Kinematic Synthesis of Adjustable Mechanisms*

(Part 2, Function generators)

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A dimensional synthesis of three types of planar six-link adjustable function generators is carried out in succession to the path generators in Part 1.

The relations between adjustable parameters and input-output displacement curves are classified, and four characters are suggested in order to deal with the relations quantitatively. Then the kinematic conditions satisfying each character above are obtained, and a systematic synthesis of adjustable mechanisms has become possible.

Key Words : Mechanism, Linkage, Optimum Synthesis

1. Introduction

Adjustable mechanisms whose outputs are changeable by adjusting some mechanism parameters have such desirable characteristics that they are versatile and need no coordination control. The authors have synthesized some adjustable path generating mechanisms in Part 1⁽¹⁾, and applied those mechanisms to automatic conveying machines driven by a motor, a brake, and two clutches in the continued report⁽²⁾. In this paper, adjustable function generators are studied.

Formerly, adjustable four- and geared five-link mechanisms have been synthesized with use of the precision-point-method for each of two ideal functions⁽³⁾, and a four-link mechanism whose input-output angular relation is changeable by adjusting the length of the output link⁽⁴⁾. The mechanisms studied in this paper are three types of six-link mechanisms which are the most simple under the condition that the adjustable parameters should be the positions of the fixed joints except those on the input and output links.

2. Classification of Types of Outputs and Mechanisms

We define the rocking center as an output angle Φ_c corresponding to the rocking center θ_c of the input angle θ when both input and output links are rockers, and as one corresponding to a prescribed input angle θ_c when the input link is a crank. Then the changings of the output rocking angle are, in general, classified into the following three cases:

- (i) when the rocking range $\Delta\Phi$ changes but the rocking center Φ_c is fixed [Fig.1 (a)],

- (ii) when the rocking range $\Delta\Phi$ is constant but the rocking center Φ_c changes [Fig.(b)], and
(iii) when both the rocking range $\Delta\Phi$ and its center Φ_c change [Fig.(c)].

Now, for the purpose of obtaining the outlines of the mechanisms directly from the ideal functions, let us consider the following conditions in the above case (i), where $\psi_1, \psi_2, \dots, \psi_n$ are adjustable parameters:

[Condition 1] Symmetricity: displacement curve $\theta \sim \Phi$ is symmetric about its rocking center [$(\partial\Phi/\partial\theta)_{\theta_c} = \text{const.}$],

[Condition 2] Fixed rocking center: rocking center does not move when adjusted [$\partial\Phi_c/\partial\psi_n = 0$].

[Condition 3] Linearity: relation between adjustable parameters and rocking ranges is linear [$\Delta\Phi = \sum a_n \psi_n$],

[Condition 4] Stationary output: output link does not move for any input at an adjusting position ψ_0 [$(\partial^k\Phi/\partial\theta^k)_{\theta=\psi_0} = 0, k \geq 1$].
Let us consider the following conditions instead of the above conditions 2 and 3 in the case (ii) :

[Condition 2'] Constant rocking range: rocking range does not change when adjusted [$\partial\Delta\Phi/\partial\psi_n = 0$],

[Condition 3'] Linearity: relation between adjustable parameters and rocking center is linear [$\Phi_c = \sum b_n \psi_n$],

Fig. 1 Types of output rocking angles

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and let consider the conditions 1, 3, 3', and 4 in the case (iii). The condition 1 should be satisfied at any adjusted positions in every case.

The adjustable parameters are, in this paper, limited to only the positions of the fixed joints except on the input and output links in order to simplify the adjusting process in motion of the mechanism. Then there are three types of the simplest mechanisms: Watt in Fig.2(a), Stephenson in Fig.(b), and the degenerated type in Fig.(c), where b and f are the input and output links and the position of the joint D is adjustable.

3. Kinematic Conditions for Adjusting Rocking Angles

3.1 Conditions for symmetricity

The ratio of the input and output angular velocities should at least be constant at the rocking center for the displacement curve to be symmetric. And a sufficient condition is that the joint E of the mechanism shown in Fig.2 should follow a straight line or circle at a constant velocity and the velocity ratio of the joints E and F should be constant when the input link b turns uniformly, namely:

- (i) the point E is Ball's point⁽⁵⁾ or an inflection point and its acceleration is zero when E follows a straight line, and
- (ii) the point E is on the cubic of stationary curvature⁽⁶⁾ and its tangential acceleration is zero when E follows a circle.

Fig. 2 Adjustable six-link mechanisms

Let p_1, p_2 be the distances from the relative pole P_{bd} to the fixed joints A, D in a four-link mechanism $ABCD$, then one of the following relationships must hold for the above condition (ii).

- (1) $p_1, p_2 = \text{const.}$, (2) $p_1 = p_2$,
- (3) $p_1, p_2 \rightarrow \infty$

The condition (1) is rewritten as $P_{ac} P_{bd} \perp C$ with use of Bobillier construction as shown in Fig.3, which is equivalent to that the pole normal of the coupler point P_{bd} on the coupler b coincides with the link a when the link d is fixed. Then, given c, d , and ϕ , the lengths a and b are obtained as follows.

Put $A(X, Y), P_{bd}(\xi, 0)$, then the above relation is given by

$$\begin{aligned} \{(X-c)\tan\phi - Y\}\xi &= -cY \\ (d\sin\phi - Y)\xi &= dX\sin\phi - dY\cos\phi \end{aligned} \quad \dots(1)$$

Eliminating ξ , we have

$$\begin{aligned} d\{X\sin\phi - Y\cos\phi\}^2 - cY^2\cos\phi \\ - cd\sin\phi(X\sin\phi - 2Y\cos\phi) &= 0 \end{aligned} \quad \dots\dots\dots(2)$$

which is given by the quadratic curve

$$\begin{aligned} \frac{\sin(\phi - \phi)}{\cos\phi} X^2 + \frac{\cos(\phi - \phi)}{\sin\phi} Y^2 \\ - c\{(\cos\phi\sin\phi - 2\sin\phi\cos\phi)X \\ - (\sin\phi\sin\phi + 2\cos\phi\cos\phi)Y\} &= 0 \end{aligned} \quad \dots\dots\dots(3)$$

turned through the angle

$$\phi = 1/2 \tan^{-1}\{d\sin 2\phi / (c\cos\phi - d\cos 2\phi)\}.$$

Obtaining the intersection A of this curve with a circle whose center is D and radius is a , the mechanism is determined. The condition (2) means that two fixed joints coincide with each other, and (3) is satisfied by a parallel four-link mechanism.

Similarly let p_1 and p_2 be the distances from the center of curvature O_E of the locus of E and the point G to the relative pole P between f and $O_E E$, then the same conditions (1)~(3) as the above are necessary for the ratio of velocities of the points E and F to be constant. Applying such conditions (i)~(iii) to each type of mechanism, the results are obtained as shown in Table 1.

Fig. 3 Conditions for four-link uniform motion mechanisms

3.2 Conditions for fixed rocking center

In order that the rocking center does not move while the position of the fixed joint D shown in Fig.4 is adjusted, one of the following conditions should be satisfied:

- (i) The point E does not move (E_c),
- (ii) The point E moves along a circle whose center is the fixed point F_c ,
- (iii) The points B_c and F_c coincide with each other.

The mechanisms and the methods for adjusting the joint D are obtained as shown in Table 2 by applying the above conditions to three types of six-link mechanisms.

3.3 Conditions for stationary output

The position of the stationary output is a standard position of an adjustable function generator, which is realized by the following condition:

- (i) The point E_0 does not move, in Fig. 4, or
- (ii) The point E moves on a circle whose center is the fixed point F_0 for any input,

where the suffix "0" means the state of stationary output. Three types of six-link mechanisms satisfying the above condition are shown in Table 3. Moreover, the conditions for satisfying the symmetry at

the position of the stationary output are that the links $b(c)$ and d or $d(e)$ and f should intersect at right angles under the condition (i) or (ii), respectively, in Watt and degenerated mechanisms, and that the condition (i) should be satisfied in Stephenson mechanism.

3.4 Conditions for symmetry at any adjusted positions

3.4.1 At the neighbourhoods of general positions

Although the symmetry of Stephenson mechanism is not satisfied, in general, by adjusting the position of D, it may be approximately satisfied if the angle between the links b and d (or EO_e and f) is almost 90° and $DE \gg b, d$ (or $EO_e \gg f$) in the mechanism E-(ii) (or F-(ii)). On the other hand, the symmetry of Watt mechanism can be precisely satisfied at any adjusted position, and the problem becomes one of adjusting the four-link uniform motion mechanism (E-(ii), in Table 1), but is hard to solve as a general case.

Figure 5(a) shows the adjusting locus of D for satisfying the uniformity of the four-link mechanism ABCD when $b=c$ and the

Table 1 Mechanisms satisfying symmetry

	Watt mech. and degenerated mech.	Stephenson mech.
E	(ii) ₁ $P_1, P_2 = \text{const.}$ 	(i) E is Ball's point
	(ii) ₂ $P_1 = P_2$ 	(ii) A coincides with D
	(ii) ₃ $P_1, P_2 = \infty$ 	
F	(i) (ii) (iii)

Fig. 4 Mechanism for fixed rocking center or stationary output

Table 2 Mechanisms satisfying fixed rocking center

Watt mech.	Stephenson mech. and degenerated mech.
(i) Impossible 	(i) On the circle C_c
(ii) On the 4-bar coupler curve 	(ii) Approximate: E is on the cubic of stationary curvature and F is its center of curvature
(iii) On the circle B_c 	(iii) Arbitrary

Table 3 Mechanisms satisfying stationary output

Watt mech. and degenerated mech.	Stephenson mech.
(i) 	(i) ₁ Precise
(ii) 	(i) ₂ Approximate
	(ii) Approximate: E is on the cubic of stationary curvature and F is its center of curvature

links b , c , d are collinear at the initial position, which is a circle whose center is the point A . Figure (b) shows a similar locus when $b=c$ and the links b and c are parallel with each other and they intersect with d at right angles, which is approximately a circle too. Joining the above four-link uniform motion mechanisms by considering the conditions for the fixed rocking center, a Watt mechanism satisfying the symmetry even when adjusted may be obtained.

3.4.2 At the neighbourhoods of the positions of stationary output

Both the symmetry at the position of stationary output mentioned in section 3.3 and that at the general position under the adjustment mentioned in paragraph 3.4.1 should be taken into consideration. The conditions for Watt mechanism are that the directions of velocity and adjustment of the coupler point D of the mechanism whose arms are c and e and coupler is d in Table 3 (i) should coincide with each other.

Since it is very hard to obtain similar kinematic conditions about the linearity of condition 3 as the above, these will be considered numerically at the steps of synthesis.

Fig. 5 Adjusting methods for keeping symmetries

4. Synthesis of Adjustable Function Generators and Examples

Adjustable six-link mechanisms are synthesized such as to satisfy the conditions in Chapter 3 in the case (i) where only the rocking range changes among the three cases classified in Chapter 2.

The procedure is split into two steps as shown in Fig. 6: one of determining the kinematic conditions for satisfying the symmetry, fixed rocking center, and stationary output, and the other of optimizing the mechanism such as to satisfy a given input rocking range, adjusting range, maximum pressure angle, maximum ratio of link lengths, and linearity.

4.1 Determination of kinematic conditions

The conditions for adjustable rocking range in each type of mechanism are obtained from Tables 1~3 as follows, when the adjusting locus of the joint D is a circle.

(1) Watt mechanism The adjusting method in order to satisfy the fixed rocking center is (iii) in Table 2. But, since the points A and C , B and D and F respectively coincide with each other considering the stationary output in Table 3 in addition to the above condition, there is no Watt mechanism.

(2) Stephenson mechanism When the point E describes a straight line uniformly (E -(i) in Table 1), let the condition for F be F -(i) and the adjusting method for the fixed rocking center be (i) in Table 2. Such mechanisms will satisfy the stationary output (i)₂ in Table 3 at the position where the link d lies upon c , and an

Fig. 6 Procedure for synthesis

example is shown in Fig.7 and Table 4 (Example 1). A mechanism drawn by the broken lines satisfies $B_c E_c // f \perp e$ and the symmetry and fixed rocking center, and one drawn by the solid lines is a mechanism corresponding to the position of the above mechanism whose input link and joint D turn through θ and ψ respectively.

When the point E describes a circle uniformly (E-(ii) in Table 1), let the adjusting method for the fixed rocking center be (iii) in Table 2, and the output link precisely stops at the position of E-(ii) in Table 1 where the symmetry is satisfied (i), in Table 3). Then the mechanism shown in Fig.8 and Table 4 (Example 2, Initial) is obtained, which is similar to the mechanism suggested in the reference (6) qualitatively but satisfies the condition that the links b, c, e coincide with each other, and f and these links, d and d' intersect respectively at right angles at the position drawn by broken lines. The solid lines mean the mechanism obtained by turning the point D about O_b and making the input angle θ .

(3) Degenerated mechanism If the symmetry is E-(ii)₂, F-(ii) in Table 1, the adjusting method of the point D for satisfying the fixed rocking center should be, in general, (i) in Table 2. The stationary output in Table 3 may be satisfied by adjusting the point D. An example of such mechanisms is shown in Fig.9, in which the mechanism drawn by the broken lines satisfies the symmetry. Putting $d=e$ and so adjusting the point D as to coincide with F_c , the stationary output is satisfied.

Table 4 Results of syntheses

Parameters	Example 1	Example 2	
		Initial	Optimized
b	1.000	1.000	1.000
c	0.620	0.400	0.450
c'	1.940	1.000	1.000
v	107.0°	66.0°	94.7°
d	1.130	0.917	1.130
d'	1.130	0.700	0.600
e	2.000	1.000	1.000
f	1.940	1.000	0.703
λ	26.0°	—	—

Fig. 7 Example 1

4.2 Appreciation of characteristics and optimization

The optimum synthesis is carried out by investigating the symmetries and linearities, and considering the pressure angles and ratios of link lengths about the mechanism obtained in section 4.1.

The input-output displacement curves $\theta-\psi$ of the mechanism in Fig.7 under the conditions of $-25^\circ \leq \theta \leq 25^\circ$, $0^\circ \leq \psi \leq 70^\circ$ are shown in Fig.10, which shows that the symmetries after adjusted are approximately satisfied and that the rocking center is fixed. The $\psi-\phi$ representations of the above relationships are shown in Fig. 11, which shows that the output link approximately stops at $\psi = 41.5^\circ$.

Fig. 8 Example 2

Fig. 9 Example 3

Fig. 10 Example 1, displacement curves

The relations Ψ - Φ of the mechanism in Fig.8 are also obtained under the conditions of $-25^\circ \leq \theta \leq 25^\circ$, $0 \leq \Psi \leq 50^\circ$, and those curves at $\theta = \pm 25^\circ$ are shown in Fig.12 by the broken lines. Although the output link stops at $\Psi = 28.7^\circ$, the linearity is worse than that of the mechanism in Fig.7. The same relations of the degenerated mechanism in Fig.9 are shown in Fig.13. Although the output stops at $\Psi = 20^\circ$, the symmetry and linearity become worse at $0 \leq \Psi \leq 20^\circ$ and $\Psi > 60^\circ$, respectively.

Next, as an example, the mechanism in Fig.8 is optimized about the linearity and symmetry when adjusted according to the procedure shown in Fig.6 under the condition that the ranges of input and adjusting angles are the same as the above, and the maximum pressure angle and ratio of link lengths are less than 70° and 5, respectively. The results are shown in Table 4 (Example 2, optimized), and the actual Ψ - Φ curves at $\theta = \pm 25^\circ$ are shown in Fig.12 by solid lines, and further the total amplitude $\Delta\Phi = \Phi^+ - \Phi^-$ and the ratio of amplitudes $|\Phi^+/\Phi^-|$ are shown in Fig.14. As the result, the Ψ - $\Delta\Phi$ curves and $|\Phi^+/\Phi^-|$ become approximately straight lines and unity, respectively, and therefore both the linearity and symmetry when adjusted become better by the optimization.

5. Conclusions

The results obtained in this paper are summarized as follows:

(1) The input-output angular relationships of adjustable function generators have been classified and four conditions have been suggested for the purpose of dealing with these relationships quantitatively.

(2) The kinematic conditions for satisfying the above four conditions have been obtained about three types of planar six-link mechanisms having three arms by limiting the adjustable parameters only to the positions of the fixed joints except those on the input and output links, the type and dimensional synthesis of such mechanisms have become easy without any repeating computations.

(3) When the ranges of the inputs and adjusting angles, the maximum allowable pressure angles, etc., are given, precise adjustable mechanisms have been obtained through optimization considering the conditions of the input, output, and adjustable angles suggested in (2).

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Fig. 11 Example 1

Fig. 12 Example 2

Fig. 13 Example 3

Fig. 14 Example 2, comparison of characteristics