On Tooth Form Accuracy of Roll-finished Involute Helical Gears*

2nd Report; Influence of Addendum Modification Factor

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This paper deals with tooth form accuracy of roll-finished involute helical gears with addendum modification. The calculated and measured results indicate that a difference in flexural rigidity between the die-wheel tooth and the work-gear tooth induces a pressure angle error on the roll-finished tooth form: when the tooth rigidity of die-wheel is too large, a positive pressure angle error is induced; on the contrary, when the tooth rigidity of die-wheel is too small, a negative pressure angle error is induced. It is concluded, therefore, that the difference in flexural rigidity between the die-wheel tooth and the work-gear tooth must be kept minimum, in order to improve the tooth form quality of profile shifted helical gears made by roll-finishing.

Key Words: Gear, Roll-finishing, Addendum Modification Factor, Pressure Angle Error

1. Introduction

The complicate status in tooth contact condition of helical gears and the difficulty of calculation for load distributions on the tooth flanks make many investigations complex concerning the tooth form accuracy of helical gears produced by roll-finishing. It is, therefore, difficult to decide the optimum tooth form of die-wheel and the optimum working conditions for the roll-finishing operation. In order to improve such a status of roll-finishing of involute helical gears, the authors have developed a computer simulation program of roll-finishing of helical gears in the previous report⁽¹⁾, which can well reproduce the roll-finishing process of helical gears. In this report, in the case of roll-finishing of profile shifted helical gears, the influence of addendum modification factor of die-wheel and of work-gear on the tooth form accuracy is qualitatively discussed by use of this computer simulation program and many experimental data. Some fundamental conditions, especially, the influence of mating combination in addendum modification factors of die-wheel and work-gear to improve the quality of roll-finished helical gears are investigated.

2. Generation Mechanism of Tooth Form Error Caused by Addendum Modification

In order to make clear the difference between the nature of tooth form error induced on a profile shifted gear and that on gears without addendum modification, a computer simulation of the roll-finishing of helical gears was carried out. In this simulation, the distributed load on tooth flank is calculated under the conditions that the component of total forces on all meshing tooth flanks in pressing direction (i.e. load applying direction) of die-wheel is equal to the magnitude of the applied load for roll-finishing and the torque on work-gear owing to the distributed loads on tooth flanks is zero, when both tooth flanks i.e. driven and followed flanks of die-wheel and work-gear are meshed and pressed. And it was assumed that the roll-finishing of the tooth flank of a work-gear proceeds only with plastic deformation of the work-gear owing to indentation of die-tooth flanks under this distributed load on tooth flank calculated.⁽¹⁾

Figures 1, 2 and 3 show the transverse tooth forms of work-gears with addendum modification factors of $x_w = -0.5$, 0 and $+0.5$ respectively and which were roll-finished by die-wheel without addendum modification ($x_d = 0$) under a roll-finished load of 15kN (1kN/mm per unit face width). Figures (a) indicate calculated transverse tooth forms and figures (b) indicate measured transverse tooth forms. Among the calculated tooth forms, S1, S2 and S3 show the transverse tooth form at a distance of 3.2mm from leading side end of meshing, at the center of face width, and at a distance of 3.2mm from trailing side end of meshing respectively. The specifications of die-wheels and work-gears are shown in Table 1.

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Fig.1 Transverse tooth forms of roll-finished gears with negative addendum modification of $x_w = -0.5$

Fig.2 Transverse tooth forms of roll-finished gears without addendum modification of $x_w = 0$

Fig.3 Transverse tooth forms of roll-finished gears with positive addendum modification of $x_w = +0.5$

Table 1 Specifications of die-wheel and work-gear

	DIE	WORK
tooth form	normal	
module	2 mm	
press. angle	20	
no. of teeth	80	34
prof. mod. factor	-1, 0, +1	-1, -0.5, 0, +0.5, +1
helix angle	23° LH	23° RH
face width	20 mm	20 mm
material	SKS 3	S45C
flank hardness	HRC=60	HRC=12
quality	JIS MO-M1	(hobbed)

As seen in Figs. 1, 2 and 3, the most remarkable influence of the addendum modification factor on the tooth form accuracy of roll-finished gears is that a considerably large positive pressure angle error is induced when the addendum modification factor x_w of work-gear is negative. When x_w is zero, almost no pressure angle error is induced, and when x_w is positive, a negative pressure angle error is induced, although the amount of the pressure angle error in this case is less than that in the case when x_w is negative.

Following presumptions are laid down for explaining of this phenomenon:

(1) In the case of gears without addendum modification⁽¹⁾, it is observed that the magnitude of concave shape of tooth form which has valley near pitch point becomes larger when the applied load for roll-fining is larger than the optimum value.

But the tooth forms shown in Figs. 1 and 3 do not indicate much of the sign of concave tooth form, which indicate the applied load for roll-finishing does not differ much from the optimum loading value. The value of applied load for roll-finishing cannot be taken, therefore, for the cause of the pressure angle error.

(2) In spite of the different contact condition in sliding and rolling directions on the driven and followed tooth flanks of work-gear, almost same pressure angle error is induced on both tooth flanks. The plastic flow of the tooth flank material of work-gear in the direction tangential to tooth flank cannot, therefore, be the cause of the pressure angle error. In this computer simulation, the plastic flow of material of work-gear tooth flank in the tangential direction is not considered and the roll-finishing is assumed to proceed only with plastic deformation in the normal direction, which is brought about by the indentation of die-tooth flank. The calculated tooth forms show almost the same pattern of pressure angle error as observed in the tooth form obtained by the experiments. This fact indicates also that the plastic flow of material in direction tangential to tooth flank is not a main cause of the pressure angle error.

(3) Neglecting the influence of sliding between tooth flanks, the influence of addendum modification factor on the calculated tooth forms of roll-finished gears is reflected in the influence factor, that is the compliance of tooth for the bending and shearing deflection and the contact deformation at the meshing tooth flanks.

These facts indicate that the difference in the tooth rigidity of die-wheel and work-gear must be the main cause of pressure angle error. When die-wheel and work-gear are in meshing as shown in Fig. 4, the positions near tooth tip and tooth root of work-gear are indented by die-teeth. When the tooth rigidity of work-gear is smaller than that of die-wheel, bending deformation of work-gear teeth is large, but the amount of bending of work-tooth at the point a in Fig. 4 is less than the amount at the point c , because the work-gear tooth at the point a is also loaded on the back tooth flank at

the point b . Therefore the amount of contact approach at the point a is larger than at the point c . The plastic deformation at the position near tooth tip is, therefore, larger than at the position near tooth root, and then a positive pressure angle error is induced. Contrary to this, when the tooth rigidity of work-gear is greater than that of die-wheel, the amount of bending of die-tooth at the point c becomes less than that at the point a , and the negative pressure angle error is induced. Particularly when the addendum modification factor of work-gear is negative, the decrease in the tooth rigidity is even larger than in the other case, and therefore, a very large amount of positive pressure angle error is induced.

3. Change in Tooth Form Due to the Combination of Addendum Modification Factors

In order to reveal the problems concerning the tooth form accuracy induced by roll-finishing of profile shifted helical gears, experiments of roll-finishing with die-wheels and work-gears of various combinations of addendum modification factors were carried out. The conditions of the experiments covered various combinations of different rigidities of die-wheel and of work-gear, and the tooth form error and pressure angle error induced by roll-finishing were investigated. The specifications of die-wheels and work-gears used in this study are shown in Table 1. Die-wheels with JIS quality in the range of MO-M1 and with addendum modification factors of $x_d = -1, 0$ and $+1$ were used. Work-gears with addendum modification factors of $x_w = -1, -0.5, 0, +0.5$ and $+1$ were used. The gears were cut by hobs with and without protuberance on their cutting tooth tip. Height of the scaffolding in protuberances of the hobs amounts to 0.075 or 0.10 mm. Tooth rigidity of work-gears cut by a hob with protuberance seems to decrease by about 8%. A roll-finishing machine⁽²⁾ with two die-wheels was operated under the conditions applying of a constant load. The revolution speed of die-wheel was 25 r.p.m. and the roll-finishing time was 5 sec.

Figure 5 shows typical examples of roll-finished tooth forms of work-gears, which were cut by a conventional hob with $x_w = -1$. The die-wheel used for this roll-finishing operation has three values of addendum modification factor. Numbers in the figure mean the total amplitude of tooth form error curves, which contain the tooth form error and the pressure angle error. This total amplitude is called in this paper the "tooth form error" in broad sense. The notation $\blacktriangle$ indicates that there was a strong interference between the tooth tip of die-wheel and the root and bottom of work-gear tooth during roll-finishing operation. The notation $\triangle$ indicates that there was an interference but it was not so strong as in the former case. This

Fig. 4 Meshing of die-wheel and work-gear (transverse section)

order of interference was judged by the noise which was heard during operation of roll-finishing. An increase in roll-finishing load, of course, causes an increase of such interference, and in the case of $x_d=-1$ and $x_w=-1$ especially the tip of die-wheel collided only with root and bottom of work-gear, and roll-finishing operation did not proceed at all. In the cases of $x_d=0$ and $+1$, some signs of interference, a dull-rounding down of tooth form curve at tooth tip and pressure angle error were also observed. An improvement in tooth form accuracy could not therefore be achieved.

Figure 6 shows the roll-finished tooth forms of work-gears cut by a hob with protuberance. The tooth form of the work-gear has a recess at tooth root, which acts to prevent the interference. When roll-finishing was operated on this work-gear with $x_w=-1$ by a die-wheel with $x_d=-1$, a fairly high quality of tooth form was obtained, because there occurred no interference between tooth tip of die-wheel and tooth root and bottom of work-gear, and moreover the difference in tooth ri-

gidity was small. On the contrary, in the case of roll-finishing by a die-wheel with $x_d=0$ and $+1$ on the same work-gear with $x_w=-1$, the difference in tooth rigidity of the die-wheel and the work-gear adversely worked, and the roll-finished tooth form showed a large pressure angle error. In this case, the tooth form accuracy rather decreased to a great extent in the roll-finishing operation. Even when the addendum modification factor of work-gear decreased a little to $x_w=-0.5$, an improvement of tooth form accuracy was not either observed due to the meshing interference when the work-gears were ones cut by conventional hob. In the case of work-gears cut by a hob with protuberance, however, an improvement of roll-finished tooth form was expected: Fig.7 shows the roll-finished tooth forms of such work-gears with $x_w=-0.5$. Roll-finishing by die-wheel with $x_d=-1$ brings about some improvement of tooth form accuracy, though a little negative pressure angle error is observed, which is induced by the greater tooth rigidity of work-gear than that of die-wheel. In the case of $x_d=0$ and $+1$, on the

Fig.5 Prehobbed and roll-finished tooth forms of gears with $x_w=-1$

Fig.6 Prehobbed and roll-finished tooth forms of gears with $x_w=-1$
(given undercut)

contrary, a positive pressure angle error is induced because of greater tooth rigidity of die-wheel than that of work-gear. This cannot be said definitely, because the experiments of roll-finishing were not carried out on die-wheel with $x_d = -0.5$ and work-gear with $x_w = -0.5$, but as general experiences in this investigation show when the difference between addendum modification factors of the die-wheel and the work-gear is greater than about 0.5, an improvement in the tooth form accuracy is almost impossible: a considerably large amount of pressure angle error will be induced by roll-finishing operation.

Figures 8 and 9 show the roll-finished tooth forms of work-gears without addendum modification ($x_w = 0$) which were cut by hobs with and without protuberance respectively. In both cases, the pressure angle error is hardly observed on the tooth forms after roll-finishing when a combination of $x_d = 0$ and $x_w = 0$ was used. However, when a die-wheel with $x_d = -1$ was used, a slight negative pressure angle error was induced, because the tooth rigidity of work-gear was greater than that of

die-wheel. In contrast to this in the case of $x_d = +1$, a positive pressure angle error is induced. Roll-finishing by a die-wheel with $x_d = 0$ shows some improvement of tooth form accuracy of work-gear with $x_w = 0$, but the improvement of the tooth form accuracy is not obtained when a die-wheel with $x_d = -1$ or $+1$ is used with this work-gear.

As shown in Figs. 6, 7 and 9, when work-gears cut by a hob with protuberance were roll-finished by a die-wheel which had a greater addendum modification factor than that of work-gear, a dull-rounding down of tooth form curve at the root of work-gear became significant. This comes from the fact that a plastic deformation of work-tooth flank easily occurs at the tooth root due to an intentional undercut of tooth form. Furthermore, due to this intentional undercut, the sum of the simultaneous contactlines at the meshing at tooth tip of work-gear becomes shorter than in the case of work-gears cut by a conventional hob, whereby the distributed load near tooth tip increases. The dull-rounding down of tooth profile near tooth tip of work-gears cut by a hob with

Fig.7 Prehobbed and roll-finished tooth forms of gears with $x_w = -0.5$ (given undercut)

Fig.8 Prehobbed and roll-finished tooth forms of gears with $x_w = 0$

protuberance becomes slightly larger than that of work-gears cut by a conventional hob.

Figure 10 shows roll-finished tooth forms of work-gears ($x_w=+0.5$) cut by a conventional hob. When a load of 0.7 kN/mm was applied, roll-finishing did not proceed enough, because the radius of curvature on tooth flanks of the work-gear became larger, which brought about a greater resistance to the plastic deformation of work-tooth flank. When the load increased to 1 kN/mm roll-finishing proceeded enough, but the tooth form roll-finished by die-wheel with $x_d=-1$ showed a negative pressure angle error because the tooth rigidity of the die-wheel was smaller than that of the work-gear. In the case of $x_d=0$, a slight amount of negative pressure angle error is observed, but an improvement of tooth form accuracy was recognized. In the case of $x_d=+1$, a slight pressure angle error was observed on the roll-finished tooth form. The improvement of tooth form accuracy of gears roll-finished by the die-wheel with $x_d=0$ was somewhat better than that obtained with the die-wheel with

$x_d=+1$.

Figure 11 shows roll-finished tooth forms of work-gears with $x_w=+1$ cut by a conventional hob. In this case also, roll-finishing at a load of 1 kN/mm especially with $x_d=+1$ did not proceed due to insufficient applying load. In both cases of $x_d=-1$ and 0, a pressure angle error was hardly induced and a better tooth form was obtained by roll-finishing. In these experiments, the tooth form roll-finished by the die-wheel with $x_d=0$ was the best. There was also a probability that the improved degree of the tooth form roll-finished by die-wheel with $x_d=+1$ became better when the roll-finishing load was increased.

Summarizing the results mentioned above, the "tooth form errors" in broad sense after roll-finishing operation and the mutual influences of addendum modification factors of die-wheel x_d and work-gear x_w are illustrated in Figs. 12 and 13. These figures show the results obtained under an applied load of 1 kN/mm. Qualitatively similar results are also obtained in the case of applying 0.7 kN/mm load.

Fig.9 Prehobbed and roll-finished tooth forms of gears with $x_w=0$ (given undercut)

Fig.10 Prehobbed and roll-finished tooth forms of gears with $x_w=+0.5$

From Fig. 12 which shows the tooth form errors after roll-finishing of work-gears cut by a conventional hob, it is recognized that a very large tooth form error is induced when work-gears with negative addendum modification factor are used, and this state is found to be independent of the selection of addendum modification factor of die-wheel. Lack of data on the combination of $x_d = -1$ and $x_w = -1$ means that roll-finishing under this condition can not proceed because of a very strong interference between the tooth tip of the die-wheel and the tooth root and bottom of the work-gear. On the contrary, in the case of roll-finishing of positive profile shifted gear, a fairly good tooth form accuracy is usually obtained.

Figure 13 shows the tooth form errors of work-gears cut by a hob with protuberance. It is seen from this figure that the tooth form accuracy becomes worse with an increasing negative addendum modification

factor of work-gear, the same as in the former case. However there is a possibility to get a fairly good tooth form when the tooth rigidity of die-wheel is selected almost the same as that of work-gear as observed in the case in which work-gear with $x_w = -1$ is roll-finished by a die-wheel with $x_d = -1$, and also the interference at tooth root and bottom of work-gear is prevented by giving an intentional undercut under the optimum applying of load for roll-finishing. However as mentioned above, there are some problems in that when undercut is intentionally provided on the work-gear, this causes a dull-rounding down of tooth form curve near the tooth tip and root of work-gear. Roll-finishing of gears with negative addendum modification factor is in general difficult in comparison with that of gears with positive addendum modification factor. It would better to avoid a roll-finishing of gears with negative addendum modification

Load (kN/mm)	x_d	- 1		0		+ 1	
		Driven Side	Followed Side	Driven Side	Followed Side	Driven Side	Followed Side
0.7	Hobbed
	Finished	20	19	16	22	24	14
1	Hobbed
	Finished	11	9	9	14	19	15
1	Hobbed
	Finished	14	13	20	16	15	15
1	Hobbed
	Finished	5	6	4	8	11	13

Fig.11 Prehobbed and roll-finished tooth forms of gears with $x_w = +1$

Fig.12 Relation between prof. mod. factor and tooth form error

Fig.13 Relation between prof. mod. factor and tooth form error
(given undercut)

factor.

4. Conclusions

Experimental investigation of roll-finishing of involute helical gears with addendum modification was carried out. The results are summarized as follows:

- (1) Difference in the tooth rigidity between the die-wheel and the work-gear causes a pressure angle error on roll-finished tooth form, i.e., a positive pressure angle error is induced when the tooth rigidity of work-gear is smaller than that of die-wheel; and a negative pressure angle error is induced when the tooth rigidity of work-gear is greater than that of die-wheel.
- (2) Increase in negative addendum modification of work-gear usually causes a large amount of pressure angle error, because the decrease in tooth rigidity of work-gear in this case is far larger than the increase in tooth rigidity when a positive addendum modification is given.
- (3) In order to avoid the pressure angle error and to obtain a good tooth form accuracy by roll-finishing of profile shifted helical gear, the tooth rigidity of die-wheel should be balanced with that of work-gear. This is possible when the die-wheel with almost same amount of addendum modification as that of the work-gear is used. However, with a negative profile shifted gear, an interference between tooth tip of die-wheel and tooth root and bottom of work-gear may occur, and therefore, a counterplan against the interference must be considered.
- (4) In order to prevent the interference in roll-finishing operation, an intention-

al undercut of tooth form of work-gear helps. Such an intentional undercut can be provided by cutting the work-gear by a hob with protuberance. To give a relief at the tooth tip of die-wheel can serve almost as well the same purpose to prevent the interference. However, a large dull-rounding of tooth form curve at tooth tip and root results when the undercut is given at the tooth root of work-gear. Therefore, great care should be taken in the selection of the tooth form of the die-wheel and the applied load when this method is implemented.

(5) In roll-finishing of gears without addendum modification and positive profile shifted gears, the tooth form accuracy is improved more easily than in the case of negative profile shifted gears.

(6) To work out the roll-finishing of positive profile shifted gears, the applied load should be greater than in the cases of negative profile shifted gears and/or gears without addendum modification, because the radius of curvature of tooth flank is larger.

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