplet has the same value and is independent of time.

(3) Relative distribution of weight fractions at initial condition is given by Rosin-Rammler distribution function.

(4) A hypothetical spray consists of fifteen logarithmically spaced groups. For any one of these groups, we assume that there is a uniform size distribution, which has a representative size of the groups.

(5) A number of droplets within any group is constant until the representative size droplet has vaporized and vanished.

Fig.8 shows the model explained above, in which fifteen spaced groups are determined by this particle analyser. At $f=300$ mm, these groups are calculated and

Fig.9 Time dependence of relative weight distribution

Fig.10 Time dependence of relative weight distribution

Diameter range μm	
562.86	— 261.71
261.71	— 160.29
160.29	— 112.86
112.86	— 84.29
84.29	— 64.57
64.57	— 50.29
50.29	— 38.86
38.86	— 30.29
30.29	— 23.71
23.71	— 18.57
18.57	— 14.57
14.57	— 11.43
11.43	— 9.14
9.14	— 7.14
7.14	— 5.71

Table 1 Diameter range

Fig.8 Vaporizing model of cloud spray particles

Fig.11 Time dependence of relative total diffracted light energy

Fig.12 Effect of measurement position Z on the relative total diffracted light energy (experimental result)

shown in Table 1. The representative size of any one of these groups is the arithmetic mean of the upper and lower limits of the particle diameter.

The change of relative distribution of weight fractions, which depends on time, is calculated by using this model. Figs. 9 and 10 show the calculated results for two conditions $C_e = 0.1 \text{ mm}^2/\text{s}$ and $0.5 \text{ mm}^2/\text{s}$ respectively with initial condition $\bar{X}_R = 50 \mu\text{m}$ and $W = 2.0$.

On the other hand, minimum and maximum of the detector ring radius are about

$$\left. \begin{aligned} s_{\min} &= 0.15 \text{ mm} \\ s_{\max} &= 14.55 \text{ mm} \end{aligned} \right\} \dots\dots\dots (10)$$

respectively. Therefore, the relative total energy is calculated by Eq. 5, using the above values of $s_{\max}$, $s_{\min}$, the change of the weight fraction and diameter shown in Figs. 9 and 10. The result is shown in Fig. 11. In the calculated results, it is expected that the relative total energy decreases rapidly as evaporation rate constant C_e increases.

Thus, droplets evaporation rate constant can be estimated by making a comparison of the measured total light energy with L_R , which is calculated by C_e expected from experiment and is shown in Fig. 11. The value obtained by this procedure is called the observed evaporation rate constant.

The experimental relative total energy obtained from Figs. 4-7 is shown in Figs. 12 and 13 which are the cases for offset angle of 0° and 12° respectively.

6. Estimating Observed Evaporation Rate Constant

Fig. 15 Time dependence of relative total diffracted light energy (offset 0°)

Fig. 16 Time dependence of relative total diffracted light energy (offset 0°)

Fig. 13 Effect of measurement position Z on the relative total diffracted light energy (experimental result)

Fig. 14 Particle velocity and flying time in hot spray

Fig. 17 Time dependence of relative total diffracted light energy (offset 12°)

Fig. 18 Time dependence of relative total diffracted light energy (offset 12°)

It is necessary that the measurement position Z be represented by a function of time in order to compare the calculated relative total energy with the experimental results shown in Figs.12 and 13. Now, if the particle velocity is V_d , the following relationship is valid.

$$\left. \begin{aligned} V_d &= \frac{dZ}{dt} \\ dt &= \frac{1}{V_d} dZ \\ t_z &= \int_0^z \frac{1}{V_d} dZ \end{aligned} \right\} \dots\dots\dots(11)$$

Where, t_z means the time span to reach from the spray nozzle to the measurement position. Therefore, if V_d is measured, t_z is determined from the relation of z and $1/V_d$.

V_d is measured by two-beam laser correlation method. The result for offset angle of 0° is shown in Fig.14. The particle velocity V_d is an experimental value and t_z is calculated by z and V_d . There was no difference of the result between the cases for offset angles of 0° and 12° .

The results shown in Figs.12 and 13 are arranged as relation of t_z , so that it seems that the evaporation rate constant can be estimated by making a comparison of the experimental results with the calculated relative total energy. Initial condition used in the calculation is the experimental data. The results obtained by this procedure are shown in Figs.15-18. Figs.15 and 16 are the case for offset angle of 0° , and Figs.17 and 18 are the case for offset angle of 12° . From these figures, it seems that observed evaporation rate constant is about $0.1 \text{ mm}^2/\text{s}$ ($T_a=473\text{K}$) for offset angle of 0° , $0.2 \text{ mm}^2/\text{s}$ ($T_a=573\text{K}$) for offset angle of 0° , $0.15 \text{ mm}^2/\text{s}$ ($T_a=473\text{K}$) for offset angle of 12° and $0.2 \text{ mm}^2/\text{s}$ ($T_a=573\text{K}$) for offset angle of 12° .

7. Comparison of Droplets Evaporation Rate Constant with Single Droplet Evaporation Constant Rate

It is well known that the single droplet evaporation rate constant is given theoretically and experimentally by Eq. (9). Here, C_e in Eq.(9) set C_{eth} , C_{eth} is equivalent to the value of $Nu=2$ ($Re=0$). On the other hand, it seems that Re is about 50 in this experiment. Therefore, it is not adequate to compare the observed evaporation rate constant C_e , which is obtained in this experiment, with C_{eth} which is the calculated evaporation rate constant. The limited evaporation rate constant C_{eo} ($Re=0$) is given by

$$C_e = C_{eo}(1 + 0.3Re^{1/3}Pr^{1/3}) \dots\dots\dots(12)$$

The above equation considers the effect of the relative velocity between droplet and air. It is considered that C_{eth} corresponds to C_{eo} in Eq.12. Limited evaporation rate constant C_{eo} is calculated by Eq.12 and the observed evaporation rate constant C_e which is obtained in previous section. In calculation, the representative diameter s arithmetic mean diameter $\bar{X}_s$ which is

Fig.19 Comparison of evaporation rate constant between single droplet and spray

Fig.20 Comparison of evaporation rate constant between the pioneers result and author's result

calculated by measured $\bar{X}_R$ and W , and the droplet velocity is a value which is measured by two-beam laser correlation method[7]. The kinematic viscosity of air is a value at arithmetic mean of the air temperature T_a , the liquid droplet temperature T_w (measured by $60 \mu\text{m}$ CA thermocouple inserted into the spray droplets), $Pr=0.7$, and also $T_o = T_a$. Fig.19 shows the relation of C_{eo} and C_{eth} . In this figure, the straight line explains $C_{eo} = C_{eth}$. From this figure, C_{eo} for offset angle of 12° (around the outer edge of the spray) almost agrees with the evaporation rate constant which is calculated by assuming a single droplet. But for inner part of the spray, C_{eth} is smaller than the evaporation rate constant of a single droplet because of the difference in the assumption of $T_o = T_a$, where T_o in inner part of the spray decreases due to the poor evaporation, that is, the temperature difference $(T_o - T_w)$ is overestimated.

Fig.20 shows the comparison of the present results with the other researchers's data on a single droplet and the

spray droplets. The curve lines are an approximate line which is rewritten by the authors using their data. Kumagai & Isoda[1] and Kobayashi[2] presented the results of the single droplet and Araki [10] reported on the result of droplets. As mentioned above, there still remain some problems in the determination of the ambient temperature T_g . But, it is considered that the present data are suitable for comparing with these results.

8. Conclusions

A method which can quantitatively estimate the evaporation rate constant of the liquid droplets in hot spray is proposed using the particle analyser based on the Fraunhofer diffraction theory, and measurements are made within a quasi-two dimensional spray flow generated by Fan Spray nozzle.

The following conclusions appear to be valid:

(1) Diffracted total light energy from particles is defined, and the relation of the diffracted total light energy and spatial number density and diameter of the spray droplets is made clear. The range where this relation is applicable is verified by using glass beads which have a known diameter.

(2) In the range where the diffracted total light energy is proportional to the product of the number density and diameter, the relative total light energy is defined, and the change of the relative weight distribution at any point is estimated by the change of the relative total light energy.

(3) The d^2 -Law for single droplet evaporation process is applied to the droplets vaporization model. The observed droplets evaporation rate constant is determined by the comparison of the experimental results with the relative total energy calculated by droplets vaporization model. This estimation method is applied to the cloud spray droplets of methanol and the observed evaporation rate constant of droplets is determined.

(4) Limited evaporation rate constant for ethanol is calculated from the observed evaporation rate constant. The appropriateness of the present results is verified by the comparison of it with the results of pioneers.

(5) It is confirmed that the spray around the outer part of the spray has a larger limited evaporation rate constant than that of the inner part.

(6) It is expected that the evaporation rate constants of various liquids will be determined by this estimation method.

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