1. Introduction

Many experimental and theoretical studies have been made on the vortex shedding from a straight circular cylinder (two-dimensional) in a uniform stream. These studies reveal fairly well the mechanics of the formation of periodic wakes and the dependence of various parameters on the vortex shedding frequency. In practical problems, the flows are mostly nonuniform, like an atmospheric boundary-layer or a pipe flow. On the other hand, the diameter of a cylinder varies along its axis such as television towers, smoke stacks and multi-stage rockets. In such cases, the vortex shedding frequency varies along the cylinder axis and therefore it may be considered that the structure of a vortex filament becomes complicated. When two vortex streets with different frequencies are formed adjacent to a tapered cylinder, it may be expected that the local frequency at a given position along the axis differs from that of a straight cylinder with the same diameter in a uniform stream because of the interaction of vortex filaments. Gaster(1) measured the vortex shedding frequency behind a tapered cylinder in a uniform flow, and Yagita et al. (2)(3) or Chen and Mongione(4) measured it behind a straight cylinder in a shear flow. All these experiments show the same result. Both results of the tapered cylinder in a uniform flow and the straight cylinder in a shear flow indicate that their respective frequencies are lower than that of a straight cylinder in a uniform flow. In such cases, the diameter of a cylinder or flow velocity varies successively along the cylinder axis, and consequently it may be considered that the vortex filaments connect periodically with each other or vary complexly along its axis.

The purpose of this experimental study is to clarify the three-dimensional structure of a vortex street and the relation between Strouhal number and various parameters using circular cylinders with a step in a uniform flow. In these cylinders the position of the interaction of vortex filaments is restricted to the step.

Nomenclature

\( x, y, z \) : Cartesian coordinates with origin at step (see Fig.1)
\( d \) : diameter of thin cylinder
\( D \) : diameter of thick cylinder
\( d_w \) : diameter of disk
\( l \) : length of thin cylinder
\( L \) : length of thick cylinder
\( N_w, N_b \) : vortex shedding frequencies behind thin cylinder and thick one, respectively
\( u \) : friction velocity
\( U \) : velocity of uniform free stream
\( \delta \) : boundary-layer thickness of test section wall
\( Re \) : Reynolds number \( DU/\nu \)
\( St_1 \) : Strouhal number \( D N_b/U \)
\( \nu \) : kinematic viscosity

2. Experimental apparatus and method

The wind tunnel used in this experiment is the N.P.L. blow-down type with a 400mm x 400mm square working section and 3000mm in length (Fig.1). The upper wall of the working section is adjusted so that there would be no pressure gradient in the flow direction. The free stream velocity in the section is uniform within 0.2% except in the proximity of the wall. The intensity of turbulence of the free stream is about
0.25% at $U=15\text{m/s}$. The lower wall of the working section having a sharp leading edge is set at 25mm distance above the contraction nozzle wall as shown in Fig.1 to reduce the effect of the boundary-layer on the lower wall. The velocity profile of the boundary-layer at the position of a cylinder with a step agrees well with Blasius' exact solution for the laminar boundary-layer and its thickness is about 3.0mm at $U=15\text{m/s}$.

The cylinder with a step is spanned between the upper and the lower wall at a position 250mm downstream from the leading edge of the lower wall. The dimensions of the cylinders are shown in Tables 1-3. Six different diameter ratios were used, namely $d/D=0$ (finite circular cylinder), 0.2, 0.4, 0.6, 0.8 and 1.0 (straight cylinder). The length-diameter ratios $L/D$ were varied between 10 and 27 for each diameter ratio by moving the cylinder along Z-axis. Three thick cylinders, the diameters of which were 3mm, 5mm and 10mm, were used in the range of Reynolds numbers $800 < \Re < 10^4$.

The vortex shedding frequency was measured by a hot-wire anemometer of constant-temperature type having a tungsten wire of 0.005mm diameter. After examining the region in which $N_r$ becomes constant along the cylinder axis, the measuring position of $N_r$ was chosen at ten times the diameter $D$ from the step except at six times the diameter for $L/D=10$. The signal from the hot-wire was sent to a frequency meter through a bandpass filter to count the frequency. An electromagnetic oscillograph was also used to ascertain that there was no difference in waveform between the filtered and the non-filtered signal.

In order to examine the three-dimensional character of the vortices at the step, the signals representing the vortices were obtained at the same instant by six hot-wire anemometers arranged like a comb along Y or Z-axis. Hot-wire space was adjustable from 0 to 20mm.

The separation of flow and vortex formation were made visible by black ink introduced into the flow from the surface of the cylinder in a water tank having a working section 250mm high, 400mm wide and 1500mm long.

### Table 1 Dimensions of circular cylinder with a step

<table>
<thead>
<tr>
<th>$d$ (mm)</th>
<th>$D$ (mm)</th>
<th>$d/D$</th>
<th>$L/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>9.996</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>1.996</td>
<td>9.987</td>
<td>0.200</td>
</tr>
<tr>
<td>3</td>
<td>3.994</td>
<td>9.985</td>
<td>0.600</td>
</tr>
<tr>
<td>4</td>
<td>5.991</td>
<td>9.980</td>
<td>0.800</td>
</tr>
<tr>
<td>5</td>
<td>7.995</td>
<td>9.998</td>
<td>0.800</td>
</tr>
</tbody>
</table>

### Table 2 Dimension of circular cylinder with a step

<table>
<thead>
<tr>
<th>$d$ (mm)</th>
<th>$D$ (mm)</th>
<th>$d/D$</th>
<th>$L/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>4.996</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.992</td>
<td>5.022</td>
<td>0.198</td>
</tr>
<tr>
<td>3</td>
<td>1.994</td>
<td>5.036</td>
<td>0.396</td>
</tr>
<tr>
<td>4</td>
<td>2.991</td>
<td>5.035</td>
<td>0.594</td>
</tr>
<tr>
<td>5</td>
<td>3.993</td>
<td>5.038</td>
<td>0.793</td>
</tr>
</tbody>
</table>

### Table 3 Dimension of circular cylinder with a step

<table>
<thead>
<tr>
<th>$d$ (mm)</th>
<th>$D$ (mm)</th>
<th>$d/D$</th>
<th>$L/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.595</td>
<td>2.990</td>
<td>0.199</td>
</tr>
<tr>
<td>2</td>
<td>1.191</td>
<td>2.991</td>
<td>0.398</td>
</tr>
<tr>
<td>3</td>
<td>1.797</td>
<td>2.993</td>
<td>0.600</td>
</tr>
<tr>
<td>4</td>
<td>2.395</td>
<td>2.995</td>
<td>0.800</td>
</tr>
</tbody>
</table>

3. Results and Discussion

It is expected that the vortex shedding frequencies of both the thin cylinder and the thick cylinder (named cylinder d and cylinder D, respectively hereafter) are affected by various parameters. However, this paper presents only the results for cylinder D. In advance, preparatory experiments were performed and it was ascertained that $S_d$ of cylinder d was independent of various parameters and nearly equal to $S_d$ of the two-dimensional cylinder within the present experimental conditions.

3.1 Parameters to vortex shedding frequency

According to dimensional analysis, Strouhal number $S_d$ is represented as follows:

$$S_d = f(\Re, d/D, L/D, 1/d, \delta/L, u_U/u) \quad (1)$$

In this study, the cylinder with a step is set in the uniform stream and $d/L$ is much smaller than unity because of thin boundary-layer at the position of the cylinder. Furthermore, it may be considered that effects of $u_U$/$u$ and $1/d$ are assumed to be negligible as described above. Therefore, Eq.(1) is reduced to the following relation:

$$S_d = f(\Re, d/D, L/D) \quad (2)$$

3.2 Vortex shedding frequency

Figs.2, 3 and 4 show the experimental results of the vortex shedding frequency.
In these figures, the result of a straight circular cylinder is shown by a solid line and this result agrees well with that of Hoshko. According to Eq.(2), \( S_t \) is governed by \( Re, d/D \) and \( L/D \).

At first, considering the effect of \( Re \) on \( S_t \), \( S_t \) is nearly constant with \( Re \) for long cylinders \( L/D \geq 23 \) in each case of \( d/D \). But the value of \( S_t \) doesn't always agree with \( S_t \) of the straight cylinder. For \( 15 \leq L/D \leq 20 \),
L/D=5.19 and diameter ratio d/D=0.2, S_t decreases rapidly with a decreasing Re for 800 ≤ Re ≤ 2500. For 10≤ L/D ≤ 15 in each case of d/D, S_t decreases with a decreasing Re for 800 ≤ Re ≤ 5000. For instance, S_t is 0.162 for Re=2000, L/D=10 and d/D=0.4, and it is about 77% of the straight cylinder S_t. Figs. 5 and 6 show the effect of L/D on S_t for constant Re and d/D. It is seen that S_t decreases with a decreasing L/D, and becomes nearly constant for values of L/D smaller than 0.2. It is noteworthy that S_t is slightly larger than that of the straight cylinder for d/D=0.8, L/D=23 and 800 ≤ Re ≤ 5000. For L/D=15, S_t decreases considerably with a decreasing d/D in the region 800 ≤ Re ≤ 5000 and it is suspected that the difference between S_t of cylinder D and that of the straight cylinder becomes smaller for Re ≥ 5000.

Figs. 7 and 8 show the effect of L/D on S_t for constant Re and d/D. S_t decreases with a decreasing L/D when L/D is reduced to a certain value. The decreasing rate is larger for smaller values of d/D. The value of L/D at which S_t begins to decrease, is larger for smaller values of Re. The difference among the values of S_t caused by d/D and L/D becomes smaller for larger values of Re. It has been found from Ref. (6) that a periodic vortex street exists for L/D ≤ 2 and disappears for L/D > 6 in the case of d/D=0. However, it is expected that the lower limit of L/D permitting the existence of a vortex street shifts by d/D in the case of d/D > 0.

3.3 Structure of vortex street

The structure of the vortex street was examined to find out the cause of a decrease of S_t with a decreasing d/D and L/D. Figs. 9, 10 and 11 show the results of flow visualization with black ink issuing from the neighborhood of the front stagnation point of the cylinder in a water tank. The ink visualized one side of the vortex street. The upper and the lower vortex streets separate at the step of the cylinder for d/D=0.7 as shown in Fig. 9. On the other hand, these vortex streets join at the step for d/D=0.86 as shown in Figs. 10 and 11 and produce periodically a Y-shape connection of vortex filaments.

Fig. 12 shows simultaneous signals from the six hot-wire probes in the wake of the neighborhood of the step for d/D=0.9. The longitudinal solid lines in this figure, which join the peaks of signals in Z-direction, represent the connection of vortex filaments. The Y-shape connection occurs Nu/(Nu−Nw) times per second on an average on the side of cylinder D. Purthermore, the dependence of d/D and Re on the Y-shape connection was examined by this method. As the result it was found that the Y-shape connection occurs when d/D is large (d/D > 0.8) or, even if d/D=0.5, when Re is large (Re ≥ 5000).

From the signals in Fig.12, we can obtain the changes of periods and amplitudes of signals for d/D=0 (signal on the side of cylinder d) and Z/D=0 (signal on the side of cylinder D) as shown in Figs. 13 and 14. In these figures, an arrow shows a point of the time of Y-shape connection.

The period and amplitude of the vortex street on the side of the cylinder d have nearly a minimum value at the time of Y-shape connection. On the other hand, the period on the side of cylinder D has a maximum value just after Y-shape connection, and the amplitude has a minimum value just before and a maximum value just after Y-shape connection. This result can be considered to be due to the fact that when two vortex filaments of cylinder d connect like a Y-shape with a vortex filament of cylinder D, these vortices of cylinder d shed from the cylinder in an undeveloped condition of vortex formation. On the other hand, the vortex of cylinder D sheds quickly because it is entrained by the vortex of cylinder d in a shorter period. Then, the D vortex just after it has a fractionally longer period making up for the fractionally shorter previous period and sheds in a fully developed condition of vortex formation.

Fig. 9 Vortex street visualized with black ink in a water tank. The upper and the lower vortex streets separate at the step. (d/D=0.70, Re =1700)

Fig. 10 Vortex street visualized with black ink in a water tank. The upper and the lower vortex streets join at the step like a Y-shape. (d/D=0.86, Re =1700)

Fig. 11 This photograph was taken at about one second after the photograph in Fig.10.
It seems that when Y-shape connection occurs $S_t$ of cylinder D scarcely decreases in comparison with that of the straight cylinder. In other words, $S_e$ of cylinder D is smaller than that of the straight cylinder when the vortex street separates at the step for small $d/D$ as shown in Fig.9. This is due to the fact that the recovery of the back pressure of the cylinder is caused by the downwash of a free stream from the step and therefore the width of the vortex formation region increases with a forward shifting of the separation point. It can be considered that the effect of the downwash is more significant for smaller values of $L/D$ and $Re$. This is the reason why $S_t$ is smaller for smaller values of $L/D$ and the effects of $d/D$ and $L/D$ on $S_t$ are small for large $Re$.

3.4 Effect of disk at step
In a wind tunnel or water tank testing, end plates have been used for obtaining a two-dimensional flow around a circular cylinder of finite length normal to a free stream. Thus, the scale effect of the end plate on the surface pressure and drag coefficient of the cylinder has been examined. As there is a good correlation between variations of drag coefficient and $S_t$ with $Re$, it is said that if drag coefficient of a cylinder with end plate has a two-dimensional value, $S_t$ may also have one. Concerning the effect of the end plate on vortex shedding frequency, however, we cannot find any papers. It is thought that the end plate is useful for us to consider the flow around the step, although these plates are usually used to remove the end effects of the cylinder.
In this section, the interaction between the upper and the lower flows of the step is discussed for various diameters $D_4$ of thin disks mounted concentrically at the step. The measurements of vortex shedding frequency were performed for $L/D=10$ and 19 of $d/D=0.6$ and for $L/D=10$ of $d/D=0$, and the disk diameters $D_4$ were varied between 20 and 60mm. The thickness of the disk was 1mm in all cases and the disk had a sharp edge.

Fig.15 shows the results of $D_4/D=2$, 4, and 6. In the case of $L/D=19$ for $d/D=0.6$, $S_t$ is nearly constant with $Re$ and $D_4/D$ for the range of $D_4/D=0.1 \sim 4$ For $D_4/D=5$ and 6, $S_t$ increases for $D_4/D$, but it is smaller than that of the straight cylinder. In the case of $L/D=10$ for $d/D=0.8$, $S_t$ increases for $D_4/D$, which increases from 0 to 4, $S_t$ increases for a large value of $Re$. For $Re=10^6$ of $D_4/D=4$, $S_t$ of $L/D=10$ agrees with that of $L/D=15$. As $D_4/D$ increases from 5 to 6, the value of $Re$, for which $S_t$ is independent of $L/D$, shifts from a large value to a small one. As mentioned in section 3.2, the effect of $L/D$ on $S_t$ is larger for smaller $Re$. It is found from the result that the effect of $L/D$ disappears from a larger value of $Re$ with an increasing $D_4/D$. In other words, the disk blocks the downwash of a free stream when $D_4/D$ is large. Also, it seems that the effect of the scale of disk diameter is associated with the scale of vortex formation region behind the cylinder.

Comparing $S_t$ of $d/D=0.6$ with that of $d/D=0$ (disk is set up at the free end of the cylinder) to examine the effect of $D_4$, $S_t$ of $d/D=0$ increases rapidly with $D_4/D$, and they agree well for $D_4/D=4$ within the present experimental range of $Re$. Therefore, if $D_4/D=4$, there is no effect of $D_4/D$ on $S_t$.