Single Surface Removal-Both Surface X-Ray Diffraction Method for Residual Stress Measurement in a Hollow Cylinder*

By Osamu DOI** and Takashi IMURA***

The authors proposed in this paper a new method using X-ray diffraction to measure the distributions of three principal residual stresses in a hollow cylinder. The procedure involves the uniform removal of successive thin layers from an inner or outer surface of a hollow cylinder and surface stress measurements by X-ray on the inner and outer surfaces of it. Residual stress distributions are calculated from both surface stresses.

The new method has a special feature of simple formulas.

The authors give an example of measurement by applying the new method to a heat treated hollow cylinder (STPT 38).

Key Words: Strength, X-ray Diffraction, Measurement of Residual Stress, Hollow Cylinder

1. Introduction

The theory of residual stress measurement with X-ray applied on an inner or outer surface of a hollow cylinder after removal of layers from one surface has been proposed.⁽¹⁾⁽⁴⁾

In this paper the authors derived two kinds of new methods using X-ray diffraction to measure residual stresses in a hollow cylinder.

The procedure includes successive thin layer removals from one surface of a hollow cylinder and surface stress measurements by X-ray on both surfaces. Residual stresses can be calculated from both surface stresses.

Assumptions for the analysis are as follows. (1) Residual stresses are symmetric with respect to longitudinal axis.

(2) Residual stresses are functions of radius r . Analytical formulas are derived with a theory of thick-walled cylinder.

2. Inner Surface Removal and Both Surface X-ray Diffraction Method

Residual stress distributions are calculated from the surface stresses measured by X-ray on both surfaces after thin layer removal from an inner surface of a hollow cylinder. In Fig.1, r_1, r_2 : the internal and external radius; ρ, r : arbitrary radius ($r_1 \leq \rho \leq r_2$); $\sigma_i(r), \sigma_t(r), \sigma_r(r)$: axial, tangential and radial residual stress

at radius r ; $\sigma_L(r), \sigma_T(r)$: axial and tangential stress measured by X-ray on an inner surface after layer removal up to radius r ; $\bar{\sigma}_{L2}(r), \bar{\sigma}_{T2}(r)$: axial and tangential stresses measured by X-ray on an outer surface after layer removal up to radius r ; $\bar{\sigma}_{L20}, \bar{\sigma}_{T20}$: axial and tangential residual stresses at an initial outer surface; $\sigma'_i(r), \sigma'_t(r)$: the changes of axial and tangential stresses in an inner surface; and $\bar{\sigma}'_{i2}(r), \bar{\sigma}'_{t2}(r)$: the changes of axial and tangential stresses in an outer surface.

Fig.1 Removal from inner surface

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** Professor, Faculty of Engineering Hokkaido University, N13, W18, Kita-ku Sapporo.

*** Instructor, Faculty of Science and Technology Meijo University, 69 Yagotourayama Tenpaku-ku Nagoya.

From Fig.1,

$$\sigma_i(r) + \sigma_i'(r) = \sigma_L(r) \quad \dots\dots\dots (1)$$

$$\bar{\sigma}_{L20} + \bar{\sigma}_{i2}'(r) = \bar{\sigma}_{L2}(r) \quad \dots\dots\dots (2)$$

$$\sigma_i(r) + \sigma_i'(r) = \sigma_T(r) \quad \dots\dots\dots (3)$$

$$\bar{\sigma}_{T20} + \bar{\sigma}_{i2}'(r) = \bar{\sigma}_{T2}(r) \quad \dots\dots\dots (4)$$

Where

$$\sigma_i'(r) = \bar{\sigma}_{i2}'(r) = \frac{2}{r_2^2 - r^2} \int_{r_1}^r \sigma_i(\rho) \rho d\rho \quad \dots\dots\dots (5)$$

$$\sigma_i'(r) = \frac{r_2^2 + r^2}{r_2^2 - r^2} \sigma_r(r) \quad \dots\dots\dots (6)$$

$$\bar{\sigma}_{i2}'(r) = \frac{2r^2}{r_2^2 - r^2} \sigma_r(r) \quad \dots\dots\dots (7)$$

$$\sigma_r(r) = \frac{1}{r} \int_{r_1}^r \sigma_i(\rho) d\rho \quad \dots\dots\dots (8)$$

2.1 Both Surface Subtraction Method

From eqs.(1),(2) and (5), the axial residual stress $\sigma_i(r)$ is obtained.

$$\sigma_i(r) = \sigma_L(r) - (\bar{\sigma}_{L2}(r) - \bar{\sigma}_{L20}) \quad \dots\dots\dots (9)$$

From eqs.(3),(4),(6) and (7), the tangential residual stress is obtained.

$$\sigma_i(r) = \sigma_T(r) - \frac{r_2^2 + r^2}{2r^2} (\bar{\sigma}_{T2}(r) - \bar{\sigma}_{T20}) \quad \dots\dots\dots (10)$$

From eqs.(4) and (7), the radial residual stress is obtained.

$$\sigma_r(r) = \frac{r_2^2 - r^2}{2r^2} (\bar{\sigma}_{T2}(r) - \bar{\sigma}_{T20}) \quad \dots\dots\dots (11)$$

2.2 Both Surface Addition Method

From eqs.(1),(2) and (5), we have

$$\begin{aligned} \sigma_i(r) + \frac{4}{r_2^2 - r^2} \int_{r_1}^r \sigma_i(\rho) \rho d\rho \\ = \sigma_L(r) + \bar{\sigma}_{L2}(r) - \bar{\sigma}_{L20} \\ = \delta_i(r) \quad \dots\dots\dots (12) \end{aligned}$$

The solution of the integral equation (12) is

$$\begin{aligned} \sigma_i(r) = \delta_i(r) - 4(r_2^2 - r^2) \int_{r_1}^r \frac{\rho \delta_i(\rho)}{(r_2^2 - \rho^2)^2} d\rho \\ \dots\dots\dots (13) \end{aligned}$$

From eqs.(3),(4),(6),(7) and (8), we have

$$\begin{aligned} \sigma_i(r) + \frac{(r_2^2 + 3r^2)}{r(r_2^2 - r^2)} \int_{r_1}^r \sigma_i(\rho) \rho d\rho \\ = \sigma_T(r) + \bar{\sigma}_{T2}(r) - \bar{\sigma}_{T20} \\ = \delta_i(r) \quad \dots\dots\dots (14) \end{aligned}$$

The solution of the integral equation (14) is

$$\begin{aligned} \sigma_i(r) = \delta_i(r) - \frac{(r_2^2 - r^2)(r_2^2 + 3r^2)}{r^2} \\ \times \int_{r_1}^r \frac{\rho \delta_i(\rho)}{(r_2^2 - \rho^2)^2} d\rho \quad \dots\dots\dots (15) \end{aligned}$$

From eqs.(8) and (15), $\sigma_r(r)$ is obtained.

$$\sigma_r(r) = \frac{(r_2^2 - r^2)^2}{r^2} \int_{r_1}^r \frac{\rho \delta_i(\rho)}{(r_2^2 - \rho^2)^2} d\rho \quad \dots\dots\dots (16)$$

3. Outer Surface Removal-Both Surface X-ray Diffraction Method

Thin layers are successively removed

from the outer surface of a hollow cylinder and surface stresses are measured by X-ray on both surfaces as shown in Fig.5($r_1 \leq r \leq r_2$).

$\sigma_L(r)$, $\sigma_T(r)$: axial and tangential stresses measured by X-ray on an outer surface; $\bar{\sigma}_{L1}(r)$, $\bar{\sigma}_{T1}(r)$: axial and tangential stresses measured by X-ray on an inner surface; $\bar{\sigma}_{L10}$, $\bar{\sigma}_{T10}$: axial and tangential residual stresses at initial inner surface. $\sigma_i'(r)$, $\sigma_i'(r)$: the changes of axial and tangential stresses in an outer surface. $\bar{\sigma}_{i1}'(r)$, $\bar{\sigma}_{i1}'(r)$: the changes of axial and tangential stresses in an inner surface.

Fig.2 Removal from outer surface

From Fig.2, we have

$$\sigma_i(r) + \sigma_i'(r) = \sigma_L(r) \quad \dots\dots\dots (17)$$

$$\bar{\sigma}_{L10} + \bar{\sigma}_{i1}'(r) = \bar{\sigma}_{L1}(r) \quad \dots\dots\dots (18)$$

$$\sigma_i(r) + \sigma_i'(r) = \sigma_T(r) \quad \dots\dots\dots (19)$$

$$\bar{\sigma}_{T10} + \bar{\sigma}_{i1}'(r) = \bar{\sigma}_{T1}(r) \quad \dots\dots\dots (20)$$

Where

$$\sigma_i'(r) = \bar{\sigma}_{i1}'(r) = \frac{2}{r_1^2 - r^2} \int_{r_1}^r \sigma_i(\rho) \rho d\rho \quad \dots\dots\dots (21)$$

$$\sigma_i'(r) = \frac{r_1^2 + r^2}{r_1^2 - r^2} \sigma_r(r) \quad \dots\dots\dots (22)$$

$$\bar{\sigma}_{i1}'(r) = \frac{2r^2}{r_1^2 - r^2} \sigma_r(r) \quad \dots\dots\dots (23)$$

$$\sigma_r(r) = \frac{1}{r} \int_{r_2}^r \sigma_i(\rho) d\rho \quad \dots\dots\dots (24)$$

3.1 Both Surface Subtraction Method

From eqs.(17),(18) and (21), $\sigma_i(r)$ is obtained.

$$\sigma_i(r) = \sigma_L(r) - (\bar{\sigma}_{L1}(r) - \bar{\sigma}_{L10}) \quad \dots\dots\dots (25)$$

From eqs.(19),(20),(22) and (23), $\sigma_i(r)$ is obtained.

$$\sigma_i(r) = \sigma_T(r) - \frac{r_1^2 + r^2}{2r^2} \{ \bar{\sigma}_{T1}(r) - \bar{\sigma}_{T10} \} \dots (26)$$

From eqs.(20) and (23), $\sigma_r(r)$ is obtained.

$$\sigma_r(r) = \frac{r_1^2 - r^2}{2r^2} \{ \bar{\sigma}_{T1}(r) - \bar{\sigma}_{T10} \} \dots (27)$$

3.2 Both Surface Addition Method

From eqs.(17),(18) and (21), we have

$$\begin{aligned} \sigma_i(r) + \frac{4}{r_1^2 - r^2} \int_{r_2}^r \rho \sigma_i(\rho) d\rho \\ = \sigma_L(r) + \bar{\sigma}_{L1}(r) - \bar{\sigma}_{L10} \\ = \bar{\delta}_i(r) \dots (28) \end{aligned}$$

The solution of eq.(28) is

$$\sigma_i(r) = \bar{\delta}_i(r) - 4(r_1^2 - r^2) \int_{r_2}^r \frac{\rho \bar{\delta}_i(\rho)}{(r_1^2 - \rho^2)^2} d\rho \dots (29)$$

From eqs.(19),(20),(22),(23) and (24), we have

$$\begin{aligned} \sigma_i(r) + \frac{1}{r} \frac{(r_1^2 + 3r^2)}{(r_1^2 - r^2)} \int_{r_2}^r \sigma_i(\rho) d\rho \\ = \sigma_T(r) + \bar{\sigma}_{T1}(r) - \bar{\sigma}_{T10} \\ = \bar{\delta}_i(r) \dots (30) \end{aligned}$$

The solution of eq.(30) is

$$\begin{aligned} \sigma_i(r) = \bar{\delta}_i(r) - \frac{(r_1^2 - r^2)(r_1^2 + 3r^2)}{r^2} \\ \times \int_{r_2}^r \frac{\rho \bar{\delta}_i(\rho)}{(r_1^2 - \rho^2)^2} d\rho \dots (31) \end{aligned}$$

From eqs.(24) and (31), $\sigma_r(r)$ is obtained.

$$\sigma_r(r) = \frac{(r_1^2 - r^2)^2}{r^2} \int_{r_2}^r \frac{\rho \bar{\delta}_i(\rho)}{(r_1^2 - \rho^2)^2} d\rho \dots (32)$$

4. Example of Application

As an application of this new method, the authors measured residual stress of a hardened steel pipe(STPT 38). After the heat treatment under the conditions as shown in Table 1, a slit is cut in axial and tangential directions with an end-mill as shown in Fig.3. X-ray is irradiated on the inner surface through the slit with the angle of X-ray incidence varied within 0° to 30° .

The successive removal of thin layers is done by electropolishing and the apparatus and condition are shown in Fig.4 and Table 2.

The X-ray measurement is done by Geiger Counter method with an apparatus of parallel beam type under the condition as shown in Table 3.

Measured stresses by X-ray and residual stress distributions calculated from the authors' formulas are shown in Figs.5 - 8. Measured values of point A in Fig.3 are marked with black triangles in Figs.5 and 6 to show that there is no effect of the slit.

The residual stress distributions can be also calculated by former method(Front Surface Method and Back Surface Method^{(1)~(4)}) and the calculated values are in good agreement with the results obtained by the new method.

Table 1 Conditions of heat treatment

Heating temperature	890°C
Hold time	45 min
Coolant, temperature	Water, 22°C

Table 2 Conditions of Electropolishing

Electrolyte	1% HCl+H ₂ O
Current density	2.2 - 2.7 A/dm ²
Temperature	21 - 24°C

Table 3 Conditions of X-ray stress measurement

Apparatus	Parallel beam type
Characteristic X-ray	Cr-K α
Tube voltage	30 Kv
Tube current	10 mA
Time constant	16 s
Slit divergent angle	1.02°
Irradiated area	4 x 2 mm ²
Determination of peak point	Half width method

Fig.3 Test piece

Fig.4 Electropolishing apparatus

Fig.5 Measured surface stress by X-ray(Axial)

Fig.7 Residual stress distribution in a hollow cylinder(Axial)

Fig.6 Measured surface stress by X-ray(Tangential) Radial)

Fig.8 Residual stress distributions in a hollow cylinder (Tangential and Radial)

5. Conclusions

- (1) The authors proposed a new method using X-ray to measure three principal residual stress in a hollow cylinder.
- (2) The formulas of the Both Surface Addition Method have no differential term as the formulas which the authors have presented for a plate.⁽⁵⁾
- (3) The formula of the Both Surface Subtraction Method has no differential and no integral term, and residual stresses can be calculated by four rules of arithmetic only.
- (4) The new method is applied to a heat treated steel pipe(STPT 38) and the results agree well with those of the former method.⁽⁴⁾

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