On the Temperature Rise Caused by Moving Heat Sources*

(1st Report, Calculation of the Temperature
under Three-dimensional Heat Flow)By Yoshio TERAUCHI**, Hiromasa NADANO***
and Masaki KOHNO****

To investigate surface failure by scoring or pitting, it is necessary to exactly evaluate the temperature distributions at the surface and in the contacting bodies. In this report, the temperature rise caused by the moving heat sources having various distributions of heat intensity was calculated under the assumption of a three-dimensional heat flow. Further, the degree of the interference between the surface temperature rises when two heat sources move with a constant velocity was examined.

From the calculated results it is shown that a considerable degree of the interference between the surface temperature rises caused by the respective heat sources is recognized, and that the surface temperature rise has to be evaluated by means of the three-dimensional heat flow since the effect of the width of the heat source on the surface temperature rise is remarkable.

Key Words : Friction, Three-dimensional Heat Flow, Moving Heat Source, Temperature Rise, Thermal Interference

1. Introduction

Machine elements are currently run under high loads and at high speeds, and a failure of the rubbing surfaces is a significant factor to be examined. It is also considered that the temperature of the contacting surfaces in such a working process as friction welding significantly affects the welding strength of the contacting surfaces. Besides, these problems are similar to the problems in cutting and case-hardening of materials by means of laser beam, and then it is an important problem for increasing the accuracy in the machining process to estimate the temperature distribution in the vicinity of the heat sources. Therefore, it is necessary to exactly evaluate the temperature distributions at the surface and in the contacting bodies.

There are a great number of investigations on the temperature rises on and below the surface when the heat source moves with a constant velocity on the surface⁽¹⁾. One of the authors has investigated the temperature rises on and below

the surface under a two-dimensional heat flow⁽²⁾. However, since the calculation of the temperature rise under the three-dimensional heat flow is complicated, the investigations with respect to this problem are few. Francis has derived an equation of the surface temperature rise caused by an ellipsoidally distributed heat source⁽³⁾.

In this report, the authors derived an equation of the temperature rise caused by the moving heat sources of which the distributions of heat intensity are assumed to be parabolic and uniform over the areas of a circle and a rectangle. Further, the degree of the interference between the surface temperature rises when two heat sources move with a constant velocity was examined. Therefore, it is considered that the calculating results are applicable to the analysis of the surface temperature when the contacting bodies with surface roughness are rubbed.

2. Nomenclature

- 2a : Width of heat source in the direction of the y -axis m
 c : Specific heat J/(kg K)
 K : Thermal conductivity W/(m K)
 l : Distance between the centers of two heat sources m
 q_0 : Maximum value of heat intensity generated per unit area and unit time W/m²
 $2R$: Diameter or width of heat source m
 t : Time s
 V : Moving velocity of heat source m/s
 θ : Temperature K
 κ : Thermal diffusivity m²/s
 ρ : Density kg/m³

* Received 5th April, 1982.

** Professor, Faculty of Engineering, Hiroshima University, Shitami, Saijocho, Higashi-Hiroshima-shi, Hiroshima.

*** Professor, Kure Technical College, 2-11, Aga-Minami 2-chome, Kure-shi, Hiroshima.

**** Assistant, Kure Technical College, 2-11, Aga-Minami 2-chome, Kure-shi, Hiroshima.

Dimensionless parameters

$$\begin{aligned}
L &= VR/(2\kappa) \\
T &= \pi KV\theta/(\kappa q_0) \\
X &= Vx/(2\kappa) \\
Y &= Vy/(2\kappa) \\
Z &= Vz/(2\kappa) \\
r &= Vr'/(2\kappa L) \\
u &= V(x-x')/(2\kappa L) \\
v &= V(y-y')/(2\kappa L) \\
\xi &= x/R=X/L \\
\eta &= y/R=Y/L \\
\zeta &= z/R=Z/L
\end{aligned}$$

3. Equation of Temperature Rise

The temperatures on and below the surface when a heat source generated by friction moves with a constant velocity on the surface of the contacting bodies, such as gear tooth and roller, will be evaluated. As the radii of curvature of the contacting bodies are very large compared with that of the conjunctural area, the contacting bodies can be assumed to be semi-infinite solids. Here, the temperature rises on and below the surface which are caused by the moving heat sources of which the distributions of heat intensity are assumed to be parabolic or uniform over the areas of a circle and a rectangle will be calculated under a three-dimensional heat flow. The initial and the boundary conditions are given as follows:

(1) The movement of the heat source steadily continues for an infinitely long time, and the initial temperature of the body is assumed to be zero.

(2) There is no heat loss from the surface of the body.

(3) Thermal conductivity, thermal diffusivity, specific heat and density are assumed to be constant against the variation in the temperature.

At the surface of the semi-infinite solid, the x -axis is in the direction of

Fig.1 Moving heat source and coordinates (semi-parabolic cylindrically distributed heat source)

the movement of the heat source, the y -axis crosses the x -axis at right angles and the z -axis is in the direction of the depth from the surface. Now, it is supposed that an instantaneous point heat source with heat quantity Q occurs at a point $(x', y', 0)$ on the surface at the time $t=0$. In this case, the differential equation of heat conduction is given by,

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} = \frac{1}{\kappa} \frac{\partial \theta}{\partial t} \quad (1)$$

As for the solution of Eq.(1) which satisfies the conditions (1)~(3), the temperature rise θ at a point (x, y, z) at a time t due to the heat quantity Q is expressed as follows:

$$\begin{aligned}
\theta &= \frac{Q}{4(\pi\kappa)^{3/2} \rho c t^{3/2}} \\
&\times \exp\left\{-\frac{(x-x')^2 + (y-y')^2 + z^2}{4\kappa t}\right\} \quad (2)
\end{aligned}$$

Therefore, when a heat source of which the distribution of heat intensity is expressed by $q(x', y')$ moves with the velocity V as shown in Fig.1, the temperature rise θ in the steady state at the point (x, y, z) becomes as follows:

$$\begin{aligned}
\theta &= \frac{1}{4(\pi\kappa)^{3/2} \rho c} \iint_D \frac{q(x', y')}{t^{3/2}} \exp\left\{-\frac{(x-x'+Vt)^2 + (y-y')^2 + z^2}{4\kappa t}\right\} dt dx' dy' \\
&= \frac{1}{8\pi^{3/2} \kappa^2 \rho c} \iint_D q(x', y') \exp\left\{-\frac{V(x-x')}{2\kappa}\right\} \int_0^\infty S^{-3/2} \exp\left(-S - \frac{A^2}{4S}\right) dS dx' dy' \quad (3)
\end{aligned}$$

where D stands for the region of the heat source, $S=V^2 t/(4\kappa)$ and $A=V\sqrt{(x-x')^2 + (y-y')^2 + z^2}/(2\kappa)$. Now,

$$\int_0^\infty S^{-3/2} \exp\left(-S - \frac{A^2}{4S}\right) dS = \frac{2\sqrt{2} K_{1/2}(A)}{\sqrt{A}} = \frac{2\sqrt{\pi}}{A} \exp(-A) \quad (4)$$

where $K_{1/2}(A)$ stands for a modified Bessel's function of second kind of order 1/2. Therefore, Eq.(3) can be rewritten as follows:

$$\theta = \frac{V}{4\pi\kappa^2 \rho c} \iint_D \frac{q(x', y') \exp\left[-\frac{V}{2\kappa}(\sqrt{(x-x')^2 + (y-y')^2 + z^2} + x-x')\right]}{\frac{V}{2\kappa}\sqrt{(x-x')^2 + (y-y')^2 + z^2}} dx' dy' \quad (5)$$

When the distribution of heat intensity is assumed to be parabolic over the area of a rectangle with the width $2R$ in the direction of the movement of the heat source and with the length $2a$ in the direction of the y -axis, the distribution of heat intensity is given as,

$$q(x', y') = q_0 \left(1 - \frac{x'^2}{R^2}\right) \quad (-R \leq x' \leq R, -a \leq y' \leq a) \quad (6)$$

Therefore, substituting Eq.(6) into Eq.(5), the equation of the temperature rise caused by a semi-parabolic cylindrically distributed heat source is expressed in terms of the dimensionless parameters as follows:

$$\frac{\pi K V}{\kappa q_0} \theta = L \int_{-a/R}^{a/R} \int_{-1}^{+1} \frac{(1-\xi^2+2\xi u-u^2) \exp\{-L(\sqrt{u^2+v^2+\xi^2}+u)\}}{\sqrt{u^2+v^2+\xi^2}} du dv \quad (7)$$

Similarly, for the case of a rectangular heat source of uniform intensity,

$$q(x', y') = q_0 \quad (-R \leq x' \leq R, -a \leq y' \leq a) \quad (8)$$

In this case, the equation of the temperature rise is given as,

$$\frac{\pi K V}{\kappa q_0} \theta = L \int_{-a/R}^{a/R} \int_{-1}^{+1} \frac{\exp\{-L(\sqrt{u^2+v^2+\xi^2}+u)\}}{\sqrt{u^2+v^2+\xi^2}} du dv \quad (9)$$

In contrast, when the distribution of heat intensity is assumed to be parabolic over a circle of radius R , the distribution of heat intensity is expressed by using cylindrical coordinates as follows:

$$q(r', \varphi) = q_0 \left(1 - \frac{r'^2}{R^2}\right) \quad (0 \leq r' \leq R, 0 \leq \varphi \leq 2\pi) \quad (10)$$

Therefore, the equation of the temperature rise caused by a paraboloidally distributed heat source is given as,

$$\frac{\pi K V}{\kappa q_0} \theta = L \int_0^{2\pi} \int_0^1 \frac{r(1-r^2) \exp\{-L[\sqrt{(\xi-r\cos\varphi)^2+(\eta-r\sin\varphi)^2+\xi^2}+\xi-r\cos\varphi]\}}{\sqrt{(\xi-r\cos\varphi)^2+(\eta-r\sin\varphi)^2+\xi^2}} dr d\varphi \quad (11)$$

Similarly, for the case of a circular heat source of uniform intensity,

$$q(r', \varphi) = q_0 \quad (0 \leq r' \leq R, 0 \leq \varphi \leq 2\pi) \quad (12)$$

In this case, the equation of the temperature rise is given as,

$$\frac{\pi K V}{\kappa q_0} \theta = L \int_0^{2\pi} \int_0^1 \frac{r \exp\{-L[\sqrt{(\xi-r\cos\varphi)^2+(\eta-r\sin\varphi)^2+\xi^2}+\xi-r\cos\varphi]\}}{\sqrt{(\xi-r\cos\varphi)^2+(\eta-r\sin\varphi)^2+\xi^2}} dr d\varphi \quad (13)$$

Since these equations of the temperature rises can not be analytically calculated, the calculation is carried out by numerical integration. Since the integrand is not infinite, the numerical analysis of the integrand can be carried out by the method of the double finite integral (Gauss's integration). In this case, to increase the accuracy of the calculation, the number of divisions of the interval

for the integral is increased within the region of the heat source.

4. Calculated Results

4.1 Temperature rises caused by the semi-parabolic cylindrically distributed heat source and the rectangularly distributed heat source
Figures 2 (a)~(f) show the typical

Fig. 2 Relation between T and X/L (semi-parabolic cylindrically distributed heat source)

calculated results of the temperature rises at the surface ($Z/L=0$) and in the subsurface ($Z/L=0.25$) when the heat source of which the distribution of heat intensity is semi-parabolic over a square with the sides $2R$ moves with a lower velocity $L=0.1$, $L=1$ and a higher velocity $L=10$. In these figures, the ordinate indicates the temperature rise in terms of the dimensionless parameter T , and the abscissa indicates the position in the direction of the movement of the heat source in terms of the dimensionless parameter X/L . In this case, the center of the heat source moves from the negative portion in the direction of the x -axis to the origin of the coordinates. Similarly, the dimensionless parameter Y/L stands for the position in the direction normal to the moving direction of the heat source, and the dimensionless parameter Z/L stands for the position in the direction of the depth from the surface.

From these figures it can be seen that the surface temperature rise at the boundaries of the heat source tends to be lower than that at the center of the heat source, and the position generating the maximum value of the temperature rise moves toward rear of the heat source, according as the velocity of the movement of the heat source increases. It is also revealed that the difference between the temperature rises at the surface and in the subsurface increases with an increasing velocity of the movement of the heat source. In the case of $L=10$, particularly, the maximum value of the temperature rise

in the subsurface occurs outside behind the heat source.

Figures 3 (a) and (b) show an isothermal line of the surface temperature rises caused by the semi-parabolic cylindrically distributed heat source. In these figures, the broken line indicates the boundary of the heat source. The temperature rise outside the region of the heat source in the case of $L=0.1$ is more significant than that in the case of $L=1$.

For example, the temperature rises at the surface ($Z/L=0$) and in the subsurface

(a)

(b)

Fig.3 Isothermal line of the surface temperature rise

(a)

(a)

(a)

(b)

(b)

(b)

Fig.4 Relation between T and X/L (rectangularly distributed heat source)

Fig.5 Relation between T and X/L (paraboloidally distributed heat source)

Fig.6 Relation between T and X/L (cylindrically distributed heat source)

($Z/L=0.25$), when the heat source of which the distribution of heat intensity is uniform over a square with the sides $2R$ moves with the velocity $L=1$, are shown in Figs.4 (a) and (b). It can be seen that the distribution of the temperature rise caused by the rectangularly distributed heat source considerably differs from that caused by the semi-parabolic cylindrically distributed heat source. Further, the maximum value of the temperature rise caused by the rectangularly distributed heat source occurs in the vicinity of the boundary of the heat source, and it is higher than that caused by the semi-parabolic cylindrically distributed heat source.

4.2 Temperature rises caused by the paraboloidally distributed heat source and the cylindrically distributed heat source

The temperature rises on and below the surface when the heat source of which the distribution of heat intensity is parabolic over a circle with the radius R moves with the velocity $L=1$ are shown in Figs.5 (a) and (b). Similarly, Figs.6 (a) and (b) show the temperature rises on and below the surface which is caused by the heat source of which the distribution of heat intensity is uniform over a circle with the radius R in the case of $L=1$. From these figures it can be seen that the temperature rise significantly differs depending on the shape of the distribution of heat intensity. The maximum value of the temperature rise caused by the paraboloidally distributed heat source occurs just behind the center of the heat source. In contrast, the maximum value of the temperature rise caused by the cylindrically distributed heat source occurs in the vicinity of the boundary of the heat source.

4.3 Temperature rise when the sum of the heat quantities from various kinds of the distributed heat sources is assumed to be equal

Fig.7 Relation between $T_{\max}$ and Y/L when the sum of the heat quantities for the distributed heat sources is assumed to be equal

For example, the relation between the dimensionless parameter $T_{\max}$ of the maximum value of the surface temperature rise and Y/L when the sum of the heat quantities from the above-mentioned distributed heat sources is assumed to be equal is shown in Fig.7 in the case of $L=1$. Here, the regions of the paraboloidally and the cylindrically distributed heat sources are assumed to be expressed by a circle with the radius R . On the other hand, the regions of the semi-parabolic cylindrically and rectangularly distributed heat sources are assumed to be expressed by a square with the sides $2R$. Therefore, the maximum values of heat intensities are given by $q_0' = 3\pi q_0/16$ in the case of the semi-parabolic cylindrically distributed heat source, $q_0'' = q_0/2$ in the case of the cylindrically distributed heat source and $q_0''' = \pi q_0/8$ in the case of the rectangularly distributed heat source, where q_0 indicates the maximum value of heat intensity in the case of the paraboloidally distributed heat source.

From this figure it is revealed that the maximum value of the temperature rise caused by the paraboloidally distributed heat source is larger than those caused by other distributed heat sources at the center of the heat source. However, the maximum value of the temperature rise in the case of the paraboloidally distributed heat source significantly decreases with an increasing Y/L , and then it is the lowest at the boundary of the heat source.

4.4 Effect of the width of the heat source on the surface temperature rise

In the case of the semi-parabolic cylindrically distributed heat source, the relation between the maximum value of the temperature rise $T_{\max}$ occurring at the center ($Y/L=0$) of the heat source and the dimensionless parameter a/R (the width of the heat source in the direction of the y -axis) is shown in Fig.8. In this case, the maximum value of heat intensity q_0 is assumed to be constant regardless of the value of a/R .

From this figure it can be seen that the maximum values of the surface temperature rise increase with an increasing

Fig.8 Relation between $T_{\max}$ and a/R

width of the heat source. In the case of $\alpha/R \rightarrow \infty$, these values agree with the values calculated under the two-dimensional heat flow, that is, there is no heat flow in the direction at right angles to the moving direction of the heat source. When the heat source moves with a comparatively high velocity $L=10$, the effect of the width of the heat source on the surface temperature rise is very small. In contrast, when the heat source moves with a low velocity $L=0.1$, the effect of the width of the heat source on the surface temperature rise is very large. Therefore, in the case of $\alpha/R < 10$, it is necessary to exactly calculate the temperature rise under the three-dimensional heat flow.

4.5 Interference between the surface temperature rises caused by two heat sources

On the actual rubbing surfaces, there are many microscopic heat sources (real contacting areas) in an apparent contacting area. Therefore, it can be considered that the temperature rises due to the heat sources generated at the respective real contacting areas affect each other. Here, when two heat sources of which the shape of the distribution of heat intensity is identical move with the same velocity, the degree of the interference between the temperature rises caused by the respective heat sources is examined. With regard to the shape of the distribution of heat intensity, the temperature rise due to the rectangularly distributed heat source of which the degree of the interference between the temperature rises is significant will be mainly calculated.

For comparison, Fig.9 shows the relation between T and X/L , when the rectangularly distributed heat sources in which the distance between the centers of the two heat sources is l in the direction of the movement of the heat sources move with the velocity $L=1$. In this case, the values of T are calculated by superimposing the values of the temperature rise caused by the respective heat sources. The center of one heat source is located at the origin of the coordinates, and that of the other heat source is located at a distance ($X/L > 0$) of l/R from the origin. $l/R=2$ indicates the condition that the boundary of

Fig.9 Relation between T and X/L

one heat source adjoins to that of the other heat source. On the other hand, $l/R > 2$ indicates the condition that the boundary of one heat source is separated from that of the other heat source. In the case of $l/R = \infty$, there is no interference between the temperature rises caused by the respective heat sources, and then the temperature rise coincides with the calculated value of the surface temperature rise caused by a single heat source.

From this figure it can be seen that the maximum value of the temperature rise in the case of $l/R=2$ is approximately 1.3 times that in the case of $l/R=\infty$, and a significant interference between the temperature rises caused by the respective heat sources is recognized. However, the degree of the interference between the temperature rises decreases with an increasing value of l/R . In the case of $l/R > 2$, the maximum value of the temperature rise occurs at two positions along the x -axis, and then the maximum value of the temperature rise occurring behind the origin is higher than that occurring at the front of the origin.

Similarly, Fig.10 shows the relation between T and Y/L when the rectangularly distributed heat sources in which the distance between the centers of the two heat sources is l in the direction at right angles to the moving direction of the heat source move in parallel along the x -axis with the velocity $L=1$. The center of one heat source is located at the origin of the coordinates, and that of the other heat source is located at a distance ($Y/L > 0$) of l/R from the origin. When comparison is made of the degree of the interference between the surface temperature rises caused by the respective heat sources when the two heat sources are located in the direction of the x -axis and those caused by the respective heat sources when the two heat sources are located in the direction of the y -axis, the latter is smaller than the former. In the case of $l/R > 2.5$, particularly, little degree of the interference is recognized between the surface temperature rises caused by the respective heat sources located in the direction of the y -axis.

Next, Fig.11 shows the relation between the maximum value N of the degree of

Fig.10 Relation between T and Y/L

Fig.11 Relation between N and L/R

the interference between the surface temperature rises caused by the respective heat sources (the value of N indicates the ratio of the maximum value of the surface temperature rise in the case of the variable L/R to that in the case of $L/R=\infty$) and L/R , when the two heat sources move with the same velocity L . In this figure, the solid lines indicate the values when the distance between the centers of the two rectangularly distributed heat sources is L along the x -axis, and the one-dotted chain lines indicate the values when the distance between the centers of the two rectangularly distributed heat sources is L along the y -axis. Further, the broken lines indicate the values when the distance between the centers of the two paraboloidally distributed heat sources is L along the x -axis. Here, the value of N expressed by the one-dotted chain lines indicates the maximum value of the degree of the interference between the temperature rises generated on the y -axis ($X/L=0$).

From this figure it can be seen that when the two heat sources are located in the direction of the x -axis, the value of N increases with a decreasing distance between the centers of the heat sources and with an increasing velocity of the movement of the heat sources. Particularly, a significant interference between the temperature rises due to the rectangularly distributed heat sources is observed when the heat sources move with a comparatively high velocity $L=10$, and then the value of N in the case of $L/R=2$ becomes about 1.4. In the case of $L/R<2.3$, the value of N in the case of $L=0.1$ is slightly larger than that in the case of $L=1$. Therefore, it is seen that the effect of the velocity of the movement of the heat source on the degree of the interference between the surface temperature rises is rather complicated. In the case of $L/R\leq 3$, the position at which the degree of the interference between the temperature rises is maximum

occurs at $X/L>0$ in the case of $L=0.1$. In contrast, it occurs at $X/L<0$ in the case of $L\geq 1$.

The value of N due to the paraboloidally distributed heat source, of which the degree of the interference between the temperature rises is the smallest of the four kinds of the distributed heat sources mentioned above, is smaller than that due to the rectangularly distributed heat source. However, when the value of L/R is small, a significant interference is recognized between the temperature rises caused by the paraboloidally distributed heat sources. On the other hand, when the value of N due to the two rectangularly distributed heat sources located in the direction of the y -axis is compared with that due to same heat sources located in the direction of the x -axis, the former significantly decreases with an increasing distance between the centers of the heat source, and increases with a decreasing velocity of the movement of the heat source. Therefore, the value of N due to the latter shows a reversed tendency to the above-cited facts.

5. Conclusions

An equation for the temperature rise caused by moving heat sources of which the distributions of heat intensity are assumed to be parabolic and uniform on the areas of a circle and a rectangle is derived under the assumption of a three-dimensional heat flow, and the temperature rise is evaluated for various kinds of the calculating conditions. The calculated results are summarized as follows:

(1) When the sum of the heat quantities distributed over the heat source is assumed to be equal, the maximum value of the temperature rise caused by a paraboloidally distributed heat source is larger than those caused by other types of the distributed heat sources at the center of the heat source. However, it is the smallest at the boundaries of the heat source.

(2) A significant effect of the width of the heat source on the maximum value of the surface temperature rise is observed in the case of a low velocity $L=0.1$. Therefore, in the case of $a/R<10$, it is necessary to exactly calculate the temperature rise under the three-dimensional heat flow.

(3) When two rectangularly distributed heat sources are located in the direction of the movement of the heat source, the maximum value of the degree of the interference between the surface temperature rises caused by the respective heat sources increases with an increasing velocity of the movement of the heat source. In the case of $L=10$, the maximum value in the case of $L/R=2$ becomes about 1.4. On the other hand, when the two heat sources are located in the direction at right angles to the moving direction of the heat sources, the maximum value of the degree of the interference increases with a decreasing velocity of the movement of the heat source, and decreases with an increasing distance between the centers of the two

heat sources.

References

(1) Carslaw, H.S. and Jaeger, J.C., Conduction of Heat in Solids, (1959), p.266,

Oxford Univ. Press.

(2) Terauchi, Y and Hamamoto, T., Jour. Japan Soc. Lub. Engrs. (in Japanese), Vol. 15, No. 3 (1970), p. 133.

(3) Francis, H.A., ASLE Trans., Vol. 14, No. 1 (1971), p. 41.