A Study on Rolling Finish of Gear*

(End Effects)

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In the rolling finish of gears, the tooth profile of gear is affected by the end effect at tooth tip or root. In the present study the compression test between a die roller and rectangular bars is performed at five points having different distances from the end of test piece.

The range subjected to end effect is unaffected by the width of test piece. The increase of deformation due to the end effect varies almost linearly with the increase of compressive load at every loading position but independently of the width of test piece.

Key Words: Gear, Forming, Rolling Finish, End Effects, Compression Test, Plastic Deformation

1. Introduction

It was already reported that the tooth tip and root were deformed considerably by interferential action in rolling finish of gear.⁽¹⁾ Such end effect at tooth tip or root affects the load shared at each tooth during rolling finish. Therefore the end effect is an important factor and cannot be ignored in order to clarify the deformation process on tooth surface.

However a study of the end effect has not been done except a discussion for a compression test of a wedge⁽²⁾ and further the end effect on deformation has not been clarified yet.

In this paper compression tests between a die roller and rectangular bars are performed at some points having different distances from the end of test piece, and the end effect on deformation in a central longitudinal section or a cross section is quantitatively investigated in detail by measuring a depth of indentation at loading point, a protrusion value in cross and longitudinal direction, a crowning value and an overall length of protrusion on a flank of test piece. Moreover relations among those parameters of deformation are clarified at the same time.

2. Testing Apparatus and Conditions

An outline of testing apparatus is shown in Fig. 1. A test piece is fixed by setting bolts on both sides and upper surface. The load is imposed through two

load bars set at both ends of an arbor fitting a die roller by means of the universal testing machine. The testing apparatus is connected with the testing machine through a ball in order to avoid partial contact between a die roller and the test piece. Moreover the distance from the end of the test piece to loading points is adjusted by a positioning bolt and is measured by a linear scale.

The die roller is of the material SKD 12 and has an external diameter of 55 mm and a width of 16 mm. The hardness after heat treatment is H_v 892 and the surface is finished, as mirror surface, to $0.2 \mu\text{m}$ R_{max} in roughness.

Test pieces are rectangular bars of the material S45C, have a length of 75 mm and a height of 9 mm and three kinds of widths 5, 10 and 15 mm. All of surfaces are finished to about $3 \mu\text{m}$ R_{max} in roughness by a surface grinder. The hardness after heat treatment (heating at 850°C , opening a furnace and cooling slowly) is H_v 205 ± 5 .

The experiment is carried out by changing the compressive load and the loading position for each test piece. Loads applied in this experiment are 0.588{60}, 0.784{80}, 0.980 kN/mm {100 kgf/mm} and five points having different distances of 0, 1, 2, 4 and 10 mm from the end in the longitudinal direction of test piece are

Fig.1 Testing apparatus

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selected as loading position. Hereafter the width of test piece is represented by b , the compressive load per unit width by W or by W_L, W_M or W_H ($W_L < W_M < W_H$) for the purpose of distinguishing dimension of loads, and the loading position by x , the distance from the end to loading point.

Machine oil (ISO VG 46) is used as a lubricant and smeared on the compressed surface of the die roller and test pieces.

3. Experimental Results and Considerations

3.1 Deformation in a central longitudinal section of test piece

The profile of the indentation and of the end in a central longitudinal section, when the load of 0.980 kN/mm (100 kgf/mm) acts at each loading position on test piece of width of 10 mm, is shown in Fig. 2. The edge of indentation seems not to be piled up but to be sunken in at every distance x . The depth and the width of indentation are different with the distance x . In regard to the profiles of end-protrusion, the position showing maximum protrusion is slightly below the surface when $x=1$ mm, but is very much near the surface when $x=0$ mm and $x \geq 2$ mm. The tendency described above is observed with other widths and other compressive loads.

Profiles of the indentation and the end-protrusion are quantitatively considered by the depth of indentation δ_y at loading point, the width of indentation a , the distance from the loading point to an edge of indentation a_h and the height of end-protrusion δ_{ys} . These parameters are illustrated in Fig. 2.

An example of the relation between the depth of indentation δ_y and the loading distance x is shown in Fig. 3. δ_y becomes maximum at $x=0$ mm, decreases rapidly at

Fig.2 Profiles after loading in a central longitudinal section [$b=10$ mm, $W=0.980$ kN/mm (100kgf/mm)]

$x=1$ mm and then converges with an increase of the distance from the end. The convergent position of δ_y is affected by the compressive load W , i.e. it approaches the end with a decrease of the compressive load, but is unaffected by width. δ_y converges within about $x=8$ mm as far as we know from the experiment. δ_y at $x=20$ mm is plotted in Fig. 3 at the same time for the purpose of confirmation of the convergence.

The relation between the increase of the depth of indentation due to the end effect ($\delta_y - \delta_{ys}$) and the compressive load W is shown for each loading distance x in Fig. 4. δ_{ys} is the convergent value of

Fig.3 Relation between the depth of indentation δ_y and the loading distance x

Fig.4 Relation between the increase of depth of indentation due to end effect ($\delta_y - \delta_{ys}$) and the compressive load W

δ_v , viz. the value of δ_v when the end effect can be neglected. $(\delta_v - \delta_{vs})$ increases with an increase of the compressive load W and the increasing rate has a maximum value at $x = 0$ mm and decreases with an increase of the loading distance x . As regards the tendency of increase, $(\delta_v - \delta_{vs})$ increases parabolically when $x \geq 1$ mm but linearly when $x = 0$ mm. Moreover converging comparatively when $x \geq 1$ mm, $(\delta_v - \delta_{vs})$ does not converge when $x = 0$ mm and preserves a comparatively large value under $W = 0.588$ kN/mm. Thus the end is deformed very easily, therefore in rolling finish of gear the load shared at tooth tip or root may decrease. Consequently it is necessary to carry out an analysis of tooth profile deformation in sufficient consideration of the end effect in the case of an interference of tooth tip or root. $(\delta_v - \delta_{vs})$ is approximately constant regardless of the width. However, at $x = 0$ mm for the width of 5 mm, $(\delta_v - \delta_{vs})$ has a slightly larger value. From this result it is seen that the increase of δ_v due to the end effect is hardly dependent on the width b . Consequently the relation between δ_v and the width can be represented by δ_{vs} of the convergent value of δ_v .

The relation between δ_{vs} and the compressive load W is shown in Fig. 5. δ_{vs} increases parabolically with an increase of compressive load and the increasing rate becomes maximum for the width of 5 mm. Therefore the effect of width appears markedly as the compressive load increases, i.e. the difference between δ_{vs} for the width of 10 mm and for 15 mm is small but δ_{vs} for a width of 5 mm is comparatively large. This result is explained by considering the difference in the condition of deformation between the width of 5 mm and of 10~15 mm, and a similar difference in deformation was observed in the rolling experiment of rollers.⁽³⁾

Fig. 5 Relation between the convergent value of depth of indentation δ_{vs} and the compressive load W

Thus, the depth of indentation δ_v at loading point is affected by the loading distance x , the width of test piece b and the compressive load W , and particularly a marked change is observed in regions near the end. An experimental equation is given as follows:

$$\left(\frac{\delta_v}{\delta_{vs0}}\right) = \left(\frac{\delta_{vs}}{\delta_{vs0}}\right) + \left[\left\{1 - \left(\frac{x}{x_0}\right)\right\}10^\alpha\right]^\beta$$

where $(\delta_{vs}/\delta_{vs0}) = 10^\gamma$

$$\gamma = 1.35(W/W_0) + 0.16\{(b/b_0) - 1.92\}^2 - 1.49$$

$$\alpha = 0.206(W/W_0) - 0.089$$

$$\beta = 0.151(W/W_0) - 0.044$$

This equation is a nondimensional equation using the following standard values; $W_0 = 0.980$ kN/mm {100 kgf/mm}, $b_0 = 10$ mm, $x_0 = 10$ mm and $\delta_{vs0} = (\delta_{vs})_{W=W_0, b=b_0} = 4.6$ μm. δ_{vs0} , however, is obtained from the compression test between the die roller of 55 mm in diameter and the rectangular bar of S45C in the material, but it is influenced by the differences in material, the relative radius of curvature between a die roller and a test piece, etc. This experimental equation is applicable when $5 \text{ mm} \leq b \leq 15 \text{ mm}$, $0.588 \text{ kN/mm} \{60 \text{ kgf/mm}\} \leq W \leq 0.980 \text{ kN/mm} \{100 \text{ kgf/mm}\}$.

Tooth profile after rolling finish can be calculated under more practical condition considering the end effect by application of this experimental equation for gear.

An example of the relation between the height of end-protrusion δ_{eg} and the loading distance x is shown in Fig. 6. δ_{eg} and the overall length of end-protrusion have little difference for every width of test piece.

Fig. 7 shows the width of indentation a , measured at each loading position. a increases gradually as the loading point approaches the end and becomes maximum at $x = 1$ mm, but rather decreases at the end

Fig. 6 Relation between the height of end-protrusion δ_{eg} and the loading distance x [$W = 0.980$ kN/mm (100 kgf/mm)]

$x=0$ mm. This tendency is observed for every width and a is approximately equal for every width under the same load.

The relation between δ_v and the depth of indentation by a rigid roller $a_h^2/(2R)$, which is calculated by using the experimental a_h , is shown in Fig. 8. R is the radius of a die roller, 27.5 mm. Fig. 8 shows the experimental results in a range of $x \leq 2$ mm and symbols are not distinguished for each x . The result for only $x=0$ mm is indicated in the zone bounded by dotted lines particularly, which is clearly different from that for $x \geq 1$ mm, i.e. the result for $x=0$ mm is shown approximately as a straight line of $\delta_v = a_h^2/(2R)$ but in the case of $x=1$ and 2 mm the experimental curve is located below the straight line. The tendency for $x \geq 4$ mm is similar to that for $x=1$ and 2 mm. From these results, it is found that δ_v is influenced by elastic deformation when $x \geq 1$ mm but is hardly affected when $x=0$ mm. As a reason why the experimental curve approaches the straight

Fig. 7 Relation between the width of indentation in central longitudinal section a and the loading distance x [$b=10$ mm]

Fig. 8 Relation between the depth of indentation by a rigid roller $a_h^2/(2R)$ and the experimental value of δ_v

line of $\delta_v = a_h^2/(2R)$ with an increase of δ_v , when $x \geq 1$ mm, it can be considered that the increasing rate of plastic deformation is higher than that of elastic deformation. Since the state of deformation considerably changes with the loading distance x and dimension of δ_v , when the order of a few μm of δ_v is considered important and the contact point is continually moved as in rolling finish of gear, the tooth profile after rolling finish must be estimated in sufficient consideration of elastic deformation.

3.2 Deformation in a cross section of test piece

In section of 3.1 the deformation in the central longitudinal section is considered and the relation between the deformation and compressive conditions is clarified. In this section it is considered how the profile in cross section is deformed by compressive conditions. The deformation is different in each cross section within indentation, therefore a section which contains the loading point is considered.

Fig. 9 shows the profile after loading. The surface is deformed like a drum (it is called the crowning) and the protrusion occurs on both flanks. The profile after loading is considered by a crowning value δ_c and a height of flank-protrusion δ_w .

The relation between the height of flank-protrusion δ_w and the loading distance x is shown in Fig. 10. δ_w also, just as δ_v , decreases with an increase of the distance from the end to the loading point and then converges. This fact implies that the end effect produces a deformation not only in the longitudinal section but also in the cross section. Moreover δ_w is approximately equal for every width and the

(a) Profile of compressed surface

(b) Profile of flank-protrusion

Fig. 9 Profiles after loading in a cross section [$b=10$ mm, $W=0.980$ kN/mm (100 kgf/mm), $x=0$ mm]

difference among widths cannot be recognized clearly.

The relation between the increase of the height of flank-protrusion due to the end effect ($\delta_w - \delta_{ws}$) and the compressive load W is shown in Fig. 11. δ_{ws} is the convergent value of δ_w and both δ_w and δ_{ws}

Fig.10 Relation between the height of flank-protrusion δ_w and the loading distance x

Fig.11 Relation between the increase of height of flank-protrusion due to end effect ($\delta_w - \delta_{ws}$) and the compressive load W

Fig.12 Relation between the convergent value of height of flank-protrusion δ_{ws} and the compressive load W

Fig.13 Relation between the crowning value δ_c and the loading distance x

Fig.14 Relation between $(\delta_v - \delta_{vs})$ and $(\delta_w - \delta_{ws})$

are indicated by the average of measured values for three widths. $(\delta_w - \delta_{ws})$ increases linearly with an increase of compressive load. If the tendency of the increase in $(\delta_w - \delta_{ws})$ is compared with that in $(\delta_v - \delta_{vs})$ shown in Fig. 4, the increasing rate of $(\delta_w - \delta_{ws})$ is less than that of $(\delta_v - \delta_{vs})$ and it becomes very small when $x \geq 2$ mm, especially it can be ignored when $x = 4$ mm. However when $x = 0$ mm, the tendency of the increase in $(\delta_w - \delta_{ws})$ is similar to that in $(\delta_v - \delta_{vs})$.

The relation between δ_{ws} and the compressive load W is shown in Fig. 12. δ_{ws} increases parabolically, just as δ_{vs} , with an increase of W .

The relation between the crowning value δ_c and the loading distance x is shown in Fig. 13. δ_c is approximately constant independently of x in the case of constant width. δ_c for the widths of 5 mm and 10 mm is approximately equal, but for 15 mm δ_c is much smaller than these values. Thus the crowning is very little formed by the loads of $W = 0.588$ kN/mm and 0.784 kN/mm.

3.3 Relations among parameters of deformation

In sections of 3.1 and 3.2, each parameter which represents the plastic deformation was evaluated quantitatively. As an adequate interrelation is estimated from these results, now we shall investigate several correlations among these parameters. In this section the correlations among $(\delta_v - \delta_{vs})$, $(\delta_w - \delta_{ws})$ and δ_{eg} , which are more important to clarify the end effect on deformation than other parameters of deformation, are considered.

The relation between $(\delta_v - \delta_{vs})$ and $(\delta_w - \delta_{ws})$ is shown in Fig. 14. As $(\delta_v - \delta_{vs})$ and $(\delta_w - \delta_{ws})$ are scarcely influenced by the width of test piece, the average of measured values for widths of 5, 10 and 15 mm is shown in the figure. A linear relation is approximately observed between $(\delta_v - \delta_{vs})$ and $(\delta_w - \delta_{ws})$ but the slope of the straight line is different between $x = 0$ mm and $x \geq 1$ mm, i.e. $(\delta_w - \delta_{ws}) / (\delta_v - \delta_{vs})$ is about 0.24 when $x = 0$ mm but about 0.12 when $x \geq 1$ mm.

The relation between $(\delta_w - \delta_{ws})$ and δ_{eg} is shown in Fig. 15. A linear relation is also observed between $(\delta_w - \delta_{ws})$ and δ_{eg} and the slope of the straight line is different between $x = 0$ mm and $x \geq 1$ mm. The slope for $x = 0$ mm is about 2.4 times that

Fig.15 Relation between $(\delta_w - \delta_{ws})$ and δ_{eg}

for $x \geq 1$ mm. From these results it is found that the degree of end effect on deformation is considerably different between $x = 0$ mm, where the loading point is the end, and $x \geq 1$ mm, where the loading point is far from the end, and particularly the width is easily increased due to loading at the end.

The relation among $(\delta_v - \delta_{vs})$, $(\delta_w - \delta_{ws})$ and δ_{eg} is expressed as follows:

$$\left. \begin{aligned} \frac{\delta_w - \delta_{ws}}{17\delta_{eg}} = \frac{\delta_w - \delta_{ws}}{12(\delta_v - \delta_{vs})} = 0.02 \quad (x = 0 \text{ mm}) \\ \frac{\delta_w - \delta_{ws}}{7\delta_{eg}} = \frac{\delta_w - \delta_{ws}}{6(\delta_v - \delta_{vs})} = 0.02 \quad (x \geq 1 \text{ mm}) \end{aligned} \right\}$$

Fig.16 Relation between the overall length of flank-protrusion h_p and the loading distance x

Fig.17 Relation between the increase of overall length of flank-protrusion due to end effect $(h_p - h_{ps})$ and the compressive load W

Fig. 18 Relation between $(\delta_v - \delta_{ws})$ and $(h_p - h_{ps})$

It can be considered that the above experimental equation expresses quantitatively the end effect on deformation, and it can be applied to the evaluation of end effect in the rolling finish of gear.

3.4 Overall length of flank-protrusion

The relation between the overall length of flank-protrusion in a cross sectional plane which contains loading point, h_p , which is illustrated in Fig. 9, and the loading distance x is shown in Fig. 16. h_p is approximately constant for every width, just as δ_w , and it decreases with an increase of the distance from the end to the loading point and then converges, just as δ_v or δ_w . However, the slope between h_p and x is gentle compared with that between δ_v or δ_w and x .

The relation between the increase of the overall length of flank-protrusion due to the end effect $(h_p - h_{ps})$ and the compressive load W is shown in Fig. 17. h_{ps} is the convergent value of h_p and $(h_p - h_{ps})$ is indicated by the average of measured values for three widths. $(h_p - h_{ps})$ increases linearly with an increase of compressive load. The increasing rate is approximately equal when $x = 0$ and 1 mm but decreases when $x = 2$ mm. Thus, the end effect is clearly recognized till about $x = 1$ mm with $(h_p - h_{ps})$, as with $(\delta_v - \delta_{ws})$.

The relation between $(\delta_v - \delta_{ws})$ and $(h_p - h_{ps})$ is shown in Fig. 18. $(h_p - h_{ps})$ increases exponentially with an increase of $(\delta_v - \delta_{ws})$, but the difference in the tendency between $x = 0$ mm and $x \geq 1$ mm, as observed among $(\delta_v - \delta_{ws})$, $(\delta_w - \delta_{ws})$ and δ_{wg} , cannot be observed. Consequently the increase of h_p due to the end effect is determined by the increase of the depth of indentation independently of the loading distance x .

The relation between the convergent values of h_p , h_{ps} , and the compressive load W is shown in Fig. 19. The value of h_{ps} is indicated by the average of measured values for three widths. The tendency of h_{ps} for the compressive load is similar to that of δ_{ws} or δ_{wg} . Therefore it can be considered that the value of h_{ps} changes parabolically with an increase of compressive load.

4. Conclusions

Fig. 19 Relation between the convergent value of overall length of flank-protrusion h_{ps} and the compressive load W

The compression test between rectangular bars and a die roller is performed at five points having different distances from the end of test piece and the end effect on deformation is considered by measuring the depth of indentation at loading point, the crowning value, the height of end-protrusion and the height and the overall length of flank-protrusion. The results may be summarized as follows.

(1) The range subjected to the end effect is unaffected by width of test piece and is fixed by the compressive load. Moreover, its limit approaches the end with a decrease of the compressive load.

(2) The deformation at loading point is different between the compression at the end and that at points having a distance of more than 1 mm from the end, i.e. in the former the deformation is hardly affected by elastic deformation, but in the latter it is considerably affected. The effect of elastic deformation becomes weaker with an increase of deformation of test piece. Moreover, the width is easily increased at the end.

(3) The increase of deformation due to the end effect is produced by the expansion of deformed region, and the increasing amount varies almost linearly with the increase of compressive load at every loading position, but independently of the width of test piece. A proportional relation is observed among the increase of the depth of indentation, of the height of flank-protrusion and of the height of end-protrusion due to the end effect, but the tendency is different between $x = 0$ mm and $x \geq 1$ mm. An experimental equation quantitatively expressing the relation among those factors is obtained. In addition, an exponential relation is observed between the increase of the overall length of flank-protrusion and of the depth of indentation due to the end effect, and it is independent of the loading position.

(4) The relation between the depth of indentation at loading point and compressive

conditions, which can be applied as an estimate of the end effect in rolling finish of gear, is expressed as follows.

$$\left(\frac{\delta_y}{\delta_{yso}}\right) = \left(\frac{\delta_{ys}}{\delta_{yso}}\right) + \left[\left\{1 - \left(\frac{x}{x_o}\right)\right\} 10^{\sigma}\right]^{1/2}$$

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