Contact Stresses between Two Disks and a Sandwiched Solid-metal Flat Gasket*

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Contact stresses between two disks and a sandwiched solid-metal flat gasket are studied to discuss the sealing behavior of the gasket. Assuming that the gasket is an elastic body, the three dimensional theory of elasticity is used to analyse the contact stresses and the deformations in three laminated disks whose both sides are compressed by axisymmetrically distributed loads. By neglecting shearing stresses on the contact surfaces, an exact solution satisfying the remaining boundary conditions is obtained. Numerical calculations are carried out for various rigidities and thicknesses of the disks under various loading conditions. In order to examine the results of the theoretical analysis, the contact stress is measured experimentally by using pressure-sensitive pins attached to a disk. It is confirmed that the theoretical results agree well with the experimental ones except the vicinity of the loaded area.

Key Words: Elasticity, Experimental Stress Analysis, Contact Stress, Three Laminated Disks, Solid-metal Flat Gasket, Pressure-Sensitive Pin

1. Introduction

The stress analysis for an elastic contact problem between flat plates is important to design connected parts of machines and apparatus. In former studies, the contact problem of flat plates composed of same materials was often analysed regarding them as a single flat plate. But works on flat plates with different thicknesses and materials seem to be few⁽¹⁾⁻⁽⁴⁾. On the other hand, in relation to the contact problem, gasket joints are often used to improve a seal efficiency in the connected parts. Particularly, the solid-metal flat gasket joint is required to have a high seal efficiency and various investigations with respect to the pressure on the gasket surface, which has the largest influence on the seal efficiency, have been done until now⁽⁵⁾⁻⁽¹³⁾. But hardly any analytical and experimental studies on it have been sufficiently done yet. The author and others have studied the connected problems⁽¹³⁾⁻⁽¹⁵⁾ of a solid-metal flat gasket inserted between two strips, slabs and finite hollow cylinders. The influences of the plate thickness, Young's modulus and the load on the contact stresses and the deformations were clarified.

In this paper, the seal efficiency of a gasket joint which is important for the problem of a solid-metal flat gasket

inserted between circular flanges will be made clear. The contact problem of three laminated disks with different thicknesses and materials being subjected to axisymmetrical compressive loads is analysed by assuming that the gasket is an elastic body. It is generally difficult to analyse this problem by taking account of the separation on the contact surfaces. Therefore, the problem is analysed on the basis of the three dimensional theory of elasticity assuming the contacts are smooth and perfect, and the influences of the plate thickness, Young's modulus and the load on the contact stresses and the deformations are clarified. Moreover, the contact stress is measured by means of a previously reported pressure-sensitive pin method⁽⁶⁾ and the experimental results are compared with the calculated ones. These results are also compared with those without a solid-metal flat gasket⁽⁴⁾ and the seal efficiency of the solid-metal flat gasket is investigated.

2. Theoretical analysis

2.1 Analytical model and boundary conditions

Fig.1 Model of analysis

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As shown in Fig.1, let's consider the case where both sides of three laminated disks with different thicknesses and materials are subjected to axisymmetrically distributed arbitrary loads $p_1(r)$ and $p_2(r)$ whose each total load is W . In this analysis, Michell's stress function ϕ in a cylindrical co-ordinate system (r, θ, z) is used. Then, the stresses and the displacements u in the r -direction and w in the z -direction are expressed by the following equations,

$$\left. \begin{aligned} \sigma_r &= \frac{\partial}{\partial z} \left(\nu \nabla^2 \phi - \frac{\partial^2 \phi}{\partial r^2} \right), & \sigma_\theta &= \frac{\partial}{\partial z} \left(\nu \nabla^2 \phi - \frac{1}{r} \frac{\partial \phi}{\partial r} \right) \\ \sigma_z &= \frac{\partial}{\partial z} \left\{ (2-\nu) \nabla^2 \phi - \frac{\partial^2 \phi}{\partial z^2} \right\}, & \tau_{rz} &= \frac{\partial}{\partial r} \left\{ (1-\nu) \nabla^2 \phi - \frac{\partial^2 \phi}{\partial z^2} \right\} \end{aligned} \right\} \dots\dots\dots (1)$$

$$u = -\frac{1+\nu}{E} \frac{\partial^2 \phi}{\partial r \partial z}, \quad w = \frac{1+\nu}{E} \left\{ (1-2\nu) \nabla^2 \phi + \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} \right\} \dots\dots\dots (2)$$

where E is Young's modulus and ν is Poisson's ratio and

$$\nabla^2 \nabla^2 \phi = 0, \quad \nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The boundary conditions in Fig.1 are given by Eqs.(3)~(8) when the contacts are smooth and perfect. Here, superscripts (1)~(3) and subscripts 1~3 denote values of the disks I~III, respectively.

(i) Disk I

$$\left. \begin{aligned} z=2h_1+h_2: & \sigma_z^{(1)} = -p_1(r) = e_0 + \sum_{s=1}^{\infty} e_s J_0(\gamma_s r), \quad \tau_{rz}^{(1)} = 0 \\ z=h_2: & \sigma_z^{(1)} = e_0' + \sum_{s=1}^{\infty} e_s' J_0(\gamma_s r), \quad \tau_{rz}^{(1)} = 0 \\ r=c: & \sigma_r^{(1)} = \tau_{rz}^{(1)} = 0 \end{aligned} \right\} \dots\dots\dots (3)$$

(ii) Disk II

$$\left. \begin{aligned} z=h_2: & \sigma_z^{(2)} = e_0' + \sum_{s=1}^{\infty} e_s' J_0(\gamma_s r), \quad \tau_{rz}^{(2)} = 0 \\ z=-h_2: & \sigma_z^{(2)} = e_0'' + \sum_{s=1}^{\infty} e_s'' J_0(\gamma_s r), \quad \tau_{rz}^{(2)} = 0 \\ r=c: & \sigma_r^{(2)} = \tau_{rz}^{(2)} = 0 \end{aligned} \right\} \dots\dots\dots (4)$$

(iii) Disk III

$$\left. \begin{aligned} z=-h_2: & \sigma_z^{(3)} = e_0'' + \sum_{s=1}^{\infty} e_s'' J_0(\gamma_s r), \quad \tau_{rz}^{(3)} = 0 \\ z=-h_2-2h_3: & \sigma_z^{(3)} = -p_2(r) = e_0''' + \sum_{s=1}^{\infty} e_s''' J_0(\gamma_s r), \quad \tau_{rz}^{(3)} = 0 \\ r=c: & \sigma_r^{(3)} = \tau_{rz}^{(3)} = 0 \end{aligned} \right\} \dots\dots\dots (5)$$

(iv) On the contact surfaces

$$z=h_2: w^{(1)} = w^{(2)} \dots\dots\dots (6)$$

$$z=-h_2: w^{(2)} = w^{(3)} \dots\dots\dots (7)$$

(v) The balance of forces

$$W = \int_0^c 2\pi r p_1(r) dr = \int_0^c 2\pi r p_2(r) dr \dots\dots\dots (8)$$

where

$$e_0 = -\frac{2 \int_0^c r p_1(r) dr}{c^2}, \quad e_s = -\frac{2 \int_0^c r p_1(r) J_0(\gamma_s r) dr}{c^2 J_0^2(\gamma_s c)}$$

$$e_0''' = -\frac{2 \int_0^c r p_2(r) dr}{c^2}, \quad e_s''' = -\frac{2 \int_0^c r p_2(r) J_0(\gamma_s r) dr}{c^2 J_0^2(\gamma_s c)}$$

and e_0' , e_s' , e_0'' and e_s'' are unknown coefficients to be determined from the boundary conditions. Moreover, $J_\mu(\gamma_s r)$ is a Bessel function of the first kind of order μ and λ_s is the s -th positive root satisfying $J_1(\lambda_s) = 0$, where $\gamma_s = \lambda_s/c$.

2.2 Stress analysis of disks

In the stress analysis of the disk I, by performing co-ordinate transformation $z_1 = z - h_1 - h_2$, the stress function $\phi_1^{(16)(17)}$ satisfying the shearing stresses $\tau_{rz}^{(1)} = 0$ on $z_1 = \pm h_1$ and $r = c$ in Eq.(3) is obtained as follows,

$$\begin{aligned} \phi_1 &= \bar{A}_0 \frac{(z_1 + h_1 + h_2)^3}{6} + \bar{C}_0 \frac{(z_1 + h_1 + h_2) r^2}{2} \\ &+ \sum_{n=1}^{\infty} \frac{\bar{A}_n}{\alpha_n^3 \mathcal{A}_n(\alpha_n)} \{ [2(1-\nu_1) I_1(\alpha_n c) + \alpha_n c I_0(\alpha_n c)] I_0(\alpha_n r) - I_1(\alpha_n c) \alpha_n r I_1(\alpha_n r) \} \sin(\alpha_n z_1) \\ &+ \sum_{s=1}^{\infty} \frac{\bar{C}_s}{\gamma_s^3 \bar{\mathcal{Q}}_s} \{ -[2\nu_1 \operatorname{sh}(\gamma_s h_1) + \gamma_s h_1 \operatorname{ch}(\gamma_s h_1)] \operatorname{sh}(\gamma_s z_1) + \operatorname{sh}(\gamma_s h_1) \gamma_s z_1 \operatorname{ch}(\gamma_s z_1) \} J_0(\gamma_s r) \\ &- \sum_{n=0}^{\infty} \frac{\bar{E}_n}{\beta_n^3 \mathcal{A}_n(\beta_n)} \{ [2(1-\nu_1) I_1(\beta_n c) + \beta_n c I_0(\beta_n c)] I_0(\beta_n r) - I_1(\beta_n c) I_1(\beta_n r) \beta_n r \} \cos(\beta_n z_1) \\ &+ \sum_{s=1}^{\infty} \frac{\bar{G}_s}{\gamma_s^3 \bar{\mathcal{Q}}_s} \{ -[2\nu_1 \operatorname{ch}(\gamma_s h_1) + \gamma_s h_1 \operatorname{sh}(\gamma_s h_1)] \operatorname{ch}(\gamma_s z_1) + \operatorname{ch}(\gamma_s h_1) \gamma_s z_1 \operatorname{sh}(\gamma_s z_1) \} J_0(\gamma_s r) \end{aligned} \dots\dots\dots (9)$$

where $\bar{A}_0$, $\bar{C}_0$, $\bar{A}_n$, $\bar{C}_s$, $\bar{E}_n$ and $\bar{G}_s$ are unknown coefficients, and $I_\mu(\alpha_n r)$ is a modified Bessel function of the first kind of order μ . Moreover, ch and sh stand for cosh and sinh, respectively and $\alpha_n = n\pi/h_1$, $\beta_n = (2n+1)\pi/2h_1$, $\bar{\mathcal{Q}}_s = \operatorname{sh}(\gamma_s h_1) \operatorname{ch}(\gamma_s h_1) + \gamma_s h_1$, $\bar{\mathcal{Q}}_s = \mathcal{Q}_s - 2\gamma_s h_1$, $\mathcal{A}_s(\alpha_n) = \{ [2(1-\nu_1) + (\alpha_n c)^2] I_1^2(\alpha_n c) - (\alpha_n c)^2 I_0^2(\alpha_n c) \} / \alpha_n c$.

Now, by substituting Eq.(9) into Eq.(1), the stresses $\sigma_r^{(1)}$ and $\sigma_z^{(1)}$ on the boundary surfaces are obtained as follows,

$$\begin{aligned} (\sigma_r^{(1)})_{r=0} &= \bar{A}_0 \nu_1 + \bar{C}_0 (2\nu_1 - 1) + \sum_{n=1}^{\infty} \left[\bar{A}_n + \sum_{i=1}^{\infty} P_{in} \bar{C}_i \right] \cos \alpha_n z_1 + \sum_{n=0}^{\infty} \left[\bar{E}_n + \sum_{i=1}^{\infty} \bar{P}_{in} \bar{C}_i \right] \sin \beta_n z_1 \dots (10) \\ (\sigma_z^{(1)})_{z_1 \pm h_1} &= \bar{A}_0 (1 - \nu_1) + 2\bar{C}_0 (2 - \nu_1) + \sum_{n=1}^{\infty} \left[\sum_{i=1}^{\infty} Q_{ni}(\alpha_n) \bar{A}_n + \bar{C}_i \right] J_0(\gamma_n r) \\ &\quad \pm \sum_{n=1}^{\infty} \left[\sum_{i=0}^{\infty} Q_{ni}(\beta_n) \bar{E}_n + \bar{C}_i \right] J_0(\gamma_n r) \dots (11) \end{aligned}$$

where

$$\begin{aligned} P_{in} &= \frac{4(-1)^n \gamma_n h_1 (\pi n)^2 \operatorname{sh}^2(\gamma_n h_1) J_0(\lambda_n)}{Q_i[(\pi n)^2 + (\gamma_n h_1)^2]^2}, \quad \bar{P}_{in} = \frac{4(-1)^n \gamma_n h_1 [(2n+1)\pi/2]^2 \operatorname{ch}^2(\gamma_n h_1) J_0(\lambda_n)}{Q_i[(2n+1)\pi/2]^2 + (\gamma_n h_1)^2]^2} \\ Q_{ni}(\alpha_n) &= \frac{4(-1)^{n+1} \alpha_n c \lambda_n^2 I_1^2(\alpha_n c)}{A_n(\alpha_n) [(\alpha_n c)^2 + \lambda_n^2]^2 J_0(\lambda_n)} \end{aligned}$$

Equating Eqs.(10) and (11) to the boundary condition Eq.(3), we get the following relations.

$$\left. \begin{aligned} \bar{A}_0 &= \frac{1-2\nu_1}{1+\nu_1} e_0, \quad \bar{C}_0 = \frac{\nu_1}{1+\nu_1} e_0 \quad (e_0 = e_0') \\ \bar{A}_n &= -\sum_{i=1}^{\infty} P_{in} \bar{C}_i, \quad \bar{E}_n = -\sum_{i=1}^{\infty} \bar{P}_{in} \bar{C}_i \\ \sum_{n=1}^{\infty} Q_{ni}(\alpha_n) \bar{A}_n + \bar{C}_i + \sum_{n=0}^{\infty} Q_{ni}(\beta_n) \bar{E}_n + \bar{C}_i &= e_i \\ \sum_{n=1}^{\infty} Q_{ni}(\alpha_n) \bar{A}_n + \bar{C}_i - \sum_{n=0}^{\infty} Q_{ni}(\beta_n) \bar{E}_n - \bar{C}_i &= e_i' \end{aligned} \right\} \dots (12)$$

Next, the stress analysis of the disks II and III can be done in the same way as that of the disk I after performing co-ordinate transformations $z_2 = z$ and $z_3 = z + h_2 + h_3$, respectively. Namely, the stress functions ϕ_2 or ϕ_3 can be used as the expression which is obtained from Eq.(9) by replacing $z_1 + h_1 + h_2$ in constant terms and another z_1, h_1, ν_1 with z_2, z_2, h_2, ν_2 or $z_3 - h_2 - h_3, z_3, h_3, \nu_3$ and putting a prime or two primes to symbols $\bar{A}_n, \bar{C}_n, \dots, \alpha_n, \beta_n$ except γ_n . By using these stress functions and taking account of the boundary condition Eqs.(4) and (5), the relation Eqs.(13) and (14) similar to Eq.(12) are obtained as follows.

$$\left. \begin{aligned} \bar{A}_0' &= \frac{1-2\nu_2}{1+\nu_2} e_0', \quad \bar{C}_0' = \frac{\nu_2}{1+\nu_2} e_0' \quad (e_0' = e_0'') \\ \bar{A}_n' &= -\sum_{i=1}^{\infty} P_{in}' \bar{C}_i', \quad \bar{E}_n' = -\sum_{i=1}^{\infty} \bar{P}_{in}' \bar{C}_i' \\ \sum_{n=1}^{\infty} Q_{ni}'(\alpha_n') \bar{A}_n' + \bar{C}_i' - \sum_{n=0}^{\infty} Q_{ni}'(\beta_n') \bar{E}_n' - \bar{C}_i' &= e_i'' \\ \sum_{n=1}^{\infty} Q_{ni}'(\alpha_n') \bar{A}_n' + \bar{C}_i' + \sum_{n=0}^{\infty} Q_{ni}'(\beta_n') \bar{E}_n' + \bar{C}_i' &= e_i' \end{aligned} \right\} \dots (13)$$

$$\left. \begin{aligned} \bar{A}_0'' &= \frac{1-2\nu_3}{1+\nu_3} e_0'', \quad \bar{C}_0'' = \frac{\nu_3}{1+\nu_3} e_0'' \quad (e_0'' = e_0''') \\ \bar{A}_n'' &= -\sum_{i=1}^{\infty} P_{in}'' \bar{C}_i'', \quad \bar{E}_n'' = -\sum_{i=1}^{\infty} \bar{P}_{in}'' \bar{C}_i'' \\ \sum_{n=1}^{\infty} Q_{ni}''(\alpha_n'') \bar{A}_n'' + \bar{C}_i'' + \sum_{n=0}^{\infty} Q_{ni}''(\beta_n'') \bar{E}_n'' + \bar{C}_i'' &= e_i''' \\ \sum_{n=1}^{\infty} Q_{ni}''(\alpha_n'') \bar{A}_n'' + \bar{C}_i'' - \sum_{n=0}^{\infty} Q_{ni}''(\beta_n'') \bar{E}_n'' - \bar{C}_i'' &= e_i'''' \end{aligned} \right\} \dots (14)$$

Finally, by substituting the stress function ϕ_1, ϕ_2 and ϕ_3 into Eq.(2), the displacements required for the remaining boundary condition Eqs.(6) and (7) are obtained as the following equation.

$$\begin{aligned} (w^{(1)})_{z_1 \pm h_1} &= \frac{1+\nu_1}{E_1} \{ \bar{A}_0 (1-2\nu_1) + 4\bar{C}_0 (1-\nu_1) \} (z_1 + h_1) + \frac{1+\nu_2}{E_2} \{ \bar{A}_0' (1-2\nu_2) \\ &\quad + 4\bar{C}_0' (1-\nu_2) \} h_2 + \frac{2(1-\nu_1^2)}{E_1} \left[\pm \sum_{i=1}^{\infty} \frac{\bar{C}_i}{\gamma_i Q_i} \operatorname{sh}^2(\gamma_i h_1) + \sum_{i=1}^{\infty} \frac{\bar{C}_i}{\gamma_i Q_i} \operatorname{ch}^2(\gamma_i h_1) \right] J_0(\gamma_i r) \dots (15) \end{aligned}$$

$$\begin{aligned} (w^{(2)})_{z_2 \pm h_2} &= \pm \frac{1+\nu_2}{E_2} \{ \bar{A}_0' (1-2\nu_2) + 4\bar{C}_0' (1-\nu_2) \} h_2 \\ &\quad + \frac{2(1-\nu_2^2)}{E_2} \left[\pm \sum_{i=1}^{\infty} \frac{\bar{C}_i'}{\gamma_i Q_i'} \operatorname{sh}^2(\gamma_i h_2) + \sum_{i=1}^{\infty} \frac{\bar{C}_i'}{\gamma_i Q_i'} \operatorname{ch}^2(\gamma_i h_2) \right] J_0(\gamma_i r) \dots (16) \end{aligned}$$

$$\begin{aligned} (w^{(3)})_{z_3 \pm h_3} &= -\frac{1+\nu_3}{E_3} \{ \bar{A}_0'' (1-2\nu_3) + 4\bar{C}_0'' (1-\nu_3) \} (z_3 - h_3) - \frac{1+\nu_2}{E_2} \{ \bar{A}_0' (1-2\nu_2) \\ &\quad + 4\bar{C}_0' (1-\nu_2) \} h_2 + \frac{2(1-\nu_3^2)}{E_3} \left[\pm \sum_{i=1}^{\infty} \frac{\bar{C}_i''}{\gamma_i Q_i''} \operatorname{sh}^2(\gamma_i h_3) + \sum_{i=1}^{\infty} \frac{\bar{C}_i''}{\gamma_i Q_i''} \operatorname{ch}^2(\gamma_i h_3) \right] J_0(\gamma_i r) \dots (17) \end{aligned}$$

Substituting the above equations into Eqs.(6) and (7), the following coefficient equations are obtained.

$$\frac{1-\nu_1^2}{E_1} \left\{ -\frac{\bar{C}_i}{Q_i} \operatorname{sh}^2(\gamma_i h_1) + \frac{\bar{C}_i}{Q_i} \operatorname{ch}^2(\gamma_i h_1) \right\} = \frac{1-\nu_2^2}{E_2} \left\{ \frac{\bar{C}_i'}{Q_i'} \operatorname{sh}^2(\gamma_i h_2) + \frac{\bar{C}_i'}{Q_i'} \operatorname{ch}^2(\gamma_i h_2) \right\} \dots (18)$$

$$\frac{1-\nu_2^2}{E_2} \left\{ -\frac{\bar{C}_i'}{Q_i'} \operatorname{sh}^2(\gamma_i h_2) + \frac{\bar{C}_i'}{Q_i'} \operatorname{ch}^2(\gamma_i h_2) \right\} = \frac{1-\nu_3^2}{E_3} \left\{ \frac{\bar{C}_i''}{Q_i''} \operatorname{sh}^2(\gamma_i h_3) + \frac{\bar{C}_i''}{Q_i''} \operatorname{ch}^2(\gamma_i h_3) \right\} \dots (19)$$

By eliminating the unknown coefficients e_i, e_i''' from Eqs.(12) ~ (14), (18) and (19), we can obtain infinite simultaneous equations with coefficients $\bar{C}_i, \bar{C}_n, \bar{C}_i', \bar{C}_i'', \bar{C}_i'''$ and $\bar{C}_i''''$. All unknown coefficients are determined by solving these equations. Accordingly, the

stresses on each boundary surface of the disks I~III are obtained by substituting the stress functions ϕ_1, ϕ_2, ϕ_3 into Eq.(1) and the displacements are given from Eqs.(15)~(17).

2.3 The case of uniformly distributed annular pressure

In order to investigate the influences of the shape and the material of laminated disks on the contact stresses and the deformations, let's consider an analytical model in the case where laminated disks are connected by multiple bolts over a circle, which is often used in the stress analysis. Assuming the loads $p_1(r)$ and $p_2(r)$ are subjected to a uniformly distributed pressure p over an annular region of the width $2a$ and the diameter of load center $2b$, the coefficients of the Bessel series e_0, e_1, e_0''' and e_1''' are taken as follows.

$$e_0=e_0'''=-\frac{4abp}{c^2}, \quad e_1=e_1'''=-\frac{2p[(b+a)J_1(\gamma_1(b+a))-(b-a)J_1(\gamma_1(b-a))]}{\lambda_1 c J_0^2(\gamma_1 c)} \quad (20)$$

From Eq.(8), we have

$$W=4\pi abp \quad (21)$$

Accordingly, by substituting Eqs.(20) and (21) into Eqs.(12)~(14) and using the procedure stated in 2.2 paragraph, this problem is solved.

3. Experimental method

In order to verify the practical usefulness of an analytical model under a uniformly distributed annular pressure, the contact stress is measured by means of a previously reported pressure-sensitive pin method⁽⁶⁾.

Fig.2 shows the dimensions of the disks I, II, III used in the experiment. The disk III has holes for attaching pressure-sensitive pins so that the contact stress may be measured at intervals of 10 mm from 20 mm to 110 mm. The disks I and III are made of steel (S45C). Their contact surfaces were lapped after grinding. The flatness of them is under $2\mu\text{m}$ and the surface roughness is $1\mu\text{m } R_{\text{max}}$. Besides, the disk II corresponding to the solid-metal flat gasket is a brass plate (BsP2) of the plate thickness 2 mm and the surface roughness $1\mu\text{m } R_{\text{max}}$. The combinations of the laminated disks used in the experiment are shown in Table 1.

The measuring method of contact stress is the same as that of the previous report⁽⁶⁾. Beforehand, the stress-strain calibration curves of ten pressure-sensitive pins attached to the disk III are drawn using a calibrative block of 45 mm cube which is made of the same material as the disk III. Next, after attaching these pressure-sensitive pins to the disk III, steel rings (material: S45C, inner diameter: 140 mm, outer diameter: 180 mm, height: 50 mm) are placed on both sides of the three laminated disks. Then, the load W is applied by means of fastening nuts. Fig.3 shows the experimental apparatus. The relation between the load and the strain of each pressure-sensitive pin is adjusted with lock nuts so that each strain may

become about 70×10^{-4} under the load of 19.6 kN, and the measurement is made by loading up to 196 kN (=20 tf) after a pre-load has been applied.

Before carrying out the experiment, the surfaces of each disk were well degreased and cleaned with acetone.

Fig.2 Dimensions of disks I, II and III

Table 1 Dimensions and material properties of disks I, II and III

	2h1	2h2	2h3	E1	E2	E3	V1	V2	V3
	mm	mm	mm	GPa	GPa	GPa			
No.1	30	2	30	206	103	206	0.3	0.33	0.3
No.2	30	2	45	206	103	206	0.3	0.33	0.3
No.3	45	2	45	206	103	206	0.3	0.33	0.3

(1GPa=102kgf/mm²)

Fastening nuts

Load cell

Disk III
Disk II
Disk I

Fig.3 Experimental apparatus

4. Comparison between the numerical results and the experimental ones and considerations

In order to compare with the experimental results, numerical calculations are carried out for various plate thicknesses and Young's moduli by putting $2a=20$ mm, $\phi 2b=160$ mm, $\phi 2c=240$ mm, $\nu_1=\nu_2=0.3$ and $\nu_3=0.33$ for the case of 2.3 paragraph. Besides, the stresses and the displacements are calculated with the Bessel series cut off at 40 terms.

Fig.4 shows the calibration curves of a pressure-sensitive pin used at $r=30$ mm in the disk III. The contact stress is defined as the load divided by the cross-sectional area (45×45 mm²) of the calibrative block. Here, the curves A & B and A correspond to the calibration result where

Fig.4 Calibration curves of a pressure-sensitive pin

Fig.5 Measured results of the strain induced in pressure-sensitive pins

the plate thicknesses of the disk III are 45 mm and 30 mm, respectively.

Fig.5 shows the measured results of each pressure-sensitive pin when the load is applied to the laminated disks under the combination No.2 in Table 1. The increment of the strain with an increasing load becomes smaller as the pressure-sensitive pin moves away from the loading position ($70 \text{ mm} \leq r \leq 90 \text{ mm}$) and the curve is nearly horizontal at $r=30$ mm. Moreover, the strain decreases with an increasing load at $r=20$ mm. Namely, the surfaces are considered to be in contact at $r=30$ mm but separate at r less than 30 mm.

The stress distribution on the contact surfaces is obtained from the relation between the measured results in Fig.5 and the calibration curves of each pressure-sensitive pin. Fig.6 indicates the experimental results of the contact stress distribution with the marks $\circ$, Δ and the calculated results with the solid lines. Though it is not likely that the practical load distribution agrees with the calculated results regarded as under a uniformly distributed pressure, both the calculated results and the experimental ones can be considered to be in a fairly good agreement except the vicinity of the loaded region ($7 \leq r/a \leq 9$). The load distribution is considered not to be influenced very much on the loading conditions as is evident from Saint-Venant's principle. But the experimental results in the loaded region are found to be larger than the calculated ones. The error of measurement seems to have been caused by the facts that the load is not always a uniformly distributed pressure as shown in Fig.3 and the vicinity of the load is liable to be affected by a decreased rigidity on the circumference of holes holding pressure-sensitive pins since the deformation is large. It is fairly difficult to measure the stress near the load exactly by means of this method.

Next, the influences of the plate thickness, Young's modulus and the load on the contact stresses are explained. Fig.6(a) shows the result when Young's moduli ratio is constant and the plate thicknesses ratio h_2/h_1 changes. It is seen that the maximum contact stress becomes larger and the estimated separation point on the contact surfaces, where the sign of $(\sigma_r)_{r=a}$ changes from negative to positive, tends to come closer to the loading position as h_2/h_1 becomes smaller. Fig.6(b) shows the case where the conditions $h_2/h_1=1/15$, $h_3/h_1=1$ do not change but Young's moduli ratio E_2/E_1 changes. It is found that the maximum contact stress becomes smaller and the estimated separation point on the contact surfaces becomes distant from the loading position as E_2/E_1 becomes smaller. Fig.6(c) shows the case where the plate thicknesses ratio, Young's moduli ratio and the size of the load width are constant and only the plate thickness changes. It is shown that the maximum contact stress becomes larger and the estimated separation point on the contact surfaces comes closer to the loading position as the plate becomes thinner. Moreover, the analytical

(a) Effect of plate thicknesses ratio

(b) Effect of Young's moduli ratio

(c) Effect of the ratio of plate thicknesses and load width

Fig.6 Stress distributions on the contact surface ($z = -h_2$)

(a) Effect of plate thicknesses ratio

(b) Effect of Young's moduli ratio

(c) Effect of the ratio of plate thicknesses and load width

Fig.7 Deformations

results in Fig.6 involve an assumption that the contacts are smooth and perfect neglecting the separation on the contact surfaces. But the separation point on the contact surfaces obtained by the experiment is in a fairly good agreement with the estimated separation point on the contact surfaces obtained by the calculation. It is found that this analytical method has a considerable practical usefulness. The broken lines in the figure represent the calculated results⁽⁴⁾ without the disk II, namely, a solid-metal flat gasket. Unlike the case where the disk II is used, the maximum contact stress is large and the estimated separation point on the contact surfaces is located near the loading position.

Fig.7 shows the calculated results of the displacement on the upper end, the contact surface ($z=-h_2$) and the lower end of the disks. Each condition in Fig.7(a)~(c) corresponds to that in Fig.6(a)~(c), respectively. In Fig.7(b) and (c), only the displacement of the disk III is shown since the deformation is symmetric because of the same thickness of disks I and III.

5. Conclusions

In order to make clear the seal efficiency in the connection of a solid-metal flat gasket inserted between two disks, a contact problem in which both sides of three laminated disks with different thicknesses and materials were subjected to axisymmetrical compressive loads was analysed assuming the contacts are smooth and perfect. Numerical calculations were carried out on the case of compression by a uniformly distributed annular pressure. It is made clear that the plate thickness, Young's modulus and the load have an effect on the contact stresses and the deformations. Moreover, the contact stress was measured by means of a pressure-sensitive pin method. Thereby, the experimental results with respect to the contact stress are found in a fairly good agreement with the calculated results except the vicinity of the loaded area. From these results, the usefulness of the analytical method is shown. Further, the analytical results are compared with a case of not inserting the solid-metal flat gasket and the seal effect of the solid-metal flat gasket is

clarified.

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