The Effect of Radiation on Film Boiling Heat Transfer*  
( Plane Wall Parallel to Upward Vertical Flow )

By Tohru SHIGECHI**, Takehiro ITO***
and Kaneyasu NISHIKAWA***

The effect of radiation on forced convective film boiling heat transfer from a vertical plane to a subcooled liquid is analyzed by the integral method of boundary-layer. Numerical solutions are obtained for water under the atmospheric pressure. The effect of radiation is discussed using some parameters used in the conventional analyses of film boiling and a new parameter for radiative contribution, and the method which Bromley proposed to estimate the radiation effect on total heat transfer is examined.

Key Words: Phase Change, Film Boiling, Radiation, Forced Convection, Subcooled Liquid, Vertical Plate

1. Introduction

In the film boiling heat transfer, since the temperature of heating surface is much higher than those attained in other regimes of boiling, the radiative heat transfer may not be ignored in some cases. When there exists a substantial radiative heat transfer, the vapor film thickness increases due to an enhanced rate of evaporation and thus the rate of convective heat transfer reduces. Hence, a problem to estimate the contributions of both convection and radiation to total heat transfer arises. Bromley(1) analyzed this problem quite intuitively for the first time for pool film boiling from a horizontal cylinder to a saturated liquid, taking notice that total heat transfer is in direct proportion to the rate of evaporation, and the convective heat transfer in inverse proportion to the evaporation rate to the one-third power. He ended by suggesting the following well-known formula of the total heat transfer coefficient $h$ composed of the convective contribution $h_c$ and the radiative contribution $h_r$.

$$h = h_c + h_r$$

Later, Sparrow (2) found the above formula fairly dependable after making a simplified analysis. In addition, he examined the effect of radiatively participating vapor. The result indicates that the vapor film may be regarded as completely transparent to the radiation for steam at near atmospheric pressure.

Thus, the effect of radiation on film boiling heat transfer can be approximately estimated by Bromley's method (Eq.(1)) for saturated pool film boiling. However, some questions still remain to be answered: how should we estimate the effect of radiation for a subcooled liquid and or imposed forced convection; how does Bromley's arithmetic work when applied to such systems? When the effect of subcooling or forced convection is large, the vapor film must become thinner and the resulting convective heat transfer would be promoted. Correspondingly, the effect of radiation becomes relatively weak, and it might be actually neglected. Nevertheless, it will be worthwhile to examine the effect of radiation once in a different boiling system from that of Bromley.

In this report, the effect of radiation on forced convective film boiling heat transfer from a vertical plane to a subcooled liquid is analyzed by the integral method of boundary-layer and Bromley's procedure is examined using the same system.

2. Nomenclature

- $\alpha$: absorptivity of liquid
- $c_p$: specific heat at constant pressure
- $g$: acceleration due to gravity
- $h$: total heat transfer coefficient
- $h_r$: radiation heat transfer coefficient
- $K$: density ratio, Eq. (19)
- $L$: latent heat of vaporization
- $M$: dimensionless parameter for radiative contribution, Eq. (20)
- $Pr$: Prandtl number
- $q$: local heat flux
- $q_r$: local heat flux by radiation
- $\dot{Q}$: heat transfer rate up to $x_1$ per unit width
- $R$: $p_u$ ratio, Eq. (21)
- $Sc$: dimensionless subcooling, Eq. (22)
- $Sp$: dimensionless superheating, Eq. (23)
- $T$: absolute temperature

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** Associate Professor, Faculty of Engineering, Nagasaki University, Nagasaki.
*** Professor, Faculty of Engineering, Kyushu University, Fukuoka.
Since in the present study major attention is paid to the effect of radiation on film boiling heat transfer, the hydrodynamic formulation is simplified by introducing the integral method of a two-phase boundary-layer. The fundamental equations which govern the vapor film thickness $\delta_1$ and the liquid boundary-layer thickness $\Delta l$ (it is assumed that in the liquid the thickness of hydrodynamic boundary-layer is equal to that of thermal boundary-layer), are given by

\[
-\frac{dT_v}{dt} + \frac{q_r}{\rho} = \omega_1 \frac{d^2 T}{dx^2} + \omega_2 \frac{dT_T}{dt} + \cdots \tag{2}
\]

\[
-\frac{dT_L}{dt} + \frac{q_r}{\rho} = \omega_1 \frac{d^2 T}{dx^2} + \omega_2 \frac{dT_T}{dt} + \cdots \tag{3}
\]

Equation (2) manifests the energy balance at the vapor-liquid interface, which was derived on the assumption that the vapor film is completely transparent to the radiative heat transfer and the radiation from the wall to the liquid is completely absorbed at the interface without being transmitted into the bulk liquid. The second term on the l.h.s. of Eq.(2) is the heat flux due to the radiation $q_r$. Here $q_r$ is given by

\[
q_r = h_1(T_w - T_{wm}) \tag{4}
\]

where $h_1$ is the radiation heat transfer coefficient, which was derived on the assumption that both the heating surface (emissivity = 1) and the vapor-liquid interface (absorptivity = 1) are gray and both surfaces consist of two parallel planes. $h_1$ is defined as

\[
h_1 = \frac{\sigma(1+\varepsilon)(1+\varepsilon)}{\varepsilon} \times (T_w - T_{wm})/(T_i - T_{wm}) \tag{5}
\]

Equation (3) is an integrated form of energy equation for the liquid boundary-layer, where $\Delta l$ is eliminated by introducing the integrated form of the continuity equation and the continuity of mass flux at the vapor-liquid interface. For a saturated liquid the second term of r.h.s. of Eq.(2) vanishes and Eq.(3) is rendered needless.

Now, to determine $\Delta l$ and $\delta_1$ by Eqs.(2) and (3), we must prescribe the distributions of velocities $\omega_1$ and temperatures $T_v$ in the vapor film and of velocities $\omega_1$ and temperatures $T_L$ in the liquid boundary-layer. These were determined somehow arbitrarily as follows.

(1) Dropping the inertia term from the momentum equation for the vapor film, where the density of the vapor film is negligible compared with that of liquid ($\rho_L = \rho_v \approx \rho_0$), we have

\[
\rho_v \frac{d^2 u_v}{dx^2} + \rho_0 = 0 \tag{6}
\]

Integrating the above equation under the conditions that $\omega_1 = 0$ at $y = 0$ and $\omega_L = \omega_1$
at \( Y = \delta_1 \), we obtain

\[
\nu_1 = (\frac{u_0}{\delta_1}, \frac{p_0}{\delta_1}), + \frac{p_0}{\mu} \frac{\partial \nu_1}{\partial \delta_1}, \frac{\partial \nu_1}{\partial \delta_1} - \frac{\rho_0 \partial p_0}{\mu \delta_1} \tag{7}
\]

The energy equation for the vapor film with neglected convection term yields

\[
\lambda_1 \frac{\partial T_1}{\partial \delta_1} = 0 \tag{8}
\]

On integration under the conditions that \( T = T_1 \) (constant) at \( Y = 0 \) and \( T = T_{l_{1}} \) at \( Y = \delta_1 \), we get

\[
T = T_{l_{1}} - (T - T_{l_{1}}) \frac{\partial \nu_1}{\partial \delta_1} \tag{9}
\]

(2) Quadratic functions of \( \nu_1 \) for \( u_{l_{1}} \) and \( T_{l_{1}} \) in the liquid boundary-layer are assumed first. Then, forcing these polynomials to comply with the conditions that \( u_{l_{1}} = \delta_1 \) at \( Y = 0 \) and \( u_{l_{1}} = \nu_1 \), \( \mu_1(\nu_1/\partial \nu_1) = 0 \) at \( Y = \delta_1 \) for \( \nu_1 \) and that \( T_{l_{1}} = T_{l_{1}} \) at \( Y = \delta_1 \) and \( T_{l_{1}} = T_{l_{1}} \), \( \lambda_1(\nu_1/\partial \nu_1) = 0 \) at \( Y = \delta_1 \), we get

\[
u_1 = \nu_0 - (\nu_1 - \nu_1) \frac{\partial \nu_1}{\partial \nu_1} + (\nu_0 - \nu_0) \frac{\partial \nu_1}{\partial \nu_1} \tag{10}
\]

\[
T = T_{l_{1}} - (T - T_{l_{1}}) \frac{\partial \nu_1}{\partial \nu_1} \tag{11}
\]

From the compatibility condition for shearing stress at the vapor-liquid interface:

\[ \nu_1 = \delta_1 \tag{12} \]

\[ \nu_1 = \nu_0 \tag{13} \]

a functional relation among \( \nu_1 \), \( \delta_1 \) and \( \lambda_1 \) is deduced

\[ \nu_0 \frac{\partial \nu_1}{\partial \nu_1} + \frac{\partial \nu_1}{\partial \nu_1} \frac{\partial \nu_1}{\partial \nu_1} = \delta_1 \tag{14} \]

Substituting \( \nu_1 \), \( T_1 \), \( u_{l_{1}} \) and \( T_{l_{1}} \) into Eqs. (2) and (3) and eliminating \( \nu_1 \) by Eq. (13), we obtain the following simultaneous ordinary differential equations for \( \delta_1 \) and \( \delta_1 \):

\[
\frac{dY}{dx} + \frac{dX}{dx} = f_1 \tag{15}
\]

\[
\frac{dX}{dx} + \frac{dX}{dx} = f_2 \tag{16}
\]

where the coefficients are as follows:

\[
A = \frac{\partial B}{\partial \mu_1}, \quad B = \frac{\partial C}{\partial \mu_1}, \quad C = \frac{\partial D}{\partial \mu_1}, \quad D = \frac{\partial E}{\partial \mu_1}, \quad E = \frac{\partial F}{\partial \mu_1}, \quad F = \frac{\partial G}{\partial \mu_1}, \quad G = \frac{\partial H}{\partial \mu_1}
\]

Now, introduction of the dimensionless variables and parameters defined by Eqs. (16) through (23), reduces Eqs. (14) and (15) into Eqs. (24) and (25), respectively.

\[
X = \frac{\partial X}{\partial \mu_1}, \quad \frac{\partial X}{\partial \mu_1} \tag{17}
\]

\[
D = \frac{\partial X}{\partial \mu_1}, \quad \frac{\partial D}{\partial \mu_1} \tag{18}
\]

\[
K = \frac{\partial X}{\partial \mu_1}, \quad \frac{\partial K}{\partial \mu_1} \tag{19}
\]

\[
N = \frac{\partial X}{\partial \mu_1}, \quad \frac{\partial N}{\partial \mu_1} \tag{20}
\]

\[
R = \frac{\partial X}{\partial \mu_1}, \quad \frac{\partial R}{\partial \mu_1} \tag{21}
\]

\[
S = \frac{\partial X}{\partial \mu_1}, \quad \frac{\partial S}{\partial \mu_1} \tag{22}
\]

\[
T = \frac{\partial X}{\partial \mu_1}, \quad \frac{\partial T}{\partial \mu_1} \tag{23}
\]

\[
23 \frac{d^2 X}{dx^2} + \frac{dY}{dx} + \frac{dY}{dx} = \frac{dY}{dx} \tag{24}
\]

\[
+ \frac{dY}{dx}, \quad \frac{dX}{dx} \tag{25}
\]

where \( X \) is the inverse of local Froude number \( (u_{l_{1}}/g_{l_{1}}) \) and \( N \) a dimensionless parameter for the contribution of radiation. Density ratio \( K \), \( \nu \), \( \mu \) ratio \( R \), dimensionless superheating \( S \) and dimensionless subcooling \( S \) are parameters which have been often used in the conventional analyses of film boiling heat transfer.

By integrating the resulting final equations of Eqs. (24) and (25) for prescribed set of parameters \( K \), \( R \), \( N \), \( P \), \( R \), \( S \) and \( S \), the local heat flux \( q \) and the rate of heat transfer up to \( X = 0 \) per unit width \( Q \) are calculated as follows.

\[
q = \frac{dX}{dx}, \quad \frac{dX}{dx} \tag{26}
\]

\[
Q = \frac{dX}{dx}, \quad \frac{dX}{dx} \tag{27}
\]

Since the simultaneous differential equations, Eqs. (24) and (25), have a singular point at \( X = 0 \), the forward-integration of them with the conditions of \( 3 = 0 \) at \( X = 0 \) cannot be directly performed starting from there. In the absence of radiative contribution the boundary-layers will develop as \( \delta = X/2 \) and \( \delta = X^2/2 \) in the neighborhood of \( X = 0 \). Thus, in the presence of radiative contribution we assume the following series solutions, taking the predominance of convective contribution around \( X = 0 \) into consideration,

\[
\alpha = \frac{1}{\alpha}, \quad \frac{1}{\alpha} \tag{28}
\]

\[
\beta = \frac{1}{\beta} \tag{29}
\]

Series solutions up to the second term will suffice because of the limited use of them only at the proximity of the leading edge. The coefficients \( a_0, a_1, b_0 \) and \( b_1 \) are determined by substituting Eqs. (28)
and (29) into Eqs.(24) and (25). Needless to say, $a_i = b_i = 0$ for $N = 0$. These series solutions were employed up to $X$ of $10^{-4}$ or $10^{-5}$. Starting from there, Eqs.(24) and (25) were forward integrated numerically by Runge-Kutta-Gill method.

4. Results and Discussion

Numerical integrations were carried out for water under the standard atmospheric pressure (0.1013 MPa). This fixed three of the six parameters are as follows

$$K = 1600, R = 0.0051, Pr = 1.76$$

while the rest, $S_p$, $S_c$ and $N$ were varied systematically. Showing the parametric dependence of the ratio $Q/Q_0$ (or $Q/Q_0$) ($Q$ or $Q_0$):combined convective and radiative heat transfer, $Q_0$:pure convective heat transfer without radiation $(N = 0)$ might be preferable to demonstrating the radiative contribution. However, practically the total heat transfer coefficient $h$ in terms of $h_0$ and $h_r$ under the same conditions ($h_0$:the coefficient of purely convective heat transfer without radiation, $h_r$:that of radiative heat transfer) would be more convenient. Therefore, the expressions selected are,

local $: \frac{(h - h_0)h}{h - h_r}\frac{1}{\left[(1/N) - 1/N_0\right]}$ (30)

average $: \frac{(h - h_0)h}{h - h_r}\frac{1}{\left[(1/N) - 1/N_0\right]} + N \int_{0}^{X} \left(\frac{1}{h} \right) dX$ (31)

where $\delta_0$ is the vapor film thickness for pure convection, which is determined by putting $N = 0$ in Eqs.(24) and (25). In Figs.2 and 3, $X = (h - h_0)/h_0$ of Eq.(31) is taken as the ordinate and $X = (gX_l)/U_{\infty}$ to the one-fourth power as the abscissa, where the present results are drawn in solid lines. In these figures dashed curves indicate the values calculated by Bromley's procedure (Eq.(1)). To the best knowledge of the present authors, it is not clear whether heat transfer coefficients in Eq.(1) are to be regarded as local coefficients or average coefficients if $h$ and $h_0$ in Eq.(1) are taken as local coefficients, then Eq.(1) can be rewritten into

$$\frac{h - h_0}{h_0} = \frac{(h - h_0)/h_0^4}{1/N_0}$$ (32)

Alternatively, if $h$ and $h_0$ represent average coefficients, Eq.(1) becomes

$$\frac{h - h_0}{h_0} = \frac{\int (h - h_0)/h_0^4 dX}{1/N}$$ (33)

The solutions for $N = 0$ in this report are substituted into $\delta_0$ in Eqs.(32) and (33) to get $h/h_0$. Thus, we may evaluate the quantities corresponding to Eq.(30) or Eq.(31) as follows:

$$\frac{(h - h_0)}{h_0} = \frac{1}{(h/h_0 - 1)} \int_{0}^{X} \left(\frac{1}{h} \right) dX$$ (31)

or

$$\frac{(h - h_0)}{h_0} = \frac{1}{(h/h_0 - 1)} \int_{0}^{X} \left(\frac{1}{h} \right) dX$$ (33)

Being derived for saturated pool film boiling, Bromley's method might not be applied for a subcooled liquid and/or liquids with forced convection. However, the results calculated by Bromley's procedure ($h_0$ and $\delta_0$ are those of the pre-

![Fig.2 The effect of radiation parameter N](image)

![Fig.3 The effect of dimensionless subcooling Sc](image)
sent analysis) are also shown in these figures to examine the applicability to the present system. Since the abscissa $\lambda$, in Figs. 2 and 3 is the reciprocal of Froude number $U_{\infty}^2 / g x_1$, smaller values of $\lambda$ (the right-hand side of the figures) correspond to the region where the forced convection predominates and larger values of $\lambda$ (the left-hand side of the figures) the region where the body force predominates. Figure 2 shows the effect of parameter $N$ at $S_p = 0.5$ and $S_c = 0.01$, while Fig. 3 demonstrates the effect of parameter $S_c$ at $S_p = 0.5$ and $N = 2.0$. In these figures, the ordinate approaches a constant value in the smaller region of $\lambda$, i.e., in the region where the forced convection predominates. The constant is $2/3$ for $S_c = 0$ (a saturated liquid) regardless of $S_p$. When $S_c > 0$, it decreases with an increase of $S_c$. Incidentally, it is always equal to $3/4$ when Bromley's arithmetic is applied. In the region of large $\lambda$ the ordinate approaches unity. This means that the difference between $\delta$ and $\delta_g$ vanishes as the vapor film thickness increases and

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{The results of systematic calculations at the atmospheric pressure}
\end{figure}
the simple addition rule of \( h = h_0 + h_r \) holds. \( Sp \) in Figs. 2 and 3 is fixed at 0.5. With an increasing \( Sc \), curve moves qualitatively in the direction of a decreasing \( Sc \) (Fig.4).

Summing up, for thin vapor film, \( \{(h-h_0)/h_r\}_M \) decreases and approaches a constant value. And for thick vapor film, \( \{(h-h_0)/h_r\}_M \) increases and approaches unity. In addition, Bromley's method overestimates the effect of radiation for thin vapor film. The overestimation gets large for large degree of subcooling and free-stream velocity and for small degree of superheating, i.e., when the vapor film is thin. The asymptotic value of \( \{(h-h_0)/h_r\}_L \) for small \( X \) is the same as that of \( \{(h-h_0)/h_r\}_M \). At a same value of \( X \) \( \{(h-h_0)/h_r\}_L \) is a little larger than \( \{(h-h_0)/h_r\}_M \).

Finally, the results of the systematic calculations are summarized in Fig.4 for water at the atmospheric pressure over the range of \( Sp = 0.25, 0.50, 1.0 \) and \( Sc = 0.005, 0.01, 0.02, 0.04 \). \( N \) is, as seen from Eqs.(5) and (20), a dimensionless number including the free-stream velocity \( U_o \) and the heating surface temperature \( T_w \). The magnitude of \( N \) (1/4 \( A/a-1 \)) is shown in Table 1 for water at the atmospheric pressure.

Table 1 The magnitude of radiation parameter \( N \) for water at the atmospheric pressure.

<table>
<thead>
<tr>
<th>( S_p )</th>
<th>( T_r - T_m )</th>
<th>( U_o ) m/s</th>
<th>( N ) (1/4 ( A/a-1 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>112</td>
<td>0.108</td>
<td>0.341</td>
</tr>
<tr>
<td>0.5</td>
<td>560</td>
<td>0.443</td>
<td>1.40</td>
</tr>
<tr>
<td>1.0</td>
<td>1200</td>
<td>1.44</td>
<td>4.09</td>
</tr>
</tbody>
</table>

5. Conclusions

The effect of radiation on forced convective film boiling to a subcooled liquid from a vertical plane was analyzed and the validity of Bromley's arithmetic (Eq.(1)) was tested. It awaits for future studies to develop a procedure by which total heat transfer coefficient can be conveniently estimated (correlations equation for example) for the system dealt with here.

References