Effect of Side Grooves for Three-point Bending Fracture Toughness Specimens

— Three-dimensional Elastic-plastic Finite Element Analysis*

By Yasufumi IMAI** and Tomokazu MATAKE***

Elastic-plastic stress states on three-point bending fracture toughness specimens are analyzed by the three-dimensional finite element method. Effect of side grooves on the stress state around the crack tip is discussed on specimens about 30 % side-grooved in U, V, and slit types comparing with a regular flat specimen. Side grooves restrict the plane stress type deformation, which is commonly observed near the side face of a flat specimen. Consequently, over 80-90 % of the thickness of the side-grooved specimens, practically even distributions of the stress intensity factors, plastic zone geometries, crack tip dilatant stresses, and three-dimensional J integrals are achieved.

Key Words: Fracture, Three-point Bend Specimen, Side-groove, Plastic Zone, FEM

1. Introduction

The stress state of plane strain fracture toughness specimens has usually been analyzed two-dimensionally neglecting that the deformation mode near the side face differs from the mode at the mid-thickness region. Near-side region actually deforms in the plane stress mode and forms shear lips when fractured. In addition, it was reported that the size of the shear lip region does not reduce no matter how thick the specimen is⁽¹⁾. Therefore, using thick specimens makes the influence of side faces only relatively small. Experimentally obtained load vs. loading-point-displacement relation, J integral, and the like are only the average over the specimen thickness. Recently, some attempts have been made to employ side-grooved (SG) small specimens instead of thick ones⁽²⁾, ⁽³⁾. SG specimens were also reported to make the J test easier. But an exact analysis of the effect of side grooves has been left unresolved. In order to understand the fracture mechanism in those specimens, however, any two-dimensional analysis is no more valid. A three-dimensional analysis becomes necessary.

Previously, we examined the growth of stable cracks on several kinds of SG specimens of three-point bending (3PB) and found a proper method for restricting

shear lips and realizing a flat stable crack front throughout the thickness⁽⁴⁾: A regular flat specimen showed a so called thumb-nail crack front as shown in Fig. 1; In contrast to this, appropriately grooved specimens, such as V-grooved and slit, showed an almost straight crack front. As observed in the above example, the effect of the side grooves on the elastic-plastic stress state around a crack tip should differ depending on the groove geometry, such as its root curvature, depth, and the material properties of the specimen. In this paper, however, the effect of only a limited number of grooves on the 3PB fracture toughness specimens will be discussed following the previous study⁽⁵⁾ where the effect has been reported on the double-edge-notched tension panels.

Fig. 1 Stable crack fronts on 3PB specimens (a) without side grooves, (b) with U-grooves, (c) with V-grooves, and (d) with 0.2 mm machine slits.

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** Associate professor, Department of Mechanical Engineering, Nagasaki University, Bunkyo-machi, Nagasaki 852.

*** Professor, Department of Mechanical Engineering, Nagasaki University, Bunkyo-machi, Nagasaki 852.

Deformations are analyzed for specimens with 2 mm deep U grooves, 2 mm deep V grooves, and 0.2 mm wide 2 mm deep machine slits as well as for a regular flat specimen. In addition, from those results of the flat and machine slit specimens, J integral, which was pointed out to be a unique parameter representing the elastic-plastic stress state around the crack tip⁽⁶⁾, is estimated. Its values are compared to those from Rice's conventional formula. Variations of some fracture parameters over the specimen thickness are discussed.

2. Method of Analysis

2.1 Elastic-plastic finite element method

The material behavior was assumed to be governed by the von Mises yield criterion and Hencky's relation in the hardening state⁽⁷⁾. That is, the stress and strain tensors, σ_{ij} , ϵ_{ij} , are related as

$$\begin{aligned}\sigma_{ij} &= K\epsilon_{ij} + 2g\epsilon_{ij}, \\ \epsilon_{ij} &= \epsilon_{ij} - \frac{1}{3}\epsilon_{kk}\delta_{ij}, \\ \epsilon &= \epsilon_{rr}\end{aligned}\quad (1)$$

where K is the dilatational modulus and g is the modulus of plasticity, which is related to the tangential modulus H of the uniaxial tensile stress-strain curve as,

$$g = 3HK/(9K - H) \quad (2)$$

In the elastic state, g is to be replaced by the modulus of rigidity, G.

Deformed 20 node isoparametric brick elements⁽⁸⁾ were used to approximate curved boundaries of side grooves and also to achieve a fine sectioning around the crack tip. A stiffness matrix of each element was evaluated at 27 points using the Gaussian quadratic rule. Stresses were estimated at each nodal point as an average of the values calculated at all the elements that involve respective nodes. In the elastic-plastic state, the modulus of plasticity g varies even inside the element corresponding to the stress state. The variation of the modulus was represented, in this paper, by using the element shape functions, $N(\xi, \eta, \zeta)$ and the nodal values, $\{g\}^e$, which were known from the nodal stress state, as

$$g = \sum_{i=1}^{20} N_i(\xi, \eta, \zeta) \{g\}_i^e \quad (3)$$

The plasticity was solved by the tangential modulus method with an incremental loading. For the transition state from elastic to plastic, weighted moduli were used according to Marcal and King's method⁽⁹⁾. But no iterative calculations at each load increment were performed. When the equivalent stress σ_{eq} at some node decreased from the level attained at the last load step, the modulus g at the node was replaced by G in the next load step. To solve the linear equation systems, a kind of front solution method was employed.

2.2 Three-dimensional J integral

Within three components of a local three-dimensional J integral⁽¹⁰⁾, only one component in a crack-propagating direction was calculated as,

$$J_1 = \int_{\Gamma} W dx_2 - \int_{\Gamma} \sigma_{ij} v_j u_{i,1} d\Gamma - \int_A (\sigma_{i3} u_{i,1})_{,3} dA \quad (4)$$

where W is the strain energy density and v_j is a direction cosine of the outer normal on the boundary. The integral path Γ and the area A surrounded by Γ on the section normal to the crack front were taken as shown in Fig. 2. $u_{i,1}$ was estimated at 20 nodes on each element using the element shape functions, $N_k(\xi, \eta, \zeta)$, as,

$$u_{i,1} = \sum_{k=1}^{20} \frac{\partial N_k}{\partial x_1} \{u_i\}^e \quad (5)$$

In a similar way, using $\{\sigma_{i3} u_{i,1}\}^e$ at 20 nodes, $(\sigma_{i3} u_{i,1})_{,3}$ was estimated as,

$$(\sigma_{i3} u_{i,1})_{,3} = \sum_{k=1}^{20} \frac{\partial N_k}{\partial x_3} \{\sigma_{i3} u_{i,1}\}^e \quad (6)$$

Path and area integrations were numerically evaluated using the Gaussian quadratic rule each at three points on the side and each at nine points on the face of the element.

Fig. 2 Integral path for evaluating the three-dimensional J integral.

J_1 was calculated on each of three planes, $\zeta = -1, 0, 1$ of every element that arrays in the thickness direction although it can be calculated on any normal plane to the crack front. When two elements were adjoining at $\zeta = 1$ of one element and $\zeta = -1$ of the other, J_1 values from both elements were averaged there. The integral path was taken inside the element because the boundary of the element inclines is not perpendicular to the crack front.

3. Analyzed Specimens and the Material

Specimens shown in Fig. 3 were analyzed, which had the same geometry as the ones used in the stable crack observation in Fig. 1. Because of the symmetry of geometry and loading, only one quarter of the specimen was to be modeled. Naturally, a loading point locates at the mid-span as Fig. 3, but in this paper, in order to reduce the influence of loading distribution on the crack tip, the mid-span was fixed

Fig. 3 Specimen geometry.

Fig. 4 Side groove geometry.

Fig. 5 Approximations of the groove geometries.

and the load was applied from both end supports. The specimen had a 12 mm pre-crack. A regular flat specimen was 10 mm thick but SG specimens were 14 mm thick with side grooves of 2 mm deep on both sides so that the net thickness between groove roots became 10 mm. Each groove geometry as shown in Fig. 4 was approximated by one array of deformed elements. Their shapes, however, could not be matched completely by using only one element side. Instead of getting a complete mapping, in this paper, groove root radius ρ , depth t , and the half width on the side face a , are to be matched. That is, the coordinates of the mid-side node, (x_2, y_2) in Fig. 5 should be chosen as (cf. Appendix),

$$\begin{aligned} x_2 &= t/4, \\ y_2 &= (a + \sqrt{2\rho t})/4 \end{aligned} \quad (7)$$

Approximated curves by this method are shown in Fig. 5.

The finite element models as illustrated in Fig. 6 consisted of 4, 8, and 6 layers in the thickness, width, and longitudinal directions, respectively. The number of total nodes was 1117 and that of elements, 192. In order to make a comparison with the previous experiment where the middle and high pressure turbine generator rotor steel was used, the mechanical properties of this material were employed in the calculation. The yield strength was assumed to be 765 MPa and the stress-strain curve of the material was

(a)

(b)

Fig. 6 Finite element models of (a) the flat specimen and (b) the SG specimen. The arrow indicates the crack front.

Table 1 Stress-strain relation used in the calculation

σ_{eq} MPa	H GPa	ϵ GPa
0 - 765	211	81.1
765 - 824	58.9	20.4
824 - 883	6.54	2.19
883 - 942	4.06	1.35
942 -	2.56	0.85

$$\sigma_{eq} = \sqrt{\frac{3}{2}} s_{ij} \cdot s_{ij}, \quad s_{ij} = \sigma_{ij} - \sigma \delta_{ij}.$$

approximated by piece-wise straight lines, each of which gives tangential modulus H and hence, g as shown in Table 1. Poisson's ratio 0.3 was assumed.

4. Results and Discussion

4.1 Stress intensity factor

On the same coordinates as shown in Fig. 2 with the origin on a crack front, both normal stresses, σ_{11} and σ_{22} , have a type of singularity of $K_I/\sqrt{2\pi x_1}$. Hence, calculated stresses around the crack tip provide K_I . Fig. 7 shows K_I 's variation in the specimen thickness where the ordinate is plotted as the ratio to the value

Fig. 7 Variation of K_I across the specimen thickness.

$K_{I(2-D)}$ in the plain strain condition. In the mid-thickness region of the flat specimen, K_I value is a few percent larger than $K_{I(2-D)}$. But it becomes gradually smaller as it approaches the specimen side face and falls to 60% of the mid-thickness value near the face. This tendency is consistent with the previous study⁽¹¹⁾. On the other hand, side grooves enhance K_I near the sides and the smaller the root radius is, the larger the increase becomes. The SG specimens appear to have about 80 % of $K_{I(2-D)}$ in the mid-thickness region. But this is the result of overestimating $K_{I(2-D)}$ calculated by using the net thickness at the grooved section. If the equivalent thickness is used as pointed out by Kitagawa et al.⁽¹²⁾, consequently the ratio will become nearly equal to unity.

Here, the stress singularity has been adopted to estimate K_I , but the displacement singularity can also be adopted as well. Henshell et al.⁽¹³⁾ proposed use of the quarter node elements to represent the crack tip singularity. Using those elements, the displacements were calculated and adopted to the plane strain displacement singularity. But obtained K_I values were well consistent with the values by the stress method within a few percent.

4.2 Elastic-plastic state and the plastic zone growth

Load vs. loading-point-displacement relations obtained are shown in Fig. 8. Beyond the highest load shown in the diagram, many nodes on the ligament underwent unloading and unnatural behaviors were calculated on the whole. This probably occurred because of the fixed load increment. The SG specimens show their rigidity about 20 % higher than the flat specimen's even though the net thicknesses are all 10 mm. Their differences in the kind of groove geometry are not recognized.

In every specimen, yielding starts from the node at the crack tip on the side face. With an increasing load, the yielded zone extends around both the crack tip and the loading point. Fig. 9 compares the

Fig. 8 Calculated load vs. loading-point-displacement curves.

Fig. 9 Plastic zone growth on the middle plane and on the side face of the flat specimen.

plastic zone growth between at the middle plane and on the side face of the flat specimen. As pointed out previously⁽¹⁴⁾,⁽¹⁵⁾, much variation of the plastic zone shape along the crack tip does not arise as long as the load remains small. The shape, however, starts to vary when the load increases further. Above 10.9 kN, two plastic zones from the crack tip and the loading point join together near the side face leaving the elastic region on the ligament, which may be referred to as an isolated slip line field⁽¹⁶⁾. The plastic zone size at each loading step can be determined like Fig. 9 on all the surfaces of the elements. When limiting our attention only around the crack tip, the plastic zone growth can be represented three-dimensionally as shown in Fig. 10. As the load increases, the zone near the side face of the flat specimen extends wider as it approaches the side, which implies a plane stress deformation mode. And the plastic zone over only about 50 % of the thickness has the same shapes as on the middle plane. SG specimens, on the other hand, have zone shapes contrary to that of the flat specimen. It is obvious that side grooves restrict the plane-stress-mode deformation near the side al-

though the plastic zone turns to grow along the groove root. At a certain load level, the zone joins the other plastic zone around the loading point at the groove root as shown in Fig. 10(d) at 13.6 kN.

On the U-grooved specimen, the plastic zone shape through the thickness still varies considerably. However, on the V-grooved and machine-slit specimens, almost

invariable plastic zones are attained over about 80 % of the thickness confining the influence of the groove only to a small region at the groove root.

4.3 Dilatant stress distribution along the crack tip

As a stable crack grows through the process of void formation and its coalescence ahead of the tip, the stable crack

Fig. 10 Three-dimensional representations of the plastic zone growth around the crack tip. (a) the flat specimen, (b) the U-grooved specimen, (c) the V-grooved specimen, (d) the slit specimen. The coordinates are chosen as in Fig. 2 with the origin on the crack front.

growth may be determined by the easiness of void formation. And the dilatant stress σ ($= \sigma_{rr}/3$) at the crack tip must have large contributions to the micro-void formation there. Fig. 11 shows the dilatant stress distribution along the crack tip for the flat and slit specimens. The stress at the mid-thickness region of the flat specimen increases almost proportion-

Fig. 11 Variation of the crack-tip dilatant stress. (a) the flat specimen, (b) the slit specimen.

Fig. 12 Difference of the dilatant stress distributions with the groove geometry. Loaded about 8 kN.

ally to the applied load whereas it does not grow much at the near-side region. Accordingly, the stress difference along the tip becomes larger with an increasing load. This makes the mid-thickness region more preferable for the void formation than the other part and, consequently, may cause a thumbnail crack growth.

On the other hand, the distribution on the SG specimens is remarkably altered. The stress has almost the same value over about 90 % of the thickness although it is enhanced at a small region near the groove root. Moreover, the shape of the distribution curve does not vary much with an increasing load. This situation is suitable for the stable crack to start from the entire pre-crack tip at the same time.

Fig. 12 shows a relative distribution curve of the dilatant stress in four kinds of specimens loaded about 8 kN. The stress near the side face of the flat and U-grooved specimens is smaller than at the mid-thickness region, whereas on the V-grooved and slit specimens, the stress is larger near the side than at the mid-thickness. Each stress-distribution corresponds well to each stable crack front as seen in Fig. 1.

4.4 Three-dimensional J integral

At first, in order to see the path independence of J_I , values were obtained along several paths as shown in Fig. 13. The values on the middle plane of the flat specimen are plotted against the path number in Fig. 14. The fluctuation is wide when the path crosses the near-tip region where a steep stress gradient exists, but it becomes small on remote paths from the tip. Hereafter, therefore, an average of the values on the path numbers 3~7 will be used as J_I at the point. J integral value is also obtained in another way. Rice⁽¹⁷⁾ proposed use of the conventional formula to evaluate J experimentally from the load vs. loading-point-displacement diagram. This method can be applied similarly to the diagram obtained in Fig. 8. Fig. 15 compares the value from Rice's formula, J_R , and the actually integrated value, J_I , on the middle plane. On the flat 3PB specimen, J_I is about 10% smaller than J_R . This is contrasting to the previous study⁽¹⁸⁾ showing that the integrated value on the middle plane of a CT specimen coincides with the value from Merkle's formula. The

Fig. 13 Integral path number.

difference may be caused by the stress evaluating method employed here which is different from the method used in analyzing the CT specimen.

On the slit specimen, the difference between J_R and J_I appears to be wider. But this difference was brought about by the specimen thickness. As pointed out previously concerning the $K_{I(2-D)}$, the net thickness at the grooved section actually resulted in an overestimation of J_R . Considering the fact that the SG specimen has about 20 % higher rigidity than the flat specimen, the equivalent thickness should be used in evaluating J_R as well as $K_{I(2-D)}$.

The J_I variation in the thickness direction is shown in Fig. 16. Just as the variation of the dilatant stress, J_I near the side of the flat specimen is hard to increase whereas on the slit specimen, J_I increases almost proportionally to the load over the entire thickness. Large fluctuation near the groove root must be

due to an error in evaluating the derivative with respect to x_3 in the third term on the right hand side of Eq. (4). There exists a steep stress gradient also in the x_3 direction. To avoid this calculation error, Kikuchi et al. (19) proposed that only the value on each middle of the element should be concerned. Such values on the middle plane of the element well fit each curve of respective dilatant stresses as shown in Fig. 17.

Fig. 14 Path independence of J_I evaluated on the middle plane. (a) the flat specimen, (b) the slit specimen.

Fig. 15 Comparison of J_R , evaluated from Rice's formula and J_I , integrate on the middle plane.

Fig. 16 Variation of J_I across the specimen thickness. (a) the flat specimen, (b) the slit specimen.

Fig. 17 Comparison of the distribution curves of J_I and the crack tip dilatant stresses, σ .

5. Conclusions

From elastic-plastic finite element analysis on side-grooved 3PB specimens as well as a flat one under a monotonically increasing load, the following are concluded:

1) The stress intensity value at the mid-thickness region of the flat specimen is about the same as the value in the plane strain, $K_{I(2-D)}$, whereas at the side faces, it falls to about 60 % of $K_{I(2-D)}$. Side grooves such as V-grooves and machine slits make the intensity value larger at the groove root than at the mid-thickness region.

2) Side grooves restrict the plane-stress-mode plastic deformation near the side resulting in a uniform plastic zone shape along the crack tip over about 80 % of the whole thickness.

3) The crack-tip dilatant stress increases almost proportionally to the applied load in the mid-thickness region of both SG and flat specimens. This stress is, however, hard to increase near the side face of the flat specimen, and hence, the stress difference along the tip grows more with an increasing load. The SG specimens attain an increase equally over the whole thickness.

4) The stable crack when starting from a straight pre-crack tip has a front shape well corresponding to the crack tip dilatant stress distribution.

5) J_1 , a component in a crack advancing direction of the three-dimensional J integral, has a similar distribution in the thickness to the dilatant stress.

6) In evaluating the stress intensity factor and the J integral at the middle plane of SG specimens by using the two-dimensional formula, the equivalent thickness must be employed, which is intermediate between the net thickness at the grooved section and the real specimen thickness.

7) In our previous report about J_{IC} testing of machine slit specimens, J_I and J_{IC} values had been overestimated because the net thickness was employed in the Rice's formula. However, it was confirmed that the uniform stress state had been achieved throughout the thickness.

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Appendix Groove geometry approximation

When the groove geometry 1 - 2 - 3 on the x - y plane in Fig. A-1 (a), is mapped on the side $\eta = -1$ as in Fig. A-1 (b), its curve is expressed with a parameter ξ , as⁽⁸⁾,

$$\begin{aligned} x &= \{ (x_1 - 2x_2 + x_3) \xi^2 \\ &\quad + (x_3 - x_1) \xi + 2x_2 \} / 2 \\ y &= \{ (y_1 - 2y_2 + y_3) \xi^2 \\ &\quad + (y_3 - y_1) \xi + 2y_2 \} / 2 \end{aligned}$$

The node 1 must be the groove root, then from the condition,

$$[dx/dy]_{\xi=-1} = 0,$$

we obtain the relation,

$$4(x_2 - x_1) = (x_3 - x_1)$$

To adjust the curvature at the groove root, we have

$$\left[\frac{d^2x}{dy^2} / \left\{ 1 + \left(\frac{dx}{dy} \right)^2 \right\}^{3/2} \right]_{\xi=-1} = 1/\rho,$$

which yields

$$\begin{aligned} 4(y_2 - y_1) &= (y_3 - y_1) \\ &\quad + \sqrt{2\rho(x_3 - x_1)} \end{aligned}$$

The coordinates of nodes 1 and 3 and the groove root curvature, therefore, naturally give the coordinate of the mid-side node.

Following the similar way as above, the groove geometry may also be mapped on the side $\xi = 1$.

Fig. A-1 Mapping of the groove geometry.