Deformation and Stress in a Cylindrical Shell Hanging on Ropes*

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In this investigation, the deformations and stresses in a cylindrical shell under its own weight which is hanging on two wire ropes, are studied by an exact method using a fundamental differential equation for a thin cylindrical shell introduced by one of the authors, K. Mizoguchi. The deformations of a cylindrical pipe without end plates are said to become large. It is therefore discussed about whether it is all right or not to suspend a shell on ropes. Furthermore the problem of a cylindrical shell lifted up at a uniform acceleration is investigated.

As a result, it is seen that the deformations of the cross section of a shell are almost invariable in the axial direction, and are different from the beam-like bending deformations.

Key Words: Structural Analysis, Stress, Deformation, Cylindrical Shell, Weight of Itself, Wire Rope

1. Introduction.

In the previous report(1), as an example of the problem of a thin circular cylindrical shell under a varying radial line load distributed around the circumference, the strength and deformation of a drum supported at both ends under line load on a rope were discussed by the authors.

In this paper, the deformations and stresses in a cylindrical shell horizontally supported on two wire ropes under the weight of the shell itself are discussed, utilizing the fundamental differential equation for a thin cylindrical shell proposed by K. Mizoguchi. As the deformations of a thin cylindrical shell without end plates subjected to the rope loading are said to become large, the authors discussed the allowable limit of such deformation. Furthermore the problem of a cylindrical shell lifted up at uniform acceleration is investigated. As a result, it is seen that the deformations of the cross section of a shell are almost invariable in the axial direction

Fig.1 Cylindrical shell hanging on ropes.

, and are different from the beam-like bending deformations.

2. Analysis.

The fundamental differential equation for a cylindrical shell has already been introduced by one of the authors and is written in the case of surface loads p, q of radial and tangential direction respectively as follows(2):

$$\begin{aligned}
 & A \frac{\partial^3 w}{\partial x^3} + \frac{4B}{a^2} \frac{\partial^3 w}{\partial x^2 \partial \theta^2} + \frac{6C}{a^4} \frac{\partial^3 w}{\partial x \partial \theta^4} + \frac{8-2\nu^2}{a^4} \frac{\partial^3 w}{\partial x^2 \partial \theta^2} + \frac{6L}{a^4} \frac{\partial^3 w}{\partial x^4} \\
 & + \left(\frac{4}{a^5} \frac{\partial^2}{\partial x^2} + \frac{1}{a^5} \frac{\partial^2}{\partial \theta^2} \right) \left(\frac{\partial^2 w}{\partial \theta^2} + 2 \frac{\partial^2 w}{\partial \theta^4} + \frac{\partial^2 w}{\partial \theta^6} \right) = \frac{1}{D} \left[A \frac{\partial^4 p}{\partial x^4} \right. \\
 & + \frac{2U}{a^2} \frac{\partial^4 p}{\partial x^2 \partial \theta^2} + \frac{B}{a^4} \frac{\partial^4 p}{\partial \theta^4} \left. \right] - \frac{1}{D} \left[\frac{(2-\nu)\epsilon^2}{6(1-\nu)} \frac{\partial^3 q}{\partial x^2 \partial \theta} - \frac{(2+\nu)}{a^2} \frac{\partial^3 q}{\partial x^2 \partial \theta} \right. \\
 & \left. + \frac{(4-3\nu+\nu^2)\epsilon^2}{12(1-\nu)a^2} \frac{\partial^3 q}{\partial x^2 \partial \theta^3} + \frac{\epsilon^2}{12a^4} \frac{\partial^3 q}{\partial \theta^3} - \frac{1}{a^4} \frac{\partial^3 q}{\partial \theta^5} \right] \dots\dots\dots (1)
 \end{aligned}$$

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Fig.2 Line loads.

where, a is radius of cylinder, h is thickness of shell, ν is Poisson's ratio, $\varepsilon = h/a$, $D = Eh^3/12(1-\nu^2)$, $[A, B, C, U] = 1 + \varepsilon^2[1/3, 1/12, (1-\nu^2)/72, (2-2\nu+\nu^2)/12(1-\nu)]$, $L = 2(1-\nu^2)A/\varepsilon^2$,

In order to simplify the problem, the following assumptions are made:

- (1) The diameter and the rigidity of each rope are negligibly small and there is no friction between the ropes and the shell. Therefore the shell is subjected to radial line loads.
- (2) The cylindrical shell is suspended at the symmetrical positions with respect to the mid-section.
- (3) The cylindrical shell is of constant thickness and made of a homogeneous, isotropic elastic material obeying Hook's law. Both ends are free from constraint.

From the assumption (3), the edge conditions at $x = \pm l$ are

$$T_x = 0, \quad M_x = 0, \quad S' - \mathcal{R}/a = 0, \quad N_x + \partial \mathcal{R}/\partial \theta = 0 \quad (2)$$

Rope reactions cause radial line loads distributed uniformly around the lower half of the shell as shown in Fig.2 and are equilibrated by the total weight of the shell.

Denote the rope reaction per unit length by $-P_0$. Then the radial surface load p_r owing to the rope reactions and the gravity loads p_w , q_w owing to shell's own weight are represented in the following forms, respectively.

$$\left. \begin{aligned} p_r = & -\frac{P_0}{2l} + \frac{2P_0}{\pi l} \sum_{n=1,3}^{\infty} \frac{(-1)^{(n-1)/2}}{n} \cos n\theta - \frac{P_0}{l} \sum_{m=1}^{\infty} \cos \frac{m\pi l_1}{l} \cos \frac{m\pi x}{l} \\ & + \frac{4P_0}{\pi l} \sum_{m=1}^{\infty} \sum_{n=1,3}^{\infty} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{m\pi l_1}{l} \cos \frac{m\pi x}{l} \cos n\theta \\ m = & 1, 2, 3, \dots, \quad n = 1, 3, 5, \dots, \quad p_w = -\rho g h \cos \theta, \quad q_w = \rho g h \sin \theta \end{aligned} \right\} \quad (3)$$

$$p = p_r + p_w, \quad q = q_w \quad (4)$$

where, ρ is the density of the shell and g is the acceleration of gravity. As the total vertical components of the rope loads should be equal to the total weight of the shell, we have the following relation:

$$P_0 = \rho g \pi l h \quad (5)$$

Substituting Eqs.(3) into Eq.(1), we have a particular solution for the rope load p_r as follows:

$$\begin{aligned} w_p = & -\frac{a^2 h^2 P_0}{24(1-\nu^2)Dl} + \frac{2a^4 B P_0}{\pi D l} \sum_{n=1,3}^{\infty} \frac{(-1)^{(n-1)/2}}{n(n^2-1)^2} \cos n\theta - \frac{a^4 P_0}{D l} \sum_{m=1}^{\infty} \frac{1}{\mu^4 + 6L/A} \cos \frac{m\pi l_1}{l} \cos \frac{m\pi x}{l} \\ & + \gamma_1 \cos \theta + \frac{4a^4 P_0}{\pi D l} \sum_{m=1}^{\infty} \sum_{n=1,3}^{\infty} \frac{\xi(\mu, n)}{H(\mu, n)} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{m\pi l_1}{l} \cos \frac{m\pi x}{l} \cos n\theta \end{aligned} \quad (6)$$

where,

$$\left. \begin{aligned} \mu = & m\pi a/l \\ H(\mu, n) = & A\mu^6 + 4B\mu^4 n^2 + \{6Cn^4 - (8-2\nu^2)n^2 + 6L\}\mu^2 + (4\mu^2 + n^2)n^2(n^2-1)^2 \\ \xi(\mu, n) = & A\mu^4 + 2U\mu^2 n^2 + Bn^4 \end{aligned} \right\} \quad (7)$$

The first term of Eq.(6) is the solution for the axisymmetric deformation with the condition $T_x = 0$, and γ_1 represents a translation as a rigid body.

The axial displacement u_p and the tangential one v_p for Eq.(6) are obtained from $u-w$, and $v-w$ relations⁽³⁾, respectively, as follows:

$$\left. \begin{aligned} u_p = & \frac{\nu a h^2 P_0 x}{24(1-\nu^2)Dl} + \frac{\nu a^4 P_0}{D l} \sum_{m=1}^{\infty} \frac{1}{\mu(\mu^4 + 6L/A)} \cos \frac{m\pi l_1}{l} \sin \frac{m\pi x}{l} \\ & - \frac{4a^4 P_0}{\pi D l} \sum_{m=1}^{\infty} \sum_{n=1,3}^{\infty} \frac{f(\mu, n)}{H(\mu, n)} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{m\pi l_1}{l} \sin \frac{m\pi x}{l} \cos n\theta \\ v_p = & \left(\frac{\rho g a^4 h \varepsilon^2}{12DB} - \gamma_1 \right) \sin \theta - \frac{2a^4 P_0}{\pi D l} \sum_{n=1,3}^{\infty} \frac{(1+\varepsilon^2 n^2/12)}{n(n^2-1)^2} \frac{(-1)^{(n-1)/2}}{n} \sin n\theta \\ & - \frac{4a^4 P_0}{\pi D l} \sum_{m=1}^{\infty} \sum_{n=1,3}^{\infty} \frac{\eta(\mu, n)}{H(\mu, n)} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{m\pi l_1}{l} \cos \frac{m\pi x}{l} \sin n\theta \end{aligned} \right\} \quad (8)$$

where,

$$\left. \begin{aligned} f(\mu, n) = & \mu[\nu A \mu^2 - (1+\nu)(2-\nu)\varepsilon^2 \mu^2 n^2/12(1-\nu) - V n^2 - (1+\nu)\varepsilon^2 n^4/12(1-\nu)] \\ \eta(\mu, n) = & n[(2-\nu)\varepsilon^2 \mu^4/6(1-\nu) + (4-3\nu+\nu^2)\varepsilon^2 \mu^2 n^2/12(1-\nu) + (2+\nu)\mu^2 + \varepsilon^2 n^4/12 + n^2] \\ V = & 1 - \nu \varepsilon^2/6(1-\nu) \end{aligned} \right\} \quad (9)$$

By substituting these displacements into the stress-displacement relations⁽⁴⁾, the stress-

Fig.3 Displacements, stress-resultants and -couple on shell element.

resultants and -couples are obtained. Some of them are given as follows:

$$\begin{aligned}
 T_{xy} &= \frac{\rho g \nu a^2 \varepsilon^2}{hB} \cos \theta - \frac{2\nu a P_0}{\pi l} \sum_{n=1,3} \frac{(-1)^{(n-1)/2}}{n(n^2-1)} \cos n\theta \\
 &\quad + \frac{48a^2 P_0}{\pi l h^2} \sum_{n=1} \sum_{n=1,3} \frac{\zeta(\mu, n)}{H(\mu, n)} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{\pi x l_1}{l} \cos \frac{\pi x}{l} \cos n\theta \\
 M_{xy} &= -\frac{\rho g \nu a^2 h \varepsilon^2}{12B} \cos \theta + \frac{a^2 P_0}{l} \sum_{n=1} \frac{\mu^2}{\mu^4 + 6L/A} \cos \frac{\pi x l_1}{l} \cos \frac{\pi x}{l} \\
 &\quad - \frac{2\nu a^2 P_0}{\pi l} \sum_{n=1,3} \frac{(-1)^{(n-1)/2}}{n(n^2-1)} \cos n\theta \\
 &\quad - \frac{4a^2 P_0}{\pi l} \sum_{n=1} \sum_{n=1,3} \frac{z(\mu, n)}{H(\mu, n)} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{\pi x l_1}{l} \cos \frac{\pi x}{l} \cos n\theta \\
 S_y' - \frac{\mathfrak{M}_y}{a} &= \frac{4a P_0}{\pi l \varepsilon^2} \sum_{n=1} \sum_{n=1,3} \frac{\delta(\mu, n)}{H(\mu, n)} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{\pi x l_1}{l} \sin \frac{\pi x}{l} \sin n\theta \\
 N_{xy} + \frac{\partial \mathfrak{M}_y}{a \partial \theta} &= -\frac{a P_0}{l} \sum_{n=1} \frac{\mu^2}{\mu^4 + 6L/A} \cos \frac{\pi x l_1}{l} \sin \frac{\pi x}{l} \\
 &\quad + \frac{4a P_0}{\pi l} \sum_{n=1} \sum_{n=1,3} \frac{\Phi(\mu, n)}{H(\mu, n)} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{\pi x l_1}{l} \sin \frac{\pi x}{l} \cos n\theta
 \end{aligned} \quad (10)$$

where,

$$\begin{aligned}
 \zeta(\mu, n) &= [(2-\nu)\mu^4 n^2 + (1-\nu)^2 \mu^2 n^4 + \{12(1-\nu^2)/\varepsilon^2 + 2\nu(1-\nu)\} \mu^2 n^2 - \nu n^6 + \nu n^4] \varepsilon^2 / 12 \\
 z(\mu, n) &= A\mu^6 + (2+\nu+\varepsilon^2/3)\mu^4 n^2 + \{1+2\nu+(1-\nu^2)\varepsilon^2/12\} \mu^2 n^4 - \nu(2+\nu)\mu^2 n^2 + n^4(n^2-1)\nu \\
 \delta(\mu, n) &= [\nu A\mu^4 + 12(1-\nu^2)A\mu^2/\varepsilon^2 - (1-\nu)(3+\nu)\{1-\nu\varepsilon^2/6(3+\nu)\} \mu^2 n^2 - (2-\nu)n^4 + (2-\nu)n^2] \mu n \varepsilon^2 \\
 \Phi(\mu, n) &= A\mu^7 + \{4-\nu+(1-\nu)\varepsilon^2/3\} \mu^5 n^2 + \{5-2\nu+(1-\nu)^2\varepsilon^2/12\} \mu^3 n^4 \\
 &\quad - (4-\nu^2)\mu^3 n^2 + (2-\nu)\mu n^4(n^2-1)
 \end{aligned} \quad (11)$$

Since the solutions expressed by Eqs.(6),(8) and (10) do not satisfy the first and second expressions of the boundary conditions (2), the following homogeneous solution is added.

$$w_0 = C_0(x^2/a) \cos \theta + \operatorname{Re} \{ a k_0 \mu_0^4 \cos(\mu_0 x/a) \} + \operatorname{Re} \sum_{n=1,3} a C_{ni} \zeta(\mu_{ni}, n) \cos(\mu_{ni} x/a) \cos n\theta \quad (12)$$

where, μ_{ni} represents the solution to the following characteristic equations(5).

$$n=0 \quad \mu_0^4 + 12(1-\nu^2)/\varepsilon^2 = 0 \quad (13)$$

$$n=1 \quad A\mu_1^4 + 4B\mu_1^2 + 12(1-\nu^2)B^2/\varepsilon^2 = 0 \quad (14)$$

$$n \geq 3 \quad H(\mu_{ni}, n) = 0 \quad (15)$$

Equation (15) may be given by the following equation with sufficient accuracy(6).

$$\begin{aligned}
 &[(1+i^*/G)\mu_{ni}^4 + 2(n^2-1+i^*G)\mu_{ni}^2 + n^2(n^2-1)] \\
 &\quad \times [(1-i^*/G)\mu_{ni}^4 + 2(n^2-1-i^*G)\mu_{ni}^2 + n^2(n^2-1)] = 0
 \end{aligned} \quad (16)$$

where, $i^* = \sqrt{-1}$, $G = \sqrt{3L/2 + (1+\nu^2)n^2/2 - 1}$

μ_{ni} represent two roots not conjugate to each other in Eq.(6) and suffix i takes values 1 and 2 corresponding to these two roots. The third term of Eq.(12) stands for the summation of the expressions corresponding to $i=1,2$. Re represents the real part of a complex function. $C_0, k_0, C_{ni}(i=1,2)$ are arbitrary constants which must be determined by the boundary conditions (2). The other displacements u_0 and v_0 for Eq.(12) and the corresponding stress-resultants and -couples are obtained in the same manner as the particular solutions. Some of them are

$$\begin{aligned}
 T_{xe} &= -(24D/h^2)C_0(1-\nu^2) \cos \theta + (12D/h^2) \operatorname{Re} \sum_{n=1,3} C_{ni} \zeta_{ni} \cos(\mu_{ni} x/a) \cos n\theta \\
 M_{xe} &= (2D/a)(1-\nu^2)C_0 \cos \theta - (D/a) \operatorname{Re} \{ k_0 \mu_0^6 \cos(\mu_0 x/a) \} \\
 &\quad - (D/a) \operatorname{Re} \sum_{n=1,3} C_{ni} z_{ni} \cos(\mu_{ni} x/a) \cos n\theta \\
 S_e' - \mathfrak{M}_e/a &= (12D/h^2) \operatorname{Re} \sum_{n=1,3} C_{ni} \delta_{ni} \sin(\mu_{ni} x/a) \sin n\theta \\
 N_{xe} + \partial \mathfrak{M}_e/a \partial \theta &= (D/a^2) \operatorname{Re} \{ k_0 \mu_0^7 \sin(\mu_0 x/a) \} + (D/a^2) \operatorname{Re} \sum_{n=1,3} C_{ni} \Phi_{ni} \sin(\mu_{ni} x/a) \cos n\theta
 \end{aligned} \quad (17)$$

where, $\zeta_{ni} = \zeta(\mu_{ni}, n)$, $z_{ni} = z(\mu_{ni}, n)$, $\delta_{ni} = \delta(\mu_{ni}, n)$, $\Phi_{ni} = \Phi(\mu_{ni}, n)$

The general solutions can be expressed as the sum of the particular solutions and the complementary ones.

By substituting these expressions for the displacements and stresses into the boundary conditions (2), the undetermined constants are determined in the following forms. For the terms of $n=0$,

$$k_0' = 2F_1 \bar{\mu}_0 \sin \bar{\mu}_0 l' / \mu_0^6 (\bar{\mu}_0 \cos \mu_0 l' \sin \bar{\mu}_0 l' - \mu_0 \sin \mu_0 l' \cos \bar{\mu}_0 l') \quad (18)$$

For the terms of $n=1$,

$$\left. \begin{aligned} C_1' &= 2F_2\bar{\Phi}_1 \sin \bar{\mu}_1 l' / (\zeta_1 \bar{\Phi}_1 \cos \mu_1 l' \sin \bar{\mu}_1 l' - \bar{\zeta}_1 \bar{\Phi}_1 \sin \mu_1 l' \cos \bar{\mu}_1 l') \\ C_0' &= \frac{24}{\pi \varepsilon^2 l'} \sum_{n=1} \frac{\zeta(\mu, 1)}{H(\mu, 1)} \cos \frac{\pi n l_1}{l} (-1)^n + \frac{\nu}{2\pi B l'} + \frac{1}{2(1-\nu^2)} \operatorname{Re} C_1' \zeta_1 \cos \mu_1 l' \end{aligned} \right\} \dots (19)$$

For the terms of $n \geq 3$,

$$\left. \begin{aligned} \operatorname{Re} C_{ni}' \zeta_{ni} \cos \mu_{ni} l' &= f_n, & \operatorname{Re} C_{ni}' x_{ni} \cos \mu_{ni} l' &= f_n \\ \operatorname{Re} C_{ni}' \delta_{ni} \sin \mu_{ni} l' &= 0, & \operatorname{Re} C_{ni}' \phi_{ni} \sin \mu_{ni} l' &= 0 \end{aligned} \right\} \dots (20)$$

where,

$$\left. \begin{aligned} l' &= l/a \\ F_1 &= \frac{6L}{Al'} \left[\frac{1}{12} \left\{ 3 \left(\frac{l_1}{a} \right)^2 - \left(\frac{l}{a} \right)^2 \right\} - \frac{6L}{A} \sum_{n=1} \frac{(-1)^n}{\mu^2 (\mu^2 + 6L/A)} \cos \frac{\pi n l_1}{l} \right] \\ F_2 &= -\frac{4(1-\nu^2)}{\pi B l'} \sum_{n=1} \frac{\{\zeta(\mu, 1) - z(\mu, 1)\}}{n H(\mu, 1)} \cos \frac{\pi n l_1}{l} (-1)^n \\ f_n &= \frac{2\nu(1-\nu^2)}{\pi l'} \frac{(-1)^{(n-1)/2}}{n(n^2-1)} - \frac{48(1-\nu^2)}{\pi \varepsilon^2 l'} \sum_{n=1} \frac{\zeta(\mu, n)}{H(\mu, n)} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{\pi n l_1}{l} (-1)^n \\ f_n &= -\frac{24(1-\nu^2)\nu}{\pi \varepsilon^2 l'} \frac{(-1)^{(n-1)/2}}{n(n^2-1)} - \frac{48(1-\nu^2)}{\pi \varepsilon^2 l'} \sum_{n=1} \frac{z(\mu, n)}{H(\mu, n)} \frac{(-1)^{(n-1)/2}}{n} \cos \frac{\pi n l_1}{l} (-1)^n \\ k_0' &= h E k_0 / P_0, \quad C_0' = h E C_0 / P_0, \quad C_1' = h E C_1 / P_0, \quad C_{ni}' = h E C_{ni} / P_0 \end{aligned} \right\} \dots (21)$$

$\bar{\mu}_0$ in Eq.(18) and $\bar{\mu}_1, \bar{\zeta}_1, \bar{\Phi}_1$ in Eq.(19) are complex conjugates of $\mu_0, \mu_1, \zeta_1, \Phi_1$, respectively. The index i in Eqs.(20) indicates summation of the expressions pertaining to $i=1, 2$ as stated before. The unknown complex constants C_{ni}' can be determined by solving the simultaneous equation (20).⁽⁷⁾ By using these unknowns, the displacements and stresses can be calculated. However, among the particular solutions for the rope reactions p_r , the convergency of the expressions of stress-resultants and -couples is not good near the rope supports. In order not only to make the calculations easy but also to give an accurate distribution of the stresses near the supports, we carried out similar transformations to ones in the previous paper.⁽⁸⁾

Fig.4 Displacements w', v' .

Fig.5 $t_x, t_\theta, m_x, m_\theta$ ($l'=3$)

3. Results.

Numerical computations are carried out for such cases that the influence of the shell weight and the rope reactions on the deformations and the stresses are supposed to be large. The dimensions of the shell used for the calculation are as follows:

$$\left. \begin{aligned} a/h=1000, \quad l'=l/a=3, 5, 10 \\ l'_1=l_1/l=0.2, 0.4, 0.6, 0.8 \end{aligned} \right\} \dots\dots\dots (22)$$

Numerical calculations were also performed for the cases of $a/h < 1000$. However, in such cases, it is seen that the maximum value of the stresses is insignificant. Therefore the numerical results are shown for the case of $a/h=1000$ only in the following dimensionless values.

$$\left. \begin{aligned} w' &= Ew/P_0, \quad v' = Ev/P_0, \quad x' = x/l \\ t_x &= T_x/P_0, \quad t_\theta = T_\theta/P_0, \quad \theta' = \theta/\pi \\ m_x &= 6M_x/hP_0, \quad m_\theta = 6M_\theta/hP_0 \\ \sigma_x' &= h\sigma_x/P_0, \quad \sigma_\theta' = h\sigma_\theta/P_0 \end{aligned} \right\} \dots\dots\dots (23)$$

3.1. Displacements w' , v' .

As shown in Fig. 4, the deformations of the cross section of the shell are almost invariable in the axial direction, and the beam-like bending deformations do not occur. In the cases of short pipe $l'=3, 5$, the deformations of the shell are almost independent of the position of the rope supporting the shell. For the case of long pipe $l'=10$,

in which the wire rope shifts towards the end of the shell, the deformations of the cross section become on the whole smaller. At the locations of the supporting wire ropes, the shell is deformed inward locally. But the local deformation is too small to show in drawings.

3.2. Stresses t_x , t_θ , m_x , m_θ , σ_x' , σ_θ' .

Membrane stresses t_x , t_θ and bending stresses m_x , m_θ are shown in Figs. 5, 6, 7.

Distribution of the axial membrane stresses t_x has a different tendency according to the position of ropes. t_x is, on the whole, smaller than the axial bending stress m_x .

Circumferential membrane stress t_θ appears at the load-applied sections and at the free ends of the shell, and disappears at any other sections.

Axial bending stress m_x is, on the whole, smaller than circumferential bending stress m_θ . In the case of long pipe, m_x may be larger than m_θ locally at the cross section where rope loads are applied.

The skin stresses σ_x' , σ_θ' can be calculated by the superposition of t_x , m_x and t_θ , m_θ , respectively, as follows:

$$\sigma_{x'out} = t_x + m_x, \quad \sigma_{\theta'out} = t_\theta + m_\theta \dots\dots\dots (24)$$

They are shown in Figs. 8, 9, 10. The maximum skin stress at the load-applied section is the circumferential skin stress $\sigma_{\theta'out}$ at

Fig. 6 t_x , t_θ , m_x , m_θ ($l'=5$)

Fig. 7 t_x , t_θ , m_x , m_θ ($l'=10$)

$\theta = 0.6\pi$ in the case of short pipe. When the cylindrical shell is long, i.e. $l'=10$, the axial skin stress σ_{xout}' becomes maximum except $l_1'=0.2$. It does not always follow from this that the peak skin stresses are dangerous for practical applications, because the high skin stresses are extremely localized. The deformations of the cross section are nearly constant in the axial direction, namely they are almost two-dimensional deformations. Therefore the largest absolute magnitude of the circumferential stresses σ_θ' near the position where the maximum value of the radial displacement w' occurs, has practical significance. If the maximum stress $\sigma_{\theta max}'$ exceeds the yielding point σ_Y of the material, permanent deformations of the cross section in the shell arise. Unless the plastic deformations are produced in the shell, considerable deformations may be tolerated, because they are likely to be harmless practically.

The problem of a cylindrical shell lifted up at a uniform acceleration $\alpha(m/s^2)$ can be easily solved. The total loads consist of the gravity loads owing to the shell's own weight and the inertia forces due to acceleration. Therefore we use the following expressions in place of Eqs.(4), (5), respectively.

$$\left. \begin{aligned} p_w &= -\rho g h (1 + \alpha/g) \cos \theta \\ q_w &= \rho g h (1 + \alpha/g) \sin \theta \end{aligned} \right\} \dots\dots\dots(4)'$$

$$P_0 = \rho g \pi l h (1 + \alpha/g) \dots\dots\dots(5)'$$

When the shell is lifted up at a uniform acceleration α , the results are obtained by multiplying those in the case of $\alpha=0$ by $(1+\alpha/g)$. Namely, the stresses are given in the following forms,

$$\left. \begin{aligned} \sigma_x &= \rho g \pi l (1 + \alpha/g) \sigma_x' \\ \sigma_\theta &= \rho g \pi l (1 + \alpha/g) \sigma_\theta' \end{aligned} \right\} \dots\dots\dots(25)$$

For example, the maximum stress in the case of

$$a=1m, \quad h=1mm, \quad l=10m$$

$$l_1=2m, \quad \rho=7.8 \times 10^3 \text{ kg/m}^3$$

is $\sigma_{\theta max} = -128.5 \text{ MPa} (-13.1 \text{ kgf/mm}^2)$

which is smaller than the yielding stress

$$\sigma_Y = 206 \text{ MPa} (21 \text{ kgf/mm}^2)$$

However, when the acceleration α is larger and the length l is greater, the maximum stress is not always smaller than the yield point.

Even in the cases of $a/h < 1000$, when the shell is abruptly lifted up, the maximum stress in the shell may exceed the yielding point of the material, resulting in a significant plastic deformation.

Fig.8 Skin stresses σ_x' , σ_θ' ($l'=3$)

Fig.9 Skin stresses σ_x' , σ_θ' ($l'=5$)

4. Conclusions.

The authors studied the stresses and displacements in a circular cylindrical shell without end plates supported by two wire ropes under the shell's own weight by using the fundamental differential equation proposed by K.Mizoguchi. The important findings are summarized as follows:

- (1) The deformations of the cross section of the shell are almost invariable in the axial direction, and differ from the beam-like bending deformations.
- (2) The maximum local skin stress at the load-applied section is $\sigma_{\theta out'}$ in the case of short pipe, and $\sigma_{x out'}$ in the case of long pipe. Except at the rope supporting position, the maximum value of the circumferential outer skin stress $\sigma_{\theta out'}$ should receive attention in connection with the deformations of the cross section of the shell.
- (3) When the shell is at rest, the strength of shell may pose a problem in the case of $a/h > 1000$, but even in the case of $a/h < 1000$, the same problem may occur when the shell is lifted up abruptly.
- (4) When the shell is lifted up at a uniform acceleration α , the results will be the product of multiplying those in the case of $\alpha=0$ by $(1+\alpha/g)$.

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Fig.10 Skin stresses $\sigma_{x'}$, $\sigma_{\theta'}$ ($l'=10$)