Numerical Analysis of Pneumatic Capsule Pipeline System

by the Method of Characteristics

(Continuous Loading under a Constant Cycle)*

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The numerical analysis on the motion of the capsules are surveyed for the condition that the same capsules of 20 kg in mass are pneumatically transported at a constant loading time interval in a horizontal and straight pipeline of 0.2 m I.D. and 500 m length under a constant inlet pressure. The results show that the mean capsule velocity is decreased a decreased loading cycle and that when there is air leakage through the clearance between the capsule and the pipe wall the capsule velocity is also decreased but the transport becomes smooth.

Key Words: Compressible Flow, Unsteady Flow, Fluid Transportation System, Capsule Transport, Numerical Analysis, Continuous Loading

1. Introduction

In the previous papers⁽¹⁾⁽²⁾, we examined the motion of a single pneumatic capsule under a constant inlet pressure and showed that the method of Lur'e et al.⁽³⁾, which uses the method of characteristics and neglects the nonlinear inertial term in the equation of motion for air flow, is a good approximation in the low speed regions. The analysis of a single capsule has its own significance as a method of communication for dispatch tube in a building like hospital. However, for a large-scale transport of industrial materials over a long distance it is necessary to analyze such a case when many capsules are loaded periodically in a pipeline. Otaki⁽⁴⁾ obtained interesting results through the numerical analysis of this case assuming that the air flow is incompressible and that the capsules are driven by the air drag on them when the air stream passes through the clearance between the capsule and the pipe wall. However, it seems there is a problem in applying the continuity equation of air flow to the capsule boundary. Furthermore, change of air velocity will attain a considerably high level associated with a large pressure drop in the transport of long distance even if the air velocity at the pipe inlet is small. Therefore, in such a case, there arises a problem of accuracy in the analysis using an incompressible flow model.

In the present paper, the numerical analysis is performed considering the compressibility of air flow by the method of characteristics when capsules are loaded into the pipeline at a constant period under a constant air pressure at the pipe inlet, and the effects of capsule loading cycle and of clearance air flow between the capsule and the pipe wall on the characteristics of

capsules motion are examined. Lur'e⁽³⁾ also states that when there are enough number of capsules in the pipeline it is possible to apply a distributed mass model to the capsules motion and he derives an analytical model of the capsules motion as a function of time and space coordinates. He gives an analytical solution for a steady capsule flow assuming that the air flow is isothermal, there is no clearance air flow and the capsules are loaded into the pipeline at a constant air pressure at the pipe inlet. We compare our numerical solution with this analytical solution.

Nomenclature

- A : pipe cross sectional area
- A_c : capsule cross sectional area
- c : coefficient of discharge through the clearance between the capsule and the pipe wall
- d : inside diameter of pipe
- F^* : dimensionless friction force on the capsule $= g/M/p_o A$
- f : kinematic friction coefficient between the capsule and the pipe wall
- g : gravity acceleration
- L : pipeline length
- M : mass of capsule
- M^* : dimensionless mass of capsule $= MRT/p_o AL$
- N : number of capsule in the pipeline
- n : concentration of capsule
- p : pressure
- p_i : pressure at the pipe inlet
- p_o : pressure at the pipe exit
- Δp : pressure difference across the capsule $= p_o - p_f$
- Δp^* : dimensionless pressure difference across the capsule $= \Delta p/p_o$
- Q_i : air flow rate at the pipe inlet m^3/min
- Q_c : air mass flow rate at the clearance
- R : gas constant
- T : absolute temperature of air in the pipe
- T_r : residence time of capsule in the pipeline
- t : time

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t^* : dimensionless time $= t\sqrt{RT}/L = tu_c/L$
 Δt : time step in the computation
 u : air velocity
 x : axial coordinates
 Δx : grid length in the computation
 Z : capsule coordinates
 $\dot{Z}$: dimensionless capsule velocity $= \dot{Z}/u_c$
 $\dot{Z}_{av}$: average capsule velocity $= L/T, u_c$
 $\dot{Z}_{mean}$: mean velocity of capsule obtained from numerical analysis
 λ : pipe friction coefficient for air
 ρ : air density
 ρ_0 : air density at the pipe inlet
 $\bar{\rho}$: average air density in the clearance
 r : capsule loading cycle
 Subscripts
 F : front side of capsule
 R : rear side of capsule
 i : i -th capsule

2. Basic Equations and Procedure for Calculation

The computation is based on the following assumptions:

- (1) The air flow in the pipe is a one-dimensional isothermal flow of an ideal compressible gas.
- (2) The pipeline is straight and horizontal with constant cross sectional area and the pipe is hydraulically smooth.
- (3) The kinematic friction coefficient between the capsule and the pipe wall is constant.
- (4) All capsules have the same shape and dynamical characteristics and are injected into the pipe at zero initial velocity with a constant period.
- (5) The capsule is driven only by the pressure difference across the capsule.
- (6) The convective acceleration term in the equation of motion for the air flow is neglected.
- (7) The pressure at the pipe inlet is constant.

Under these assumptions the equations of state, continuity and motion for the air and the equation of motion for the capsule are

$$\frac{p}{\rho} = RT = \text{const.} \quad (1)$$

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \quad (2)$$

$$\frac{\partial}{\partial t}(\rho u) = -\frac{\partial p}{\partial x} - \frac{\lambda}{2d}\rho u^2 \quad (3)$$

$$M_i \dot{Z}_i = (p_{Ri} - p_{Fi})A - fM_i g \quad (4)$$

The boundary conditions at the capsule are

$$\dot{Z}_i = u_{Fi} = u_{Ri} \quad (5)$$

when there is no clearance air flow, and

$$\rho_{Ri} u_{Ri} = \rho_{Ri} \dot{Z}_i + \left(1 - \frac{A_c}{A}\right) Q_{Hi} \quad (6)$$

$$\rho_{Fi} u_{Fi} = \rho_{Fi} \dot{Z}_i + \left(1 - \frac{A_c}{A}\right) Q_{Hi} \quad (7)$$

when there is air leakage through the clearance⁽¹⁾. As shown in the previous paper⁽¹⁾, a reversal of pressure difference across the capsule can take place. Thus, we assume that the air leaks from the higher pressure side to the lower and put

$$Q_{Hi} = c\sqrt{2\bar{\rho}_i(p_{Ri} - p_{Fi})}, \quad p_{Ri} \geq p_{Fi} \quad (8)$$

$$Q_{Hi} = -c\sqrt{2\bar{\rho}_i(p_{Fi} - p_{Ri})}, \quad p_{Ri} < p_{Fi} \quad (9)$$

where $\bar{\rho}_i$ is given by

$$\bar{\rho}_i = \frac{1}{2}(\rho_{Ri} + \rho_{Fi}) \quad (10)$$

The boundary conditions at the inlet and exit of pipe are

$$x=0; p=p_i, \quad x=L; p=p_o \quad (11)$$

For a pressure system of transportation p_i is larger than p_o . The initial conditions are given as follows

$$\left. \begin{aligned} t=0, x>0; p=p_o, \quad u=0 \\ t=0, x=0; p=p_i \end{aligned} \right\} \quad (12)$$

and for the i -th capsule we give

$$t=(i-1)r; \dot{Z}_i=0, Z_i=0 \quad (13)$$

The calculation was carried out for the case of no collision of the capsules by the method of characteristics as in the previous paper⁽¹⁾, where we put Δt as sufficiently smaller than τ and the interval between capsules as sufficiently larger than Δx .

For comparison with numerical solution later we write Lur'e's analytical solution⁽¹⁾ when the capsules are injected into the isothermal air flow in a pipe with a constant period under a constant air pressure at the pipe inlet and there is no air leakage through the clearance between the capsule and the pipe wall.

$$\dot{Z}(x) = \dot{Z}(0)(1 - \lambda \dot{Z}(0)x/u_c^2 d)^{-1/2} \quad (14)$$

$$\Delta p(x) = \frac{\lambda M \dot{Z}(0)}{2u_c^2 A d} \left(1 - \frac{\lambda \dot{Z}(0)}{u_c^2} \frac{x}{d}\right)^{-2} + \frac{fMg}{A} \quad (15)$$

$$n(x) = n(0)(1 - \lambda \dot{Z}(0)x/u_c^2 d)^{1/2} \quad (16)$$

$$N = \frac{2n(0)u_c^2 d}{3\lambda \dot{Z}(0)} \left\{1 - \left(1 - \lambda \frac{\dot{Z}(0)}{u_c^2} \frac{L}{d}\right)^{3/2}\right\} \quad (17)$$

$$T_r = \tau N \quad (18)$$

$$\dot{Z}_{av} = L/T_r \quad (19)$$

$$\tau = 1/\{n(0)\dot{Z}(0)\} \quad (20)$$

Since this solution is obtained from the distributed mass model, $\dot{Z}(0)$ can not be zero in a steady flow from the continuity equation for capsules. This point differs from the present numerical solution which follows the motion of each capsule.

3. Results and Discussion

3.1 Case when there is no air leakage

The following results are obtained when we put $A=A_c, c=0, L=500\text{ m}, d=0.2\text{ m}, M=20\text{ kg}, f=0.5, T=293\text{ K}, p_o=101.3\text{ kPa}, \{1.033\text{ kg}\cdot\text{f}/\text{cm}^2\}$, and $p_i/p_o=2.5$, λ being the values for hydraulically smooth pipe. In this case dimensionless parameters are $M^*=1.057$ and $F^*=0.03081$. To see the effects of the number of capsules in the pipeline we calculated three cases of capsule loading time intervals of 6, 3 and 1.5s, and the maximum number of capsules injected into the pipeline was selected as 6, 9 and 12 correspondingly at which numbers the quantities such as Q_i change in nearly the same pattern in relation to the time.

Figure 1 shows the capsules coordinates and inlet air flow rate versus time elapsed from the start of transportation for $r=6\text{ s}$. This is an example of two capsules in the pipeline. The first capsule follows almost the same trajectory with that of a single capsule. Since the air stream at the pipe inlet is instantaneously shut at every capsule injection, Q_i correspondingly falls to zero and repeats a pulsating change. Figure 2 shows the mean velocity of each

capsule over the pipeline versus the order of injection, where the total number of capsules used for the calculation is six. It is seen that each capsule makes almost the same behavior after the 3rd capsule.

Figure 3 shows a relation between Z^* and Z , that is Z^* and r . The first capsule ($i=1$) makes almost the same behavior with a single capsule. After the second capsule nearly the same orbit is observed, which is similar to that obtained when the capsule is injected into the steady flow under the same inlet pressure. The velocity of each capsule increases toward the exit of pipeline. However, unlike the first capsule, Z^* has no extreme value, which comes from the fact that the air velocity increases with expansion of air. This is a reverse tendency to the result obtained by Otaki⁽⁴⁾. Figure 4 shows the relation between the pressure difference across the capsule and Z^* . Except the first capsule almost the same behavior is seen and a reversal of the pressure difference which is seen for the single or the first capsule does not occur. The chain lines in the figures after Fig. 3 show the analytical solution given by Lur'e. The initial velocity is finite in this solution, then the virtual initial velocity should be given to compare with the present calculation. It is given by applying Eq.(14) at $r=L$ and substituting Z of the numerical solution at this point, where λ is that for the steady flow. Using this $Z(0)$, N becomes 1.293. Substituting this N for

Eq.(18) to obtain T , the average velocity is given as $\bar{Z}_{av}=64.46$ m/s. Though the number of capsules in the pipeline is considered to be considerably small, it is said that the analytical solution agrees comparatively well.

Figures 5-8 show similar results for $r=3$ s. The number of capsules injected is 9. According to the above-mentioned method, N is 2.621, $\bar{Z}_{av}$ is 63.59 m/s and this an example of three capsules in the pipe. This value of $\bar{Z}_{av}$ is a little higher than $\bar{Z}_{mean}$ s for $i=2\sim 8$ in Fig. 6. After the third or fourth capsule Q_i and others repeat nearly the same behaviors. Since after the last capsule there are no succeeding capsules, the behavior of that capsule differs from the others. The dotted lines in the figures show the case of a single capsule. It is seen that the deviation from behavior of the single capsule becomes larger as the capsule flows down the pipeline even if it is the first capsule. Figure 5 shows that the motions of capsules after the second one are not so simple as shown in Fig. 1; that is, there is an oscillation in Z , which is also seen from the comparison of Fig. 7 with Fig. 3. From Fig. 8 it is seen that near $Z^*=0.2$ a reversal of Δp clearly appears in the case of the first capsule, but this is not explained by the analytical solution.

Figures 9-12 show similar relations for $r=1.5$ s. The capsules injected are 12. There are 6 capsules in the pipe at most,

Fig. 1 Displacement of each capsule and inlet air flow rate versus time ($r=6$ s).

Fig. 2 Mean capsule velocity versus order of loading ($r=6$ s).

Fig. 3 Velocity-displacement diagram of each capsule ($r=6$ s).

Fig. 4 Pressure difference across the capsule ($r=6$ s).

Fig. 5 Displacement of each capsule and inlet air flow rate versus time ($r=3s$).

Fig. 6 Mean capsule velocity versus order of loading ($r=3s$).

Fig. 7 Velocity-displacement diagram of each capsule ($r=3s$).

Fig. 8 Pressure difference across the capsule ($r=3s$).

N is 5.349 and $\dot{Z}_v$ is 62.32 m/s which is larger than $\dot{Z}_{mean}$ in Fig. 10. The relation Z and t in Fig. 9 is considerably distorted compared with that in Fig. 5. With a further decrease of r , the intervals between the capsules become shorter so that there appears a phenomenon relating to collision of capsules. After the first capsule goes out of the pipeline, the mean value of Q_i over the time seems to get constant. In this sense the timing for the steady state initiation is the same as in the case of $r=6$ and $3s$, but the deviation from the mean value of

Q_i is increased. With an increase in number of the capsules per unit length, the condition for the distributed mass model is likely to be satisfied better, then a better agreement with the numerical solution is expected. However, as far as Figs. 10-12 are concerned, such phenomenon does not occur. The analytical solution coincides with the average of the numerical solutions, but it can not explain the appearance of a reversal of Δp .

According to the distributed mass model, the mass flow rate of

Fig. 9 Displacement of each capsule and inlet air flow rate versus time ($r=1.5s$).

Fig. 10 Mean capsule velocity versus order of loading ($r=1.5s$).

Fig. 11 Velocity-displacement diagram of each capsule ($r=1.5s$).

capsules is M/r in the steady flow. Then the mass flow rate increases when r becomes smaller. Furthermore, since Q_i is given by $AZ(0)$, it is easily conceivable that for a given transport pressure Q_i decreases with reduction of $Z(0)$. Figures 3, 7 and 11 show actually that $Z(0)$ obtained by this model decreases a little but surely as r becomes small. Thus, within the present calculation, the mass flow rate increases with a decrease of r and the power consumption becomes correspondingly favourable.

3.2 Case when there is air leakage

The data for calculation are selected the same as those in 3.1. The diameter of capsule is 0.19m and the discharge coefficient of the clearance between the capsule and the pipe wall is set at 0.5. Examples of the results for $r=1.5s$ are shown in the following except Fig. 14 where Z_{mean} of other cases are shown. The dotted lines in Fig. 13 indicates the case of a single capsule and the chain lines that of a single capsule without the air leakage. It is seen from these results that when there is

Fig. 12 Pressure difference across the capsule ($r=1.5s$).

Fig. 13 Displacement of each capsule and inlet air flow rate versus time ($r=1.5s$, with air leakage).

Fig. 14 Mean capsule velocity versus order of loading (with air leakage).

Fig. 15 Velocity-displacement diagram of each capsule ($r=1.5s$, with air leakage).

air leakage the capsule velocity drops. As compared with Fig. 9, the relation Z_i is more smooth than that in the case without air leakage. Also from the comparison of Figs. 11 and 15, the fluctuation of velocity is seen to become small in the case of existence of the air leakage. The dotted lines in Figs. 15 and 16 also show the case of a single capsule with air leakage. It is seen from Fig. 13 that due to the air leakage Q_i does not fall to zero even at the moment of the capsule loading, though Q_i seems to be a little small compared with that of Fig. 9. However, Fig. 14 shows that the total number of capsules, 12, is not enough to make the charac-

Fig. 16 Pressure difference across the capsule ($r=1.5s$, with air leakage).

teristics of capsule motion free from the effects of transient section in the case of $\tau=1.5$ s. A reversal of pressure difference across the capsule appears clearly even if the air leakage exists, but it becomes smaller than that without the air leakage. This is caused by the fact that due to the air leakage through the clearance, the discontinuity of pressure difference across the capsule is somewhat reduced.

Figure 17 shows the relation between the mean velocity and the capsule loading cycle in a period without the transient intervals at the start and end of transportation. It is observed that the mean velocity is decreased by the air leakage or by a decreased loading cycle. On the other hand, if the loading cycle is long, the mean velocity approaches that of a single capsule injected into a steady flow under a constant transport pressure, where $\bar{z}_{mean}$ of $\tau=6$ s may be said to nearly saturate.

4. Conclusions

The motion of pneumatic capsules is numerically analyzed by the method of characteristics when capsules of the same mass and friction coefficient are continuously injected into the pipeline at a constant time interval under a constant transport pressure. The pipeline is assumed to be a straight and horizontal pipe of 0.2 m I.D., the mass of capsule is 20 kg, and the inlet pressure is 2.5 in terms of pressure ratio to the exit pressure. We calculated both cases with an air leakage through the clearance between the capsule and the pipe wall and with no air leakage for three capsules loading time intervals. The results indicate that the velocity of capsule falls with a decrease in the capsules loading time intervals and at the same time with an increase in the air leakage. The reversal of pressure difference across the capsule which is seen in the case

Fig. 17 Mean capsule velocity versus capsule loading cycle.

of a single capsule is not observed after the second capsule if the loading cycle exceeds the residence time of the capsule in the pipeline, but appears in other cases. When there is air leakage, the fluctuations of capsule velocities and pressure difference across the capsules are relaxed; otherwise they would be increased with a decrease in the loading time interval. If the loading time interval is further decreased, it is predictable that there will happen a collision of capsules.

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