Heat Transfer in a Parallelogram Shaped Enclosure *

(4th Report, Combined free convection, radiation and conduction heat transfer)

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Two dimensional free convection and radiation in parallelogram shaped enclosures which are piled up infinitely, have been studied. In the computation, the thickness of the parting plates and the heat conduction in them are taken into account. The nondimensional thickness of the parting plates ranges from 5×10^{-3} to 0.5, the nondimensional conductivity of the parting plate ranges from 1 to 500, the aspect ratios of the enclosure are 0.5 to 1, and the oblique angle of the parting plate is $\omega/3$. In addition, using several kinds of parting plates of different thicknesses and different conductivities, the heat transfer by the convection was measured. The results are compared with the computations.

Key Words: Convective Heat Transfer, Numerical Calculation, Experimental Study, Parallelogram Shaped Enclosure, Combined Heat Transfer

1. Introduction

This paper is concerned with the thermal insulation by a pair of vertical walls with air between them. In the authors' 1st report (1), it was shown that there are some possibilities of the heat transfer decreasing by convection, when parting plates are obliquely installed between two vertical plates as shown in Fig. 1.

In the authors' 2nd report (2), the results of numerical computations in a situation where an infinite number of enclosures of the same shape were piled up vertically and every parting plate had no thickness, were reported. And the results of experiments in a situation where 5 enclosures of the same shape were piled up vertically, were also reported. But there were some differences between numerical and experimental results with regard to the rate of the heat transfer.

In the authors' 3rd report (3), the radiation was considered in the calculation. It was shown that one of the causes of the difference was the influence of the radiation. And the features of the heat transfer by convection and radiation were shown.

In this report, the thickness of the parting plates and the heat conduction in them are considered in the calculation. Experiments are also performed using several parting plates of different thicknesses and conductivities. And results of the computation are compared with measurements.

Nomenclature

a	: thermal diffusivity m^2/s
d	: distance between hot and cold walls m
E	: emissivity
G, $\tilde{G}$	: radiosity W/m^2 , nondimensional radiosity $= G/\sigma T_h^4$
H, H_p	: aspect ratio $= \ell/d$, nondimensional thickness of parting plate $= p/d$

J, $\tilde{J}$	: irradiation W/m^2 , nondimensional irradiation $= J/\sigma T_h^4$
ℓ	: height of hot and cold walls of enclosure m
N_R	: nondimensional radiation number $= \lambda (T_h - T_c)/\sigma T_h^4 d$
Nu	: Nusselt number $= \alpha d/\lambda$
p	: thickness of parting plate in x-direction m
Pr	: Prandtl number $= \nu/a_a$
Ra	: Rayleigh number $= g\beta d^3 (T_h - T_c)/a_a \nu$
t	: time s
T	: temperature K
U, V	: nondimensional velocity in x and y directions $= u d/a_a = v d/a_a$
X, X^+ , Y	: nondimensional coordinates $= x/\ell, = x/p, = y/d$
Y^*	: nondimensional y' coordinate $= (y'/d) \cos \omega$
α	: heat transfer coefficient $\text{W}/\text{m}^2 \text{K}$
γ, Γ	: vorticity $1/\text{s}$, nondimensional vorticity $= \gamma d^2/a_a$
θ	: nondimensional temperature $= (T - T_c)/(T_h - T_c)$
Θ	: radiation number $= T_c/T_h$
λ	: thermal conductivity W/mK
σ	: Stefan Boltzmann constant $\text{W}/\text{m}^2 \text{K}^4$
τ	: nondimensional time $= t a_a/d^2$
ψ, Ψ	: stream function m^2/s , nondimensional stream function $= \psi/a_a$
ω	: angle between y' - and y-axis rad

Subscripts

a	: air
c	: cold wall
h	: hot wall
L	: upper surface of lower parting plate
m	: average value
p	: parting plate
r	: representative value
R	: radiation
U	: lower surface of upper parting plate

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2. Governing Equation and Method of Computation

The physical situation is pictured schematically in Fig. 1. An infinite number of enclosures are piled up

vertically. The thickness of the parting plates is $p \cos \omega$ (the thermal conductivity is λ_p). The working fluid is a nonradiating gas. The hot and cold walls are grey with an emissivity of E_h . The parting plates are grey with an emissivity of E_p . It is assumed that the flow patterns in all enclosures are the same. This is also true for the temperature distribution in the enclosures and the parting plates. Therefore, the free convection within one enclosure and the heat conduction in one parting plate are computed.

In the authors' previous reports, the steady state governing equations were solved, but in this report, the unsteady equations are solved to obtain a steady state solution. It is further assumed that the flow within the enclosure is two dimensional and thermophysical properties are constant for temperature. Under these assumptions, the nondimensional vorticity equation is

$$\frac{\partial \Gamma}{\partial \tau} + \frac{U + V \tan \omega}{H} \frac{\partial \Gamma}{\partial X} + V \frac{\partial \Gamma}{\partial Y^*} = P_r \nabla^2 \Gamma + R_a P_r \left(\frac{\tan \omega}{H} \frac{\partial \theta}{\partial X} + \frac{\partial \theta}{\partial Y^*} \right) \quad (1)$$

The nondimensional energy equation is

$$\frac{\partial \theta}{\partial \tau} + \frac{U + V \tan \omega}{H} \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y^*} = \nabla^2 \theta \quad (2)$$

The nondimensional heat conduction equation within the parting plate is

$$\frac{\partial \theta}{\partial \tau} = \frac{a_p}{a_a} \left(\frac{1 + \tan^2 \omega}{H_p^2} \frac{\partial^2 \theta}{\partial X^{*2}} + \frac{2 \tan \omega}{H_p} \frac{\partial^2 \theta}{\partial X^* \partial Y^*} + \frac{\partial^2 \theta}{\partial Y^{*2}} \right) \quad (3)$$

where

$$\nabla^2 = \frac{1 + \tan^2 \omega}{H^2} \frac{\partial^2}{\partial X^2} + \frac{2 \tan \omega}{H} \frac{\partial^2}{\partial X \partial Y^*} + \frac{\partial^2}{\partial Y^{*2}} \quad (4)$$

The derivatives with respect to Y in Eqs. (1) to (3) are replaced by the derivatives with respect to X and Y^* .

The relations between the nondimensional vorticity Γ , the nondimensional velocities U and V and the nondimensional stream function Ψ are

Fig. 1 Illustration of problem

$$\Gamma = -\nabla^2 \Psi \quad (5)$$

$$U = \frac{\tan \omega}{H} \frac{\partial \Psi}{\partial X} + \frac{\partial \Psi}{\partial Y^*} \quad (6)$$

$$V = -\frac{1}{H} \frac{\partial \Psi}{\partial X} \quad (7)$$

On the surface of the parting plate, the sum of the heat transfer by convection and radiation becomes equal to the heat conduction into the parting plate. Therefore, radiosity and irradiation, respectively, at location y on the lower surface of the upper parting plate of the enclosure ($x + y \tan \omega = 0$, this surface is called a surface "U") are defined as $G_U(y)$ and $J_U(y)$. Then the following equation is obtained

$$\begin{aligned} & -\lambda_a \left(\cos \omega \frac{\partial T}{\partial x} + \sin \omega \frac{\partial T}{\partial y} \right)_{x+y \tan \omega = 0} \\ & + \lambda_p \left(\cos \omega \frac{\partial T}{\partial x} + \sin \omega \frac{\partial T}{\partial y} \right)_{x+y \tan \omega = 0} \\ & + G_U(y) - J_U(y) = 0 \end{aligned} \quad (8)$$

where the first and second terms of the left hand side of Eq. (8), respectively, represent the heat transfer by convection and the radiation from the surface U. Radiosity and irradiation, respectively, at location y on the upper surface of the lower parting plate ($x + y \tan \omega = l$, this surface is called a surface "L") are defined as $G_L(y)$ and $J_L(y)$. Then the following equation is obtained.

$$\begin{aligned} & \lambda_a \left(\cos \omega \frac{\partial T}{\partial x} + \sin \omega \frac{\partial T}{\partial y} \right)_{x+y \tan \omega = l} \\ & - \lambda_p \left(\cos \omega \frac{\partial T}{\partial x} + \sin \omega \frac{\partial T}{\partial y} \right)_{x+y \tan \omega = l} \\ & + G_L(y) - J_L(y) = 0 \end{aligned} \quad (9)$$

where the first term of the left hand side of Eq. (9) represents the heat transfer by convection from the surface L. And the second term shows the heat conduction into the parting plate.

Being multiplied by a value $1/\sigma T_h^4$, Eqs. (8) and (9) can be rewritten in a dimensionless form. Furthermore, the derivatives with respect to Y , namely $\partial/\partial Y$, are replaced by the derivatives with respect to X and Y^* . Then we obtain

$$\begin{aligned} N_R \left\{ - \left(\frac{1}{H \cos \omega} \frac{\partial \theta}{\partial X} + \sin \omega \frac{\partial \theta}{\partial Y^*} \right)_{x+(Y/H) \tan \omega = 0} \right. \\ \left. + \frac{\lambda_p}{\lambda_a} \left(\frac{1}{H_p \cos \omega} \frac{\partial \theta}{\partial X^*} + \sin \omega \frac{\partial \theta}{\partial Y^*} \right)_{x+(Y/H_p) \tan \omega = 0} \right\} \\ + \tilde{G}_U(Y) - \tilde{J}_U(Y) = 0 \end{aligned} \quad (10)$$

$$\begin{aligned} N_R \left\{ \left(\frac{1}{H \cos \omega} \frac{\partial \theta}{\partial X} + \sin \omega \frac{\partial \theta}{\partial Y^*} \right)_{x+(Y/H) \tan \omega = l} \right. \\ \left. - \frac{\lambda_p}{\lambda_a} \left(\frac{1}{H_p \cos \omega} \frac{\partial \theta}{\partial X^*} + \sin \omega \frac{\partial \theta}{\partial Y^*} \right)_{x+(Y/H_p) \tan \omega = l} \right\} \\ + \tilde{G}_L(Y) - \tilde{J}_L(Y) = 0 \end{aligned} \quad (11)$$

where $\tilde{G}(Y)$ and $\tilde{J}(Y)$ are the nondimensional radiosity and irradiation, respectively, which can be determined by knowing wall temperatures of the enclosure. And N_R is a nondimensional radiation number.

Boundary conditions for the motion and thermal conditions of the hot and cold walls are

$$\begin{aligned}
Y=0 \text{ (cold wall)} : \\
-1 \leq X^* \leq 0; \theta=0 \\
0 \leq X \leq 1; \Psi=0, \partial\Psi/\partial Y^*=0, \theta=0 \\
Y=1 \text{ (hot wall)} : \\
-1-(\tan \omega/H_p) \leq X^* \leq -(\tan \omega/H_p); \\
\theta=1 \\
-(\tan \omega/H) \leq X \leq 1-(\tan \omega/H); \\
\Psi=0, \partial\Psi/\partial Y^*=0, \theta=1 \\
X+(Y/H)\tan \omega=0 \text{ and } 1 \\
\text{(parting plates)} ; \\
\Psi=0, \partial\Psi/\partial X=0 \dots\dots\dots (12)
\end{aligned}$$

The method of the calculations is similar to Wilkes' method⁽⁴⁾. Namely, the central difference scheme is applied to Eqs. (1) to (3), (5) to (7), (10) and (11). (i) The alternating direction implicit (ADI) method is employed to determine the value of θ from the discretization form of Eqs. (2), (3), (10) and (11). The previous iteration values of $\bar{G}$ and $\bar{J}$ are used in Eqs. (10) and (11). (ii) ADI method is also employed to determine the value of Γ of inside space from Eq. (1). (iii) The successive overrelaxation (SOR) method is employed to determine the value of Ψ from Eq. (5). (iv) The wall value of Γ is determined from Eq. (5). (v) U and V are determined from Eqs. (6) and (7). (vi) The values of $\bar{G}$ and $\bar{J}$ are renewed by the wall temperature. The time is increased to $2\Delta\tau$, $3\Delta\tau$, ... and the above calculations are repeated successively until the values of θ , Γ and Ψ converge to their steady state.

The definitions of local and average Nusselt numbers are the same as in the 3rd report, namely, Nusselt number of the convection (N_{uh} , N_{umh}) and Nusselt number of the radiation (N_{uhR} , N_{umhR}) are defined as

$$\left. \begin{aligned}
N_{uh} &= (\partial\theta/\partial Y)_{Y=1} \\
N_{umh} &= \int_{-1-(\tan \omega/H_p)}^{1-(\tan \omega/H)} (\partial\theta/\partial Y)_{Y=1} dX \\
N_{uhR} &= \{ \bar{G}_h(X) - \bar{J}_h(X) \} / N_R \\
N_{umhR} &= \frac{1}{N_R} \int_{-1-(\tan \omega/H_p)}^{1-(\tan \omega/H)} \{ \bar{G}_h(X) \\
&\quad - \bar{J}_h(X) \} dX
\end{aligned} \right\} \dots\dots\dots (13)$$

In this report, a nondimensional number of the heat conduction from the hot wall to the parting plate is defined as

$$\begin{aligned}
(\partial\theta/\partial Y)_{mhp} \\
= \int_{-1-(\tan \omega/H_p)}^{-(\tan \omega/H)} (\partial\theta/\partial Y)_{Y=1} dX^* \dots\dots\dots (14)
\end{aligned}$$

The total heat transfer from the hot wall, namely a sum of the heat transfers by convection, conduction and radiation from the hot wall is defined as

$$\begin{aligned}
N_{umhr} \\
= \frac{N_{umh} + (\lambda_p/\lambda_a)(H_p/H)(\partial\theta/\partial Y)_{mhp} + N_{umhR}}{1 + (H_p/H)} \dots\dots\dots (15)
\end{aligned}$$

The air filled space was divided into 10 equal parts in both X and Y^* directions in the same manner as in authors' previous report, and the parting plate was also divided into 10 equal parts in both X^+ and Y^* directions. In this report, the convergence was confirmed by inequality $|\Delta N_{umh}| < 5 \times 10^{-6}$. The ΔN_{umh} represents a change of N_{umh} per unit time increment $\Delta\tau$. $\Delta\tau$ was set at 10^{-4} .

3. Experimental Apparatus and Procedure

Experiments have been performed for one oblique angle $\omega = \pi/3$ and two aspect ratios $H=0.5$ ($d=0.04$ m) and 1 ($d=0.02$ m). A sketch of the apparatus is shown in Fig. 2. This apparatus is almost same as the one in authors' 2nd report. Therefore only a brief description will be given here. Five enclosures of the same shape are piled up vertically. The hot and cold walls are made of copper plate of 5×10^{-3} m thick (the emissivity $E_h = 0.1$). The length is 0.3 m. Two kinds of materials are used for the parting plate. One is a vinyl film (vinyliden chloride resin) and the other is a stainless steel (sus 304). The thicknesses of the vinyl films ($\rho \cos \omega$) are $0.5, 1, 2, 5 \times 10^{-4}$ m and the thicknesses of the stainless foils are $3, 5 \times 10^{-5}$ m. Cellulose nitrate black mat paint (lacquer) coats the surface of the parting plate (the emissivity is 0.85). Using pylex 7740 as the standard plate, the thermal conductivity of the vinyl film λ_p is measured by the standard method of testing for thermal conductivity of heat insulating materials by comparison with a standard plate of known conductivity⁽⁵⁾. The value of λ_p is found to be 0.17 W/(mK) (323K).

In this experiment, measurements were performed in so-called quasi-stable state and temperature difference between the hot and cold walls ranged from 50 to 90 K. The rate of the heat transfer by convection was measured on the hot wall which was positioned at the midpoint of five enclosures (this enclosure is called a test enclosure). The rate of the heat transfer by convection was measured at 5 points on the hot wall of the test enclosure by the Schlieren method. The measuring points are located with equal spacing of 4×10^{-3} m. Seven pairs of equally spaced thermo-couples ($40 \mu m^{\phi}$) are mounted on every parting plate. To evaluate the physical properties of air, the temperature $(T_{hr} + T_{cr})/2$ of the test enclosure was employed.

4. Results of Experiment and Computation

The values of parameters and results of measurements, N_{umh} (the rate of the heat transfer by convection from hot wall), are shown in Table 1. Several measurements were performed under the same condition. The value which is closer to the average value of

Fig. 2 A sketch of apparatus

several measurements is shown in Table 1. In Table 1, V and S beside the numbers indicate the kind of the parting plate. Namely, V and S, respectively, indicate the vinyl film and the stainless steel foil. As the thickness of the parting plate increases, the rate of the heat transfer by convection, Numh, decreases (compare the results of Runs 1 to 4). And when the heat conductivity of the parting plate is high, Numh becomes low (compare the results of Runs 1 and 10).

Results of the computation, Numh, are also shown in Table 1. Values of parameters are same as the measurement. In these computations, the value of the nondimensional heat conductivity λ_p/λ_a was set at 6 and 560⁽⁶⁾ in cases of the vinyl film and the stainless steel foil, respectively. Table 1 shows that the differences between the computation and the measurement are less than $\pm 5\%$ except in Run 12.

The local Nusselt numbers are shown in Fig. 3 together with the results of computations. In the figure, the positions of $X = -\sqrt{3}$ and $1-\sqrt{3}$ represent the upper and lower ends of the hot wall, respectively. The solid lines indicate the results of the computations and the numbers correspond to the numbers in Table 1. As the thickness of the parting plate increases, both values of Nuh near the upper and lower ends of the wall become low. A similar tendency is also obtained in case of the stainless steel foil, but the results are not shown in figures.

The temperature profile on the parting plate is shown in Fig. 4 together with the results of computations. The solid lines indicate the results of the computations and the numbers correspond to the numbers in Table 1. In the experiment, there are some differences between temperatures of the plates U and L. But the temperature of the plate L is slightly higher than that of U in the results of the computations. When the parting plate is thick or the heat conductivity of it is high, the temperature profile of the parting plate approaches a linear profile. The temperature profile of the stainless steel foil is closer to the linear profile than that of the vinyl film. As mentioned earlier, when the parting plate is thick or its heat conductivity is high, the rate of the heat transfer by convection, Numh, takes a low value. The main cause is that the temperature profile on the parting plate approaches a linear profile.

5. Heat Transfer by Convection, Radiation and Conduction and Total Heat Transfer

The computations were performed to make clear the heat transfer characteristics. The aspect ratios H are 0.5 and 1, the width d ranges from 0.01 to 0.04 [m], the angle of the parting plate ω is $\pi/3$ and Prandtl number Pr is 0.72. The combinations of the nondimensional thickness of the parting plate H_p and nondimensional thermal conductivity λ_p/λ_a are shown in Table 2. The value of λ_p/λ_a of the actual parting plate ranges from 5 (polymer) to 500 (stainless steel), but the computation was also performed for $\lambda_p/\lambda_a = 1$. The combinations of the emissivities of the walls are E_h (the emissivity of the hot and cold walls) = 0.03 $\sim$ E_U (the emissivity of the parting plate) = 0.03, $E_h = 0.03 \sim E_U = 1$, $E_h = 1 \sim E_U = 0.03$ and $E_h = 1 \sim E_U = 1$. The temperatures of the hot and cold walls, respectively, are kept at 373K and 273K (the radiation parameter $\Theta = T_c/T_h$ becomes 0.732). In evaluating the physical properties of air at 323 K, the Rayleigh number Ra and the nondimensional radiation number N_R can be expressed as a function of the width d by

$$Ra = C_1 d^3, \quad N_R = C_2/d \quad \dots\dots\dots (16)$$

Fig. 3 Local Nusselt number ($H = 1$, vinyl film)

Table 1 Parameters and rate of heat transfer by convection, Numh ($Pr=0.72$, $\omega=\pi/3$, $E_h=0.1$, $E_U=0.85$)

Run	H	Ra	H_p (pcos ω m)	Θ	N_R	Numh	
						Experimental Value	Calculated Value
1 V	1	3.37×10^4	0.005 (5×10^{-5})	0.818	0.101	1.73	1.67
2 V	1	3.33×10^4	0.01 (10^{-4})	0.818	0.100	1.68	1.66
3 V	1	2.99×10^4	0.02 (2×10^{-4})	0.831	0.0947	1.62	1.60
4 V	1	3.08×10^4	0.05 (5×10^{-4})	0.828	0.0951	1.53	1.50
5 V	1	3.60×10^4	0.005 (5×10^{-5})	0.770	0.101	1.77	1.79
6 V	1	3.67×10^4	0.01 (10^{-4})	0.774	0.103	1.74	1.70
7 V	1	3.39×10^4	0.02 (2×10^{-4})	0.781	0.0973	1.66	1.62
8 V	1	3.47×10^4	0.05 (5×10^{-4})	0.779	0.0990	1.57	1.58
9 S	1	3.24×10^4	0.003 (3×10^{-5})	0.819	0.0985	1.36	1.42
10 S	1	3.36×10^4	0.005 (5×10^{-5})	0.813	0.101	1.22	1.21
11 S	1	3.48×10^4	0.003 (3×10^{-5})	0.770	0.0979	1.37	1.43
12 S	1	3.63×10^4	0.005 (5×10^{-5})	0.778	0.103	1.24	1.36
13 V	0.5	2.85×10^5	0.0025 (5×10^{-5})	0.809	0.0526	2.27	2.30
14 V	0.5	2.81×10^5	0.005 (10^{-4})	0.813	0.0522	2.24	2.33
15 V	0.5	2.44×10^5	0.01 (2×10^{-4})	0.834	0.0476	2.18	2.21
16 V	0.5	2.69×10^5	0.025 (5×10^{-4})	0.810	0.0499	2.07	2.06
17 V	0.5	3.29×10^5	0.0025 (5×10^{-5})	0.751	0.0547	2.32	2.42
18 V	0.5	3.19×10^5	0.005 (10^{-4})	0.760	0.0539	2.29	2.35
19 V	0.5	2.79×10^5	0.01 (2×10^{-4})	0.792	0.0505	2.24	2.24
20 V	0.5	3.06×10^5	0.025 (5×10^{-4})	0.745	0.0510	2.14	2.19
21 S	0.5	2.77×10^5	0.0015 (3×10^{-5})	0.817	0.0517	1.56	1.52
22 S	0.5	2.64×10^5	0.0025 (5×10^{-5})	0.819	0.0500	1.49	1.44
23 S	0.5	3.12×10^5	0.0015 (3×10^{-5})	0.757	0.0528	1.65	1.60
24 S	0.5	3.26×10^5	0.0025 (5×10^{-5})	0.766	0.0554	1.51	1.49

Fig. 4 Temperature distributions on parting plate ($H = 1$, vinyl film)

where $C_1 = 6.21 \times 10^9 \text{ m}^{-3}$, $C_2 = 2.64 \times 10^{-3} \text{ m}$.

Table 2 Range of H_p and λ_p/λ_a

H_p	λ_p/λ_a
5×10^{-4}	$1 \sim 500$
5×10^{-3}	$1 \sim 500$
5×10^{-2}	$1 \sim 50$
10^{-1}	$1 \sim 10$

5.1 Heat transfer by convection

The relations between the rate of the heat transfer by convection, $Numh$, and the width d are shown in Fig. 5. The nondimensional thickness of the parting plate H_p is 5×10^{-4} and the aspect ratios H are 0.5 and 1 (the emissivities $E_h = E_U = 0.03$). The solid lines indicate the results of the present computations. One-dotted chain line indicates the result of the previous computation in a situation where the parting plate had no thickness (results of the 3rd report). This situation is called herein a nothickness. Two-dotted chain line indicates the results of the computation in a situation where the temperature on the parting plate varies linearly (the results of $H=1$ have been reported in the 1st report). This situation is called herein a linear profile. The present result of the computation comes in between both chain lines. When the parting plate is thin or its heat conductivity is low, $Numh$ approaches the value of the nothickness (one-dotted chain line). On the contrary, when the parting plate is thick or its thermal conductivity is high, $Numh$ approaches the value of the linear profile (two-dotted chain line).

Fig. 5 Relations between rate of heat transfer by convection, $Numh$, and width d ($E_h = 0.03$, $E_U = 0.03$)

Here, we define the rate of the heat transfer by convection of the nothickness as $Numh_1$. And we also define the rate of the heat transfer by convection of the linear profile as $Numh_2$. By using $Numh_1$ and $Numh_2$, $Numh$ can be expressed as

$$Numh = (Numh_1 - Numh_2)f_1 + Numh_2 \quad \dots\dots\dots (17)$$

where the coefficient f_1 takes a value between 0 and 1. Figs. 6 and 7 show the coefficient f_1 of the width $d = 0.04 \text{ m}$. We can estimate $Numh$ of arbitrary width d by using f_1 of $d = 0.04 \text{ m}$, since the value of f_1 changes slightly with the width d . When the aspect ratio H is 0.5, the error between this estimated and true values ranges from -4 to 14% (the difference based on the Nu

Fig. 6 Relations between coefficient f_1 of Eq. (17) and nondimensional thermal conductivity of parting plate λ_p/λ_a ($H = 0.5$, $d = 0.04 \text{ m}$)

Fig. 7 Relations between coefficient f_1 of Eq. (17) and nondimensional thermal conductivity of parting plate λ_p/λ_a ($H = 1$, $d = 0.04 \text{ m}$)

Fig. 8 Relations between rate of heat transfer by radiation, $NumhR$, and width d ($H = 0.5$, $H_p = 5 \times 10^{-4}$)

number ranges from -0.1 to 0.3). When the aspect ratio H is 1 , the error between them ranges from -11 to 10% (the difference based on the Nu number ranges from -0.1 to 0.2). The value of Num_h is already shown in Figs. 7 and 8 of the authors' 3rd report.

5.2 Heat transfer by radiation

The relations between the rate of the heat transfer by radiation, Num_h , and the width d are shown in Fig. 8, when the aspect ratio H is 0.5 and the nondimensional thickness of the parting plate H_p is 5×10^{-4} . The solid lines indicate the results of present computations. The one-dotted chain line indicates the result of the nothickness (results of the 3rd report). The two-dotted chain line indicates the result of the linear profile. The rate of the heat transfer by radiation, Num_h , changes slightly with both the nondimensional thickness of the parting plate H_p and the nondimensional thermal conductivity λ_p/λ_a . When the emissivity E_h is 0.03 and E_u is unity, the change is the largest. However, the change

Fig. 9 Relations between nondimensional number of heat conduction $(\partial\theta/\partial Y)_{mhp}$ and width d ($H = 0.5$, $\lambda_p/\lambda_a = 1$)

Fig. 10 Relations between nondimensional number of heat conduction $(\partial\theta/\partial Y)_{mhp}$ and width d ($H = 1$, $\lambda_p/\lambda_a = 1$)

of the Nu number is less than 0.2 . Then the rate of the heat transfer by radiation of the nothickness or the linear profile can be used to estimate the total heat transfer rate.

5.3 Nondimensional number of heat conduction

The relations between the nondimensional number of heat conduction $(\partial\theta/\partial Y)_{mhp}$ and the width d are shown in Figs. 9 and 10. We define the nondimensional number of heat conduction $(\partial\theta/\partial Y)_{mhp}$ of $\lambda_p/\lambda_a = 1$ as $(\partial\theta/\partial Y)_{mhp}(\lambda_p/\lambda_a = 1)$. By using $(\partial\theta/\partial Y)_{mhp}(\lambda_p/\lambda_a = 1)$, $(\partial\theta/\partial Y)_{mhp}$ can be expressed as

$$(\partial\theta/\partial Y)_{mhp} = (\partial\theta/\partial Y)_{mhp}(\lambda_p/\lambda_a = 1) f_2 \quad (18)$$

where the coefficient f_2 takes a value between 0 to 1 . Figs. 11 and 12 show the coefficient f_2 of the width $d = 0.04$ m. Since the value of f_2 changes slightly with the width d , we can estimate the value of $(\partial\theta/\partial Y)_{mhp}$ of arbitrary width d by using f_2 of $d = 0.04$ m. When the aspect ratio $H = 0.5$, the error between the estimated and true values ranges from -18 to 7% (the difference based on $(\partial\theta/\partial Y)_{mhp}$ ranges from -0.2 to 0.1). When $H = 1$, the error ranges from -25 to 5% (the difference ranges from -0.2 to 0.1).

Fig. 11 Relations between coefficient f_2 of Eq. (18) and nondimensional thermal conductivity of parting plate λ_p/λ_a ($H = 0.5$, $d = 0.04$ m)

Fig. 12 Relations between coefficient f_2 of Eq. (18) and nondimensional thermal conductivity of parting plate λ_p/λ_a ($H = 1$, $d = 0.04$ m)

5.4 Total heat transfer

The relations between the total heat transfer rate, Numh_T , which is defined by Eq. (15) and the nondimensional thermal conductivity λ_p/λ_a are shown in Figs. 13 and 14. When the nondimensional thickness of the parting plate, H_p , and the width d are kept constant, Numh_T increases for a certain value of λ_p/λ_a . When the combination of the emissivities is $E_h = E_U = 0.03$, the total heat transfer rate, Numh_T , is lower than the value of the other combinations of the emissivities. The total heat transfer rate, Numh_T , keeps a value under 1.5 in a situation where the width d is less than 0.04 m (the aspect ratio $H = 0.5$) and 0.02 m ($H = 1$) and the relation between the thickness and the thermal conductivity of the parting plate satisfies the following relation. Namely, when the nondimensional thickness of the parting plate H_p is 5×10^{-4} , the nondimensional thermal conductivity of the parting plate λ_p/λ_a is less than 10^2 ; when $H_p = 5 \times 10^{-3}$, λ_p/λ_a is less than 20; and when $H_p = 0.1$, λ_p/λ_a is less than 5. If we use a polymer for the parting plate, its nondimensional thermal conductivity λ_p/λ_a ranges from 5 to 20. Then, the total heat transfer rate Numh_T takes a value under 1.5, when its thickness of the parting plate $p (= H_p d)$ is less than $5 \times 10^{-3} d$ [m] and the emissivity and the width d are within the range mentioned above.

5.5 Effect of temperature of hot wall

The computations were also performed in a situation where the temperature of the hot wall ranged $298 \leq T_h \leq 373$ K ($T_c = 273$ K). The width d was 0.02 and 0.04 m. As a consequence, it is seen that the rate of the heat transfer by convection, Numh , the rate of radiation, Numh_R , and the rate of heat conduction, $(\partial\theta/\partial Y)_{\text{mhp}}$, at the temperature $298 \leq T_h \leq 373$ K can be estimated from Figs. 5 to 12 which show results of $T_h = 373$ K.

The value of Numh can be estimated as follows. (i) Calculating the Ra number from the hot wall temperature T_h , the values of Numh_1 and Numh_2 are read off from Fig. 5 which shows the relation between Numh and Ra. (ii) These values are substituted into Eq.

(17). (iii) At the same time, the value of f_1 is read off from Figs. 6 and 7, and then, the value of f_1 is substituted into Eq. (17). In case of $H = 0.5$, the error of this estimated value ranges from -17 to 7% (the difference based on Nu number ranges from -0.2 to 0.2). In case of $H = 1$, the error ranges from -25 to 3% (the difference ranges from -0.3 to 0.1). The value of Numh_1 can be read off from Figs. 7 and 8 in the authors' 3rd report.

The value of Numh_R can be estimated as follows. The value of Numh_R which corresponds to a given distance d , is read off from the relation between Numh_R and d in Figs. 5 and 6 of the authors' 3rd report. This value is defined as Numh_{RS} , then the value of Numh_R can be estimated from Eq. (19) in the same manner as in the authors' 3rd report.

$$\text{Numh}_R = \text{Numh}_{RS} \frac{T_h' - T_c'}{C(T_h - T_c)} \quad \dots\dots (19)$$

where the constant C is 1.382×10^8 [K³]. When the combinations of the emissivities are $E_h \sim E_U = 0.03 \sim 0.03$, $1.0 \sim 0.03$ and $1.0 \sim 1.0$, this estimated value is always higher than the real value. In case of $H = 0.5$, the error ranges from 5 to 15% (the difference based on the Nu number ranges from 0.1 to 0.3). In case of $H = 1$, the error ranges from 5 to 15% (the difference ranges from 0.1 to 0.4). When the combination of the emissivities is $E_h = 0.03$ and $E_U = 1$, the maximum error is 25% (the maximum difference based on the Nu number is 0.02) and 30% (the maximum difference is 0.03), in case of the aspect ratios $H = 0.5$ and 1, respectively.

The value of $(\partial\theta/\partial Y)_{\text{mhp}}$ can be estimated as follows. (i) Calculating the Ra number from the hot wall temperature T_h , the value of $(\partial\theta/\partial Y)_{\text{mhp}}$ of the nondimensional conductivity $\lambda_p/\lambda_a = 1$, namely $(\partial\theta/\partial Y)_{\text{mhp}} (\lambda_p/\lambda_a = 1)$, is read off from Figs. 9 and 10 which show the relation between $(\partial\theta/\partial Y)_{\text{mhp}}$ and Ra. (ii) At the same time, the value of f_2 is read off from Figs. 11 and 12 and the value of f_2 is substituted into Eq. (18). In case of $H = 0.5$, the error of this estimated

Fig. 13 Relations between total heat transfer rate and nondimensional thermal conductivity of parting plate λ_p/λ_a ($H = 0.5$)

Fig. 14 Relations between total heat transfer rate and nondimensional thermal conductivity of parting plate λ_p/λ_a ($H = 1$)

value ranges from -30 to 5% (the difference based on $(\partial\theta/\partial Y)_{\text{mhp}}$ ranges from -0.3 to 0.1). In case of $H = 1$, the error ranges from -30 to 8% (the difference ranges from -0.3 to 0.1).

Figs. 13 and 14 show the total heat transfer rate of $T_h = 373$ K. These figures show that, when both the nondimensional thickness of the parting plate H_p and the width d are kept constant, Numh_T increases for a certain value of λ_p/λ_a . This value does not depend on the hot wall temperature, when the hot wall temperature ranges from 298 to 373 K.

6. Conclusions

In this report, the thickness of the parting plates and the heat conduction in them are taken into account in the computation. And using several kinds of parting plates, the rate of the heat transfer by convection was measured.

(1) As the thickness of the parting plate increases or the thermal conductivity increases, the rate of the heat transfer by convection decreases according to the results of the experiment. The results of the experiment coincide with the computations within 10% .

(2) The computations were performed under the following range of parameters: the aspect ratio $H = 0.5$ and 1 , the angle of the parting plate $\omega = \pi/3$, the hot wall temperature $T_h = 373$ K, the cold wall temperature $T_c = 273$ K, the width $d = 0.01$ to 0.04 m, the nondimensional thickness of the parting plate $H_p = 5 \times 10^{-4}$ to 0.1 , and the nondimensional thermal conductivity of the parting plate $\lambda_p/\lambda_a = 1$ to 500 . The rate of the heat transfer by convection and radiation and the heat conduction to the parting plate are shown in figures.

(3) According to the results of the computation, the rate of the heat transfer by convection, Numh , takes a value between two values of the nothickness and the linear profile. As the nondimensional thickness H_p or the nondimensional heat conductivity λ_p/λ_a increases, Numh approaches a value of the linear profile.

(4) The rate of the heat transfer by radiation changes slightly with both the nondimensional thickness of the parting plate H_p and the nondimensional thermal conductivity λ_p/λ_a . This value is closer to the value of the nothickness or the linear profile.

(5) As the nondimensional thickness of the parting plate H_p or the nondimensional thermal conductivity λ_p/λ_a increases, the nondimensional number of the heat conduction of the parting plate decreases.

(6) When the combination of the emissivities is $E_h = E_U = 0.03$, the total heat transfer rate takes the lowest value.

(7) The total heat transfer rate has a value under 1.5 in a situation where the width d is less than 0.04 m (the aspect ratio $H = 0.5$) and 0.02 m ($H = 1$) and the relation between the thickness and the thermal conductivity of the parting plate satisfies the following relations; namely, when the nondimensional thickness of the parting plate H_p is 5×10^{-4} , the nondimensional thermal conductivity of the parting plate λ_p/λ_a is less than 10^2 ; when $H_p = 5 \times 10^{-3}$, λ_p/λ_a is less than 20 ; when $H_p = 0.1$, λ_p/λ_a is less than 5 .

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