Possibility of Various Airfoil Shape Modes and Their Steady State Stability of Single Membrane Sailwing.*

By Hitoshi MURAI** and Shigenao MARUYAMA***

The sailwing has various special characteristics due to its flexible structure. In order to estimate a variety of airfoil shape modes and steady state stability of the single membrane sailwing as a representative of the lift characteristics of a sailwing, a systematic method of analysis is presented in three different trailing edge conditions. Through calculation examples, various modes of airfoil shapes and the steady state stability of each mode of the sailwing for each trailing edge condition are shown. The relationship between these characteristics and parameters of sailwing taking account of elasticities such as initial tension is also demonstrated. These characteristics are related to an abrupt change in airfoil shape and $C_L=0$ curves of the single membrane sailwings.

Key Words: Fluid Machine Element, Numerical Analysis, Sailwing Stability

1. Introduction

A sailwing generally consists of upper and lower surfaces, and a rigid leading edge spar which has curvature and thickness, as have been shown by Sweeney(1). The authors have shown that the pressure distribution on the double membrane sailwing is different from that on a single membrane sailwing(2). However, an analysis on the sailwing taking account of elasticity and deflection of the trailing edge(3) has shown that the total characteristics of double membrane sailwing such as $C_L=0$ relation agree with those of the single membrane sailwing.

As for two-dimensional analysis on single membrane sailwings or sails, Thwaites(4), Nielsen(5), and Tuck et al.(6) have done a linearized analysis, where in Thwaites and Tuck et al. have shown that a lot of airfoil shape modes can exist in one slacksness and incidence (possibility of airfoil shape modes). These various modes of airfoil shapes or the solution when the membrane tension is small can not be obtained by the numerical iteration method(2).

In order to estimate the various modes of airfoil shapes and their steady state stability of the sailwing, the present report develops a two-dimensional analysis method of single membrane sailwing under different trailing edge conditions, in which the membrane tension constant, membrane slacksness constant, and the elasticities of trailing edge wire and membrane are considered systematically. Through calculation examples, various modes of airfoil shapes and steady state stability of each mode of the sailwing for each trailing edge condition are shown.

2. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_n$</td>
<td>coefficient of airfoil slope series</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>basic chord length</td>
<td></td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift coefficient</td>
<td></td>
</tr>
<tr>
<td>$C_M$</td>
<td>moment coefficient around the leading edge</td>
<td></td>
</tr>
<tr>
<td>$C_P$</td>
<td>pressure coefficient</td>
<td></td>
</tr>
<tr>
<td>$C_T$</td>
<td>tension coefficient of membrane</td>
<td></td>
</tr>
<tr>
<td>$C_{T_0}$</td>
<td>initial tension coefficient</td>
<td></td>
</tr>
<tr>
<td>$F_m$</td>
<td>nondimensional spring constant of the membrane</td>
<td></td>
</tr>
<tr>
<td>$F_e$</td>
<td>nondimensional spring constant of the trailing edge wire</td>
<td></td>
</tr>
<tr>
<td>$P_c$</td>
<td>static pressures on wing surface, in freestream</td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>chord wise tension per unit span</td>
<td></td>
</tr>
<tr>
<td>$T_0$</td>
<td>initial tension per unit span</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>nondimensional length of the spring at unloaded position</td>
<td></td>
</tr>
<tr>
<td>$U_c$</td>
<td>freestream velocity</td>
<td></td>
</tr>
<tr>
<td>$x,y$</td>
<td>airfoil coordinates</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>attack angle</td>
<td></td>
</tr>
<tr>
<td>$\Delta C_P$</td>
<td>pressure difference coefficient</td>
<td></td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>nondimensional excess length of membrane</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
<td></td>
</tr>
<tr>
<td>$x,t$</td>
<td>nondimensional airfoil coordinates</td>
<td></td>
</tr>
<tr>
<td>$u_l$</td>
<td>upper surface, lower surface</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>trailing edge</td>
<td></td>
</tr>
</tbody>
</table>

3. Two-dimensional Sailwing Theory

In order to simplify the problem of the single membrane sailwing theory, the following assumptions are made in the analysis of a meridional cross-section of the
sailwing,
(i) Flow across the section is two-dimensional, inviscid, and incompressible.
(ii) Slope of the membrane is small.
(iii) The membrane has no porosity.
(iv) The rigidity of the membrane against the bending moment can be neglected. Namely the airload of the membrane balances only with the chordwise tension of the membrane.

As the slope of the membrane is small from the assumption, the relation between the membrane tension and the pressure difference on the lower and upper surfaces is expressed as follows:

\[ T \frac{d^2 \eta}{dx^2} = -(p_i - p_u) \]  

Equation (1) is non-dimensionalized using a basic chord length and dynamic pressure as follows:

\[ C_T \frac{d \eta}{d^2} = -(C_m - C_m) = -\Delta C_p \]  

By transforming \( \xi \)-coordinate using Eq. (3), and expressing the membrane slope as Eq. (4), \( \Delta C_p \) can be expressed as Eq. (5) from thin airfoil theory (9):

\[ \xi = \frac{T}{2} (1 - \cos \theta) \quad (0 < \theta < \pi) \]  

\[ \frac{d \xi}{d \theta} = \frac{B_0}{\Delta C_p} \cos \theta \]  

\[ \Delta C_p = 4 \left( \frac{(a - B_0) \cot \frac{\theta}{2} + \sum B_n \sin \theta} \right) \]  

Substituting the relationships of Eqs. (4) and (5) into Eq. (2), the following equation can be obtained.

\[ -C_T \frac{\sum B_n \sin \theta}{\Delta C_p} \frac{d \xi}{d \theta} \frac{1}{2} \left( a - B_0 \right) \cot \frac{\theta}{2} + \sum B_n \sin \theta = 0 \]  

When Eq. (4) is integrated along \( \xi \), the coordinate of the trailing edge can be obtained as follows:

\[ \eta = \frac{\xi}{\Delta C_p} \left( B_0 + \frac{\sum B_n \cos \theta}{\Delta C_p} \sin \theta \right) \]  

\[ = \frac{\xi}{\Delta C_p} \left( B_0 - \frac{1}{\Delta C_p} \int \frac{d \xi}{\Delta C_p} \right) \]  

In order to solve the above boundary value problem, two-dimensional models of single membrane sailwing as is shown in Fig. 1 are considered. The problem is solved under three different trailing edge conditions as follows:

1. \( \xi \) is unknown with given \( \Delta C_p \) and the fixed trailing edge at (1,0) (Fig. 1-a).
2. \( C_T \) is unknown with given \( \xi \) and the fixed trailing edge at (1,0) (Fig. 1-a).
3. As is shown in Fig. 1-b, the trailing edge is connected with a spring which has non-dimensional spring constant \( C_T \) and whose length is \( \xi \) when unloaded. The membrane is subjected to initial tension \( C_{pi} \), and has no slackness when unloaded. If the membrane elongates with an increase of the tension, non-dimensional spring constant of the membrane is \( K_T \). The leading edge is fixed at (0,0) in every trailing edge condition.

The solution under the trailing edge condition (1) can be obtained by solving Eqs. (6) and (7) provided that the trailing edge is at (1,0). The membrane slackness \( \xi \) is obtained as follows under the condition that the slope of the membrane is small.

\[ \xi = \frac{1}{\Delta C_p} \int \frac{d \xi}{\Delta C_p} \int \left( \frac{1}{\Delta C_p} \frac{d \xi}{\Delta C_p} \right) \sin \theta d \theta \]  

This trailing edge condition is the same as the ones by Thwaites (4), and Nielsen (5). As the problem is linear, this condition is convenient to analyse the characteristics of the sailwing in an equilibrium state. However the condition is not enough to analyse the actual sailwing characteristics because the membrane tension changes according to the airload.

The problem under the trailing edge condition (2) is to solve Eqs. (6), (7), and (8), and to obtain \( \Delta C_p \) and \( B_n \) (n=1,2,3,...) with given \( \xi \), \( \alpha \), and fixed trailing edge coordinates (1,0). This problem has the same trailing edge condition as the one of the authors' numerical analysis of a double membrane sailwing (2). Because the problem becomes non-linear, more than one solution can exist with one set of given parameters.

Under the trailing edge condition (3), the elongation of the non-dimensional spring connected with one end at (1+t_1,0) and the membrane elongation balance each other with an increase of the tension. The trailing edge coordinates and the membrane slackness are defined as follows:

\[ 1 - \xi = \frac{(\xi_c - \xi)}{K_T} \]  

(a) Trailing edge conditions (1), (2)

(b) Trailing edge condition (3)

Fig. 1 Two-dimensional single membrane sailwing.
\[ \varepsilon = C_T - C_L \]  \hspace{1cm} (10) \]

As the length of the membrane is l when unloaded, the change in the membrane length becomes as follows in the same manner as Eq. (8);

\[ \begin{align*}
1 - \delta t + \varepsilon & = \frac{1}{4} \int_{\alpha}^{\beta} \left( B_K \right) \left( \frac{1}{n+1} \right) \sin \theta d\theta \\
+ \sum_{i=1}^{n} B_K \cos n\theta \sin \theta d\theta & \hspace{1cm} (11)
\end{align*} \]

Substituting Eqs. (9) and (10) into Eq. (11), the following equation is obtained.

\[ \begin{align*}
C_T - C_L & = \frac{K_L}{K_K + K_{\alpha}} \left( \frac{1}{4} \int_{\alpha}^{\beta} \left( B_K \right) \left( \frac{1}{n+1} \right) \sin \theta d\theta \right) \\
+ \sum_{i=1}^{n} B_K \cos n\theta \sin \theta d\theta & \hspace{1cm} (12)
\end{align*} \]

When only the deflection of the trailing edge is considered, Eq. (12) becomes Eq. (13). The relationship between the spring constants \( K_L \) and \( K_K \) and the reduced spring constant \( K_{\alpha} \) neglecting \( K_{\alpha} \) is expressed as Eq. (14).

\[ \begin{align*}
C_T - C_L & = \frac{K_L}{K_K + K_{\alpha}} \left( \frac{1}{4} \int_{\alpha}^{\beta} \left( B_K \right) \left( \frac{1}{n+1} \right) \sin \theta d\theta \right) \\
+ \sum_{i=1}^{n} B_K \cos n\theta \sin \theta d\theta & \hspace{1cm} (13)
\end{align*} \]

\[ \begin{align*}
C_T & = \frac{K_L}{K_K + K_{\alpha}} \\
& \hspace{1cm} (14)
\end{align*} \]

Hence only \( K_L \) is considered in the following analysis. \( \eta \)-coordinate of the trailing edge is defined using \( \xi_L \) and \( d\eta/d\xi \) as follows:

\[ \eta = -(1 - \delta t + t) \frac{d\eta}{d\xi} \]  \hspace{1cm} (15)

Since \( d\eta/d\xi \) is given from Eq. (4) for \( \delta = \pi \), the following equation is obtained by eliminating \( \eta \) from Eqs. (15) and (7).

\[ \begin{align*}
(1 + t) & B_K + \sum_{i=1}^{n} B_K \left( \frac{\xi_{1}}{n+1} \right) \left( \frac{0}{n} \right) \\
+ (1 - \delta t + t)(-1)^{j} & = 0 \hspace{1cm} (16)
\end{align*} \]

This boundary value problem is to solve Eqs. (6), (9), (13), and (16), and to obtain \( B_K \) \((n=1, 2, \ldots)\) and \( C_L \) for given \( C_{T0} \), \( K_T \), \( t_L \), and \( \alpha \). The trailing edge condition is the same as the one of the authors' numerical analysis for double membrane sailing taking account of elasticity. From this condition, the characteristics of the sailing whose camber changes with the airfoil shape can be evaluated.

When the coefficient \( B_K \) \((n=0, 1, 2, \ldots)\) is solved under the condition (1), (2), or (3), the membrane coordinates and pressure distribution of the airfoil are expressed as follows:

\[ \eta = \frac{x}{2} \int_{\alpha}^{\beta} \left( B_K + \sum_{i=1}^{n} B_K \cos n\theta \sin \theta d\theta \right) \]
The boundary value problem is to solve
Eqs. (24), (25) and N th approximation of Eq.
(8) for \( \xi_L \) and obtain \( B_i \) (i=1,2,3...N) and
\( C_T \) as unknowns. The problem is to solve
the following N+2 simultaneous equations
(24), (25), and (26) as follows:
\[
\frac{\xi_i}{4} \sum_{j=0}^{N} \left( B_j + \frac{1}{A_j} B_j \cos \vartheta_j \right) \sin \vartheta_j = 0
\]
\[
\begin{align*}
\cdots & \cdots (26) \\
\end{align*}
\]
Newton-Raphson method \(^{(10)}\) is adopted in
order to solve the nonlinear simultaneous
equations.

4.3 Trailing edge condition
taking account of elasticity

Since elasticity of the membrane can
be substituted by the reduced trailing edge
spring constant \( K_T \) in Eq.(14), only the
trailing edge elasticity is considered in
the following analysis. In this trailing edge
condition, \( C_T \), \( B_i \) (i=0,1,2,...N) and \( \xi_L \)
are unknowns. The problem is to solve
Eqs. (24), (9) and the following equations
(27) (28) from Eqs.(16) and (13).
\[
\begin{align*}
(1+i)B_0 + \sum_{j=1}^{N} \left[ C_j + (1-\xi_L + i)(-1)^j \right] B_j &= 0 \\
\begin{align*}
\frac{K_T \xi_L}{4} & \sum_{j=1}^{N} \left[ B_j + \frac{1}{A_j} B_j \cos \vartheta_j \right] \sin \vartheta_j \\
&+ C_T - C_R = 0
\end{align*} \cdots \cdots (28)
\end{align*}
\]
Upon solving these equations, Eqs. (24),
(27), (28), and Eq.(9) for \( \xi_L \) are carried
out separately. The final solution is ob-
tained by iteration of both calculations
because the change in \( \xi_L \) is small. Newton-
Raphson method used for trailing edge condition
(2) is adopted to solve this non-
linear system.

For the first approximation of the
nonlinear simultaneous equations under
the trailing edge conditions (2) and (3), linear
equations (24) and (27) are solved for
assumed \( C_T \) and \( \xi_L \), and \( B_j \) solved for
the first approximation.

5. Comparison of the Present Results with Previous ana-
lysis.

For numerical calculation, Eqs.(24)-(28)
are approximated by N-term series.
Thwaites\(^{(4)}\), Nielsen\(^{(3)}\), and Truck et al.\(^{(6)}\)
approximated 20, 18, and 30 term series
respectively in a linear analysis similar to
the one of the trailing edge condition (1).
Since the main aim of the present report is
to analyze the characteristics of \( C_L \),
\( C_M \), \( C_T \), and \( C_r \) or \( C_m \) is influenced by the first
2 or 3 terms respectively, \( N=10 \) is adopted
in the following calculation.

The difference in \( C_L (a=4^\circ) \) between
10 terms and 30 terms is 0.009% under
the trailing edge condition (1), 0.05% under
the trailing edge condition (2) for \( c=1\),
and 0.03% under the condition (3) for
\( C_{TM}=2, \xi_L=0.1, K_T=50 \).

The present calculation results using
the thin airfoil theory are compared with
the authors' previous numerical iteration
method using a panel method for the
pressure distribution. \( C_T \) relations for given
\( \varepsilon \) compared with the authors' numerical
method\(^{(2)}\) are shown in Fig.2. The dotted
lines in the figure are the linear solutions
for \( C_T=const. \), trailing edge condi-
tion (1). The present result for \( \varepsilon=4^\circ \)
shows some deviation from the numerical re-
results because the maximum camber is about
15%. However, the present result for \( \varepsilon=1^\circ \)
shows good agreement with the one of the
numerical result, as the maximum camber is
6%.

\( C_L \) and \( C_T \) relations for the trailing
edge condition (3) is compared with the
authors' numerical method taking account
of elasticity\(^{(3)}\) in Fig.3. The present re-
results show relatively good agreement with
the numerical iteration method for small
camber. When the camber becomes large with
an increase of the incidence or a decrease of
\( K_T \), the present result deviates from the
one of the numerical analysis. Also Fig.3
shows that the larger \( K_T \) is, the smaller
the change in \( C_L \) is, and the larger the

\[ \begin{align*}
\text{Fig. 2 Comparison of } C_L (a) \text{ with numerical analysis}^{(2)} \text{ when } \varepsilon \text{ is constant.}
\end{align*} \]

\[ \begin{align*}
\text{Fig. 3 Comparison of } C_L (a) \text{ and } C_T (a) \text{ with numerical analysis}^{(3)} \text{ when elasticity of trailing edge is consi-
} \end{align*} \]
change in $C_T$ becomes.


Under the trailing edge condition (1), the number of the solutions of the sailwing is one for given $C_T$, but more than one solution can exist under the trailing edge condition (2) or (3) for given parameter because the problem is a non-linear system. Figure 4 shows the change in $C_{L}/\alpha$, $C_{M}/\alpha$, and $\epsilon/\alpha^2$ versus $C_T$ under the condition (1). These values are divided at $C_T=1.727$, $C_{T2}=0.556$, $C_{T3}=0.463$, $C_{T4}=0.293$, $\cdots$. The determinant of the coefficient matrix, Eqs. (24) and (25) becomes zero at $C_T=1, 2, 3 \cdots$ and the solution is indeterminate. When we call the solution between $C_{Tn}$ and $C_{Tn+1}$ the $n$-th mode solution, the first to 4th mode solutions are shown in Fig.4.

In the first mode solution, $C_{L}/\alpha$, $C_{M}/\alpha$, and $\epsilon/\alpha^2$ are positive. Each value approaches $C_{L}/\alpha=3\pi$, $C_{M}/\alpha=\pi/2$, $\epsilon/\alpha^2=0$ asymptotically when $C_T=1$, and the characteristics become those of a rigid plate. In even mode solutions, $C_{L}/\alpha$, $C_{M}/\alpha$ and $\epsilon/\alpha^2$ are negative. There is a point where $C_{L}/\alpha=0$ in the mode. At the position, the airfoil shape and pressure distribution become symmetric around $\xi=1/2$. The ratio of $C_{L}/\alpha$ to $C_{M}/\alpha$ is the center of the load $P_L$ and $P_T$ becomes 1/4 at $C_T=1$, 1/2 at $C_T=1.727$, and 1 at a point where $C_{L}/\alpha$ line and $C_{M}/\alpha$ line intersect in every mode. The membrane slope at the leading edge is zero when $C_T=1$.

$\epsilon/\alpha^2$ curve in Fig.4 has a minimum value in each mode except in the first mode. If we call the minimum value in the $n$-th mode $(\epsilon/\alpha^2)_{\min}$, each value is $(\epsilon/\alpha^2)_{\min}=1.015$ ($C_T=0.916$), $(\epsilon/\alpha^2)_{\min}=0.507$ ($C_T=0.727$), $(\epsilon/\alpha^2)_{\min}=0.364$ ($C_T=0.727$). These values agree with Thwaites' analysis (4) of $(\epsilon/\alpha^2)_{\min}=1.020$, and $(\epsilon/\alpha^2)_{\min}=3.25$. When we estimate the number of solutions under the condition (2), if $\epsilon/\alpha^2<(\epsilon/\alpha^2)_{\min}$, only one solution exists; and if $\epsilon/\alpha^2<(\epsilon/\alpha^2)_{\min}$, three solutions. Such various airfoil shape modes have been reported by Thwaites and Tuck et al. But their stability has not been discussed by them.

Figure 5 shows $C_L(\alpha)$ characteristics and change in airfoil shape of the sailwing for $\epsilon=1/4$ when incidence is reduced from positive value. The $C_L(\alpha)$ characteristics show that the solution with positive camber can not exist for $\alpha<5.7^\circ$ and the camber reverses abruptly at $\alpha=5.7^\circ$, then $C_L$ becomes negative with negative camber as shown in airfoil shapes 3 and 4. Also the change in airfoil shape shows that the point of maximum camber shifts backward when $\alpha$ becomes small and it shifts forward abruptly after the camber reverses.

In an actual sailwing, the membrane tension and the elasticities of the membrane and the trailing edge balance each other as is shown in Fig.1-b. If actual slackness (1-$C_T$) is substituted by $\epsilon$ and $C_T=1$ provided the membrane slope is small, the following equation is obtained from Eq. (9).

$$\epsilon/\alpha^2=(C_T-C_{\alpha})/K\alpha^2$$

As is shown in Fig.6, the solution under the condition (3) is an intersection of straight line Eq. (29) and $\epsilon/\alpha^2$ curve expressed in Fig.5. If the membrane slackness changes $\Delta\alpha$ from balanced position, the increase in the membrane tension from elasticities $\Delta t\alpha$ is expressed from Eq. (9) as follows.

![Fig. 4 Change of $C_L/\alpha$, $C_M/\alpha$, and $\epsilon/\alpha^2$ with $C_T$](image)

![Fig. 5 Change of airfoil shape and corresponding $C_L(\alpha)$ when is constant](image)

![Fig. 6 Steady state stability model of sailwing with various trailing edge conditions](image)
\[ \frac{dC_T}{d\alpha} = K_\alpha \frac{de}{d\alpha} \]  

(30)

If the membrane slackness changes \( \Delta \alpha \), the increase in the tension from airload \( \Delta T_a \) is expressed from the slope of the \( \epsilon/\alpha^2 \) curve in Fig. 6 as follows provided the incidence does not change.

\[ \frac{dC_T}{d\alpha} = \frac{dC_L}{d\alpha} \frac{d\alpha}{d\alpha} \]  

(31)

If the change in \( C_L \) from the elasticity \( \Delta C_L \) is larger than the change in \( C_T \) from the airload \( \Delta C_T \), the shape of the membrane recovers the balanced position. Hence the stability against the small change in the slackness can be expressed by the following equation.

\[ \Delta C_T < \Delta C_L \]  

(32)

Equation (32) is expressed from Eqs. (30) and (31) as follows:

\[ d(\epsilon/\alpha^2)/dC_T < 0 \quad \text{or} \quad d(\epsilon/\alpha^2)/dC_T > \frac{1}{K_\alpha \alpha} \]  

(33)

If more than three solutions exist as shown in Fig. 6, a statically unstable solution can exist among them.

Under the trailing edge condition (1), the solution is an intersection of the \( \epsilon/\alpha^2 \) curve and \( C_T = \text{const.} \) line as shown in Fig. 6. Considering \( K_\alpha = 0 \) in Eq. (33) in this case, the solution is stable for \( d(\epsilon/\alpha^2)/dC_T < 0 \) and unstable for \( d(\epsilon/\alpha^2)/dC_T > 0 \). Hence the solutions expressed as \( C_T = \text{const.} \) line in Fig. 2 are unstable if \( C_T \) comes between 1.727 and 0.916.

Under the trailing edge condition (2), the solution is an intersection of the \( \epsilon/\alpha^2 \) curve and \( C_T = \text{const.} \) line as shown in Fig. 6. Since this condition is considered to be \( K_\alpha = \text{const.} \) in Eq. (30), the solution at \( (\epsilon/\alpha^2) \) min. is neutral, and the others are statically stable. However, as the solution for \( (\epsilon/\alpha^2) \) min. \( \epsilon/\alpha^2 \) can only exist in the first mode, an abrupt change in the airfoil shape occurs as shown in Fig. 5.

Figure 7 indicates the number of solutions and their steady state stability under the trailing edge condition (3) on the \( C_T = K_\alpha \alpha^2 \) plane. This figure shows that the number of solutions increases as \( C_T \) or \( K_\alpha \alpha^2 \) decreases, and only one solution can exist if \( C_T > 1.727 \) or \( K_\alpha \alpha^2 < 0.922 \). Also it can be seen that there is a region where an unsteady solution exists between the regions of one stable solution and three stable solutions.

Figure 8 shows \( C_L(\alpha) \) characteristics and change in airfoil shape of the sailwing under the condition (3) for \( K = 100 \), \( C_T = 1 \), \( t = 0.1 \). In the figure, stable solutions
are indicated because $C_{T0}=0.916$ from the result of Fig.7. When incidence is decreased in the airfoil shapes from 1 to 4, the camber decreases and the maximum camber point shifts backward like the one in Fig. 5. The camber changes abruptly at $a=1.7^\circ$. A similar change can be observed in the airfoil shapes 5 to 8, and $C_L$ characteristics show hysteresis against the incidence. Two stable solutions exist for $|a|<1.7$ in the case of the figure.

The region in Fig.7 is indicated on the $C_L(a)$ curves for $K_t=50$, $t=0$, and $C_{T0}$ as a parameter in Fig.9. $C_L(a)$ curves in the figure are stable solutions for $C_{T0}=0.916$ with positive camber. If $C_{T0}$ changes from 0 to $C_{T0}$, $C_L(a)$ comes between $C_{T0}=0$ curve and a rigid wing line indicated as dotted dash line. When $C_{T0}$ is increased, the region of one stable solution becomes large. $C_L(a)$ for $C_{T0}=1.727$ has only one solution and the curve is continuous at $a=0$ without hysteresis. A solution outside the left critical line in the figure does not exist, and if $C_{T0}$ is less than 1.727, the camber changes abruptly at the intersection of the critical line and $C_L(a)$ curve as shown in Fig.8. The critical line is not a straight line as shown in Fig.2 for the constant slowness. If $C_{T0}$ is large, the camber reverses at small $a$ and large $C_L$. In the region of two stable and one unstable solutions, all the unstable solutions are the one presented in the figure and a reversed solution which is symmetric with respect to $(0,0)$. For $1.727>C_{T0}>0.916$, only those two solutions can exist for all over $a$.

Change in the region where more than one solution exist is shown in $C_L-a$ plane for different $K_t$ in Fig.10. $C_L(a)$ curve of each $C_{T0}$ comes between the rigid wing line and $C_{T0}=0$ curve for each $K_t$. The figure shows that the larger $K_t$ is the smaller the region of more than one solution. Only one stable solution exists for all $K_t$ in the region between the two dotted dash line and the rigid wing line in the figure.

When Eqs. (24) and (27) are solved for given $C_T$ and $t_i>0$, the boundary value problem is to solve the linear equations under the trailing edge condition given by Tuck et al. (6). Figure 11 shows $c/a^2-C_T$ relations of up to the third mode solutions for different $t_i$ as a parameter. If $t_i$ is increased within 0.3, $(c_2/a^2)_{\text{min}}$ decreases. Figure 12 shows the region of more than one solution in $C_T-K_t-a^2$ plane from Fig.11 and Eq.(29) for different $t_i$ as a parameter, in similar manner to Fig.7. The figure indicates that the larger $t_i$ is the larger the one stable solution region becomes.

7. Conclusions

Two dimensional analysis method of single membrane sail wing is developed sys-

![Fig. 9 Number of possible solutions and their stability in $C_L-a$ plane with elastic trailing edge](image)

![Fig. 10 Change of one stable solution region with $K_t$.](image)

![Fig. 11 Change of $c/a^2-C_T$ relation with $t_i$.](image)

![Fig. 12 Change of one stable solution region with $t_i$.](image)
tematically using Fourier series under the trailing edge conditions of membrane tension constant, membrane slackness constant, and elasticities of the trailing edge wire and membrane are considered. Through the analysis of the theory and numerical calculation examples, various airfoil shape modes and their steady state stability are discussed. The results are summarized as follows:

1. Under the membrane tension constant condition, only one solution exists for one incidence. Among them, the solutions for $0.916 < C_T < 1.727$ are statically unstable.

2. For the membrane slackness constant problem, a stable solution with positive camber can exist in negative incidence but the camber changes abruptly at $C_L = 0.516$. The point of maximum camber shifts backward when incidence is reduced.

3. When the elasticities of the trailing edge are considered, there is one possible solution for $C_{\infty} > 1.727$ or $K_s > 0.922$. The number of solutions or airfoil modes increases when $C_{\infty}$ or $K_s$ decreases. If $C_{\infty} < 1.727$, there is a point where the camber of the airfoil changes abruptly.

4. If the length of the spring increases up to $t_1 = 0.3$, the region of more than one solution decreases.

References