Characteristics of Natural Frequencies of Steam Turbine Blades*

(2nd Report, Influence of Disk on Vibration of Grouped Blades)

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A method of calculating the natural frequencies of the axial vibrations of steam turbine blades and thick disks is presented, and the natural frequencies of blades are calculated and compared with the measured values. It is found that the difference between the calculated and measured values is within 10 per cent. The natural frequencies become higher at about 30 per cent depending on the defaults of the rotary inertia and the shear deformation. The effect of rotary inertia is small, while that of shear deformation is large. The relationship between the number of nodal diameters and natural frequencies is made clear by calculation.

Key Words : Vibration, Steam Turbine Blade, Turbine Disk, Coupled Vibration, Natural Frequency, Vibration Analysis

1. Introduction

Short blades used in the high-pressure stage of a steam turbine are excited by a force whose frequencies equal the nozzle number times the revolutions, or its harmonics. If the exciting frequency coincides with one of the natural frequencies of the blades, a resonant stress occurs on the blade and it may cause damage to the blade. Therefore, when designing blades, it is necessary to calculate their natural frequencies and to make sure that resonance does not occur.

There are two vibrations in turbine blades; tangential—along the direction of the rotation of the blades, and axial—along the shaft axis. The axial vibration of the blades and that of the disk appears to be coupled. Therefore, in calculating the characteristics of axial vibration, it is necessary to consider the effect of the disk on blade vibration. In the vibration mode shape at this time, nodal diameters appear on the blades and the disk.

Previously, the vibration of turbine blades was analyzed by taking the effect of a disk into consideration^{(1)~(3)} and recently by the FEM method⁽⁴⁾. The analytic method is better than digital calculation in reducing the cost of calculating vibration characteristics. Therefore, the transfer matrix method was used to calculate the influence of a thick disk on natural frequencies of blade vibration. Thus, shear deformation and rotary inertia, which have been neglected in the past, were considered.

The natural frequencies of a turbine blade are calculated, and compared with the measured results. Next, the effect of shear deformation and rotary inertia on natural

frequencies is discussed.

2. Calculating Method

2.1 Coordinate

A coordinate is used, as shown in Fig. 1 (a)~(e).

2.2 Boundary conditions

As for boundary conditions, it is assumed that the shaft is fixed and vibrates axially, and that the blade roots are fixed and vibrate tangentially. It is also assumed that the tips of the blades are free from axial and tangential vibrations. Therefore, the inner boundary position of the disk, that is the joint between the disk and the shaft is fixed. At this position the displacement and the slope are zero, giving the equations,

$$Y = 0, \quad \eta = 0 \quad \dots\dots\dots(1)$$

The tips of the blades are free and the moment and shearing force become zero. Therefore, at the blade tips, the following equations can be obtained.

$$M_y = 0, M_z = 0, S_y = 0, S_z = 0 \quad \dots\dots\dots(2)$$

2.3 Equation on disk

A differential equation with regard to displacement w of a disk is obtained from the following equation⁽⁵⁾.

$$\frac{r}{g} \frac{\partial^2 w}{\partial r^2} + \frac{E h^2}{12(1-\nu^2)} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} \right] w = 0 \quad \dots\dots\dots(3)$$

where

r : distance from the center of the disk
 h : plate thickness

The moment M and the shearing force S are obtained from the following equations⁽⁶⁾

$$M = D \left[\frac{\partial^2 w}{\partial r^2} + \nu \left(\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \varphi^2} \right) \right] \quad \dots\dots\dots(4)$$

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$$S = D \left[\frac{\partial^3 w}{\partial r^3} + \frac{1}{r} \frac{\partial^2 w}{\partial r^2} - \frac{1}{r^2} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^3 w}{\partial r \partial \varphi^2} - \frac{2}{r^3} \frac{\partial^2 w}{\partial \varphi^2} \right] + D(1-\nu) \left[\frac{1}{r^2} \frac{\partial^3 w}{\partial r \partial \varphi^2} - \frac{1}{r^3} \frac{\partial^2 w}{\partial \varphi^2} \right] \dots (5)$$

where

D : bending stiffness

M, S : values per unit length

It is assumed that the axial vibrations of the blades and disk vary periodically in the form of a sine wave according to the position on the circumference. If the number of nodal diameters is k , and the circular frequency is ω , the displacement w , the moment M and the shearing force S are

$$\left. \begin{aligned} w &= Y \cos k\varphi \cos \omega t \\ M &= m \cos k\varphi \cos \omega t \\ S &= v \cos k\varphi \cos \omega t \end{aligned} \right\} \dots (6)$$

Substituting Eqs.(6) into Eqs.(3),(4) and (5), the equations at a cross section of a disk are gained as follows.

$$\left. \begin{aligned} &\left(\frac{d^4 Y}{dr^4} \right)_n + \frac{2}{r_n} \left(\frac{d^3 Y}{dr^3} \right)_n - \frac{2k^2+1}{r_n^2} \left(\frac{d^2 Y}{dr^2} \right)_n + \frac{2k^2+1}{r_n^3} \left(\frac{dY}{dr} \right)_n + \left(\frac{k^4-4k^2}{r_n^4} - \frac{12(1-\nu^2)}{Eh_n^2} \frac{\gamma}{g} w^2 \right) Y_n \\ &= 0 \\ m_n &= D_n \left[\left(\frac{d^2 Y}{dr^2} \right)_n + \nu \left(\frac{1}{r_n} \left(\frac{dY}{dr} \right)_n - \frac{k^2}{r_n^2} Y_n \right) \right] \\ v_n &= D_n \left[\left(\frac{d^3 Y}{dr^3} \right)_n + \frac{1}{r_n} \left(\frac{d^2 Y}{dr^2} \right)_n - \frac{1+2k^2-\nu k^2}{r_n^2} \left(\frac{dY}{dr} \right)_n + \frac{3k^2-\nu k^2}{r_n^3} Y_n \right] \end{aligned} \right\} \dots (7)$$

With regard to the small part of the short length ℓ in the radial direction of the disk, the following relationships are obtained.

$$\left. \begin{aligned} Y_{n+1} &= Y_n + \left(\frac{dY}{dr} \right)_n \ell_n + \frac{1}{2} \left(\frac{d^2 Y}{dr^2} \right)_n \ell_n^2 + \frac{1}{6} \left(\frac{d^3 Y}{dr^3} \right)_n \ell_n^3 \\ \left(\frac{dY}{dr} \right)_{n+1} &= \left(\frac{dY}{dr} \right)_n + \left(\frac{d^2 Y}{dr^2} \right)_n \ell_n + \frac{1}{2} \left(\frac{d^3 Y}{dr^3} \right)_n \ell_n^2 \\ \left(\frac{d^2 Y}{dr^2} \right)_{n+1} &= \left(\frac{d^2 Y}{dr^2} \right)_n + \left(\frac{d^3 Y}{dr^3} \right)_n \ell_n, \quad \left(\frac{d^3 Y}{dr^3} \right)_{n+1} = \left(\frac{d^3 Y}{dr^3} \right)_n + \left(\frac{d^4 Y}{dr^4} \right)_n \ell_n \end{aligned} \right\} \dots (8)$$

(a) Blade and disk

(b) Blade

(c) Tangential vibration of the blade

(d) Axial vibration of the blade

(e) Disk

Fig.1 Coordinate

From Eqs.(7) and (8), the relationship between $Y_n, \eta_n = (dY/dr)_n, m_n, v_n, Y_{n+1}, \eta_{n+1}, m_{n+1}$ and V_{n+1} are

$$Y_{n+1} = a_{11}Y_n + a_{12}\eta_n + a_{13}m_n + a_{14}v_n \quad (9)$$

$$\eta_{n+1} = a_{21}Y_n + a_{22}\eta_n + a_{23}m_n + a_{24}v_n \quad (10)$$

$$m_{n+1} = a_{31}Y_{n+1} + a_{32}\eta_{n+1} + a_{33}Y_n + a_{34}\eta_n + a_{35}m_n + a_{36}v_n \\ = q(Y_{n+1}, \eta_{n+1}, Y_n, \eta_n, m_n, v_n) \quad (11)$$

$$v_{n+1} = a_{41}Y_{n+1} + a_{42}\eta_{n+1} + a_{43}m_{n+1} + a_{44}Y_n + a_{45}\eta_n + a_{46}m_n + a_{47}v_n \quad (12)$$

Adding a term for the effect of the rotary inertia to Eq.(11), we obtain the next equation.

$$m_{n+1} = q(Y_{n+1}, \eta_{n+1}, Y_n, \eta_n, m_n, v_n) + a_{37}(\eta_n + \eta_{n+1}) \quad (13)$$

where

$$a_{37} = -\frac{1}{2} \frac{r}{g} h_n \frac{r_{n+1} - r_{n-1}}{2} \frac{h_n^2 + \frac{(r_{n+1} - r_{n-1})^2}{2}}{12} \omega^2, \quad \text{on } n \approx 1$$

When $n=1$, $(r_{n+1} - r_n)/2$ replaces $(r_{n+1} - r_{n-1})/2$.
The displacement Y_n with the shear deformation is

$$Y_{n+1} = Y_n - \frac{2 \ell_n}{k'G(h_n + h_{n+1})} v_n = Y_n + a_{51}v_n \quad (14)$$

where, $1/k'$ is a form factor for the cross section.

From the above mentioned results, the relationships between the displacement, slope, moment and shearing force at stations n and $n+1$ become

$$\left. \begin{aligned} Y_{n+1}^I &= a_{11}Y_n^I + a_{12}\eta_n + a_{13}m_n + a_{14}v_n \\ \eta_{n+1} &= a_{21}Y_n^I + a_{22}\eta_n + a_{23}m_n + a_{24}v_n \\ m_{n+1} &= a_{31}Y_{n+1}^I + a_{32}\eta_{n+1} + a_{33}Y_n^I + a_{34}\eta_n + a_{35}m_n + a_{36}v_n + a_{37}(\eta_n + \eta_{n+1}) \\ v_{n+1} &= a_{41}Y_{n+1}^I + a_{42}\eta_{n+1} + a_{43}m_{n+1} + a_{44}Y_n^I + a_{45}\eta_n + a_{46}m_n + a_{47}v_n + a_{48}Y_n^{\Pi} \\ Y_{n+1}^{\Pi} &= Y_n^{\Pi} + a_{51}v_n \\ Y_{n+1} &= Y_{n+1}^I + Y_{n+1}^{\Pi} \\ a_{48} &= \frac{r}{g} \frac{h_n + h_{n+1}}{2} \ell_n \omega^2 \end{aligned} \right\} \quad (15)$$

where

Y^I : displacement caused by bending

Y^{Π} : displacement caused by shear deformation

The displacement and slope at the inner boundary position of the disk become zero. From this boundary condition and Eqs. (15), the displacement, slope, moment and shearing force at each position on the disk can be represented as functions of the moment m_1 and the shearing force v_1 at the inner boundary position of the disk. The results are

$$\begin{bmatrix} Y \\ \eta \\ m \\ v \end{bmatrix}_n = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix}_n \begin{bmatrix} m \\ v \end{bmatrix}_1 \quad (16)$$

The displacement, slope, moment and shearing force on the outer circumference of the disk are gained from Eq.(16).

2.4 Equation on blade

The analysis is executed for a case where tangential vibration is coupled with axial vibration, by taking blade twist into account.

To calculate the natural frequency with regard to the bending vibration of the blades, the blades are divided and treated as a concentrated mass and a massless beam. For stations n and $n+1$, and the equilibrium of the force and the moment, we have the following equations.

$$\left. \begin{aligned} B_{an} &= \frac{I_{yn}}{E(I_{yn}I_{zn} - I_{yzn}^2)}, \quad B_{bn} = \frac{I_{yzn}}{E(I_{yn}I_{zn} - I_{yzn}^2)} \\ B_{cn} &= \frac{I_{zn}}{E(I_{yn}I_{zn} - I_{yzn}^2)} \end{aligned} \right\}$$

$$\begin{aligned}
M_{yn+1} &= M_{yn} + S_{zn} \varepsilon_n + \frac{1}{2} [-\rho A_n \omega^2 W_n + \rho \Omega^2 f_n (B_{cn} M_{yn} + B_{bn} M_{zn}) - \rho \Omega^2 f'_n \mu_n] \varepsilon_n^2 \\
&\quad + \frac{r}{g} \left(\frac{\varepsilon_n^3}{12} \frac{A_n + A_{n+1}}{2} + \varepsilon_n \frac{I_{yn} + I_{yn+1}}{2} \right) \mu_n \omega^2 \\
M_{zn+1} &= M_{zn} - S_{yn} \varepsilon_n + \frac{1}{2} [\rho A_n V_n (\omega^2 + \Omega^2) + \rho \Omega^2 f_n (B_{an} M_{zn} + B_{bn} M_{yn}) + \rho \Omega^2 f'_n \theta_n] \varepsilon_n^2 \\
&\quad - \frac{r}{g} \left(\frac{\varepsilon_n^3}{12} \frac{A_n + A_{n+1}}{2} + \varepsilon_n \frac{I_{zn} + I_{zn+1}}{2} \right) \theta_n \omega^2 \\
\theta_{n+1} &= \theta_n + \frac{1}{2} (B_{an+1} M_{zn+1} + B_{an} M_{zn} + B_{bn+1} M_{yn+1} + B_{bn} M_{yn}) \varepsilon_n \\
\mu_{n+1} &= \mu_n - \frac{1}{2} (B_{cn+1} M_{yn+1} + B_{cn} M_{yn} + B_{bn+1} M_{zn+1} + B_{bn} M_{zn}) \varepsilon_n \\
V_{n+1} &= V_n + \frac{1}{2} (\theta_{n+1} + \theta_n) \varepsilon_n + \frac{2 \varepsilon_n}{k' G (A_n + A_{n+1})} S_{yn} \\
W_{n+1} &= W_n + \frac{1}{2} (\mu_{n+1} + \mu_n) \varepsilon_n + \frac{2 \varepsilon_n}{k' G (A_n + A_{n+1})} S_{zn} \\
S_{yn+1} &= S_{yn} - \frac{1}{2} \rho (\omega^2 + \Omega^2) (A_{n+1} V_{n+1} + A_n V_n) \varepsilon_n - \rho \Omega^2 (f_{n+1} \theta_{n+1} - f_n \theta_n) \\
S_{zn+1} &= S_{zn} - \frac{1}{2} \rho \omega^2 (A_{n+1} W_{n+1} + A_n W_n) \varepsilon_n - \rho \Omega^2 (f_{n+1} \mu_{n+1} - f_n \mu_n)
\end{aligned} \tag{17}$$

where

A : area of cross section
 L : blade length
 $f = \int_x^L A x dx$, $f' = df/dx$
 I_y, I_z : moments of inertia of blade cross section
 I_{yz} : product of inertia of blade cross section
 V, W : displacements
 ε_n : length of small part of blade
 θ, μ : slopes
 ρ : density
 Ω : angular velocity of turbine shaft

Equations (17) are obtained by adding the terms for the effect of the rotary inertia and shear deformation to equation (7) for the natural frequency of the turbine blade. If the blade has an additional mass, the resulting inertia force must be added to the equation of the shearing force in Eqs. (17).

From Eqs. (17), the displacement, slope, moment and shearing force at each position on the blade can be represented as functions of the displacement, slope, moment and shearing force at the blade root.

2.5 The relationship between disk and blades at their junction point

With regard to the displacement and slope of the axial direction at the joint between the disk and the blades, the values on the disk are equal to those on the blades. The displacement and slope of the blades in the tangential direction are assumed to be zero. From these conditions and the condition of equilibrium of the shearing force and the moment, we can gain the next relationship at the joint between the disk and the blades.

$$V_1 = 0, W_1 = Y_p, \theta_1 = 0, \mu_1 = \eta_p,$$

$$M_{y1} = -(2\pi R/Q)m_p, S_{z1} = -(2\pi R/Q)v_p \dots \tag{18}$$

where

R : radius of disk outer circumference
 Q : number of blades around a disk
 P : station number at outer part of disk

2.6 Calculation of natural frequency

Using the equation for the disk, the relationship at the joint between the disk and the blades, and the equation for the blade, we can find the displacement, slope, moment and shearing force at each position on the blades. From these results, the displacement, slope, moment and shearing force at the tip of the blades can be represented as functions of m_1, v_1, M_{z1} , and S_{y1} , and the results are

$$\begin{bmatrix} V \\ W \\ \theta \\ \mu \\ M_y \\ M_z \\ S_y \\ S_z \end{bmatrix}_T = \begin{bmatrix} g_{11} & \dots & g_{14} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ g_{81} & \dots & g_{84} \end{bmatrix} \begin{bmatrix} M_{z1} \\ S_{y1} \\ m_1 \\ v_1 \end{bmatrix} \dots \tag{19}$$

Substituting the boundary conditions at the tip of the blades into Eq. (19), the next frequency equation is obtained.

$$\begin{vmatrix} g_{51} & \dots & g_{54} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ g_{81} & \dots & g_{84} \end{vmatrix} = 0 \dots \tag{20}$$

The frequency, which satisfies Eq.(20), is the natural frequency. Thus, the natural frequency was decided by trial and error by computing the determinant involving assumed frequency.

3. Calculated Results and Discussion

3.1 Values used in calculations

The natural frequency of a blade and disk system of a turbine was calculated and compared with measured values. The results were obtained using the following data: $E=205.8 \text{ G Pa}$ ($2.1 \times 10^4 \text{ kgf/mm}^2$), $G=82.32 \text{ G Pa}$ ($0.84 \times 10^4 \text{ kgf/mm}^2$), $\nu=0.3$, density= 7850 kg/m^3 , $g=9.8 \text{ m/s}^2$, $1/k'=1.11$ (with the blade profile), 1.2 (without the blade profile). The values at the disk: inner radius = 30.2 cm , outer radius = 42.6 cm , $h = 11.18 \text{ cm}$. The values at the blade: $A=34 \text{ cm}^2$, $I_y=181 \text{ cm}^4$, $I_z=52 \text{ cm}^4$, $I_{yz}=0$ at the join of a blade and disk; $A=35.3 \text{ cm}^2$, $I_y=134.4 \text{ cm}^4$, $I_z=62.4 \text{ cm}^4$, $I_{yz}=0$ at the distance of 3.8 cm from the joint of the blade and the disk. Next part is the profile in which $A=12.1 \text{ cm}^2$, $I_y=37.7 \text{ cm}^4$, $I_z=11.7 \text{ cm}^4$, $I_{yz}=0$. The length of this part is 3.9 cm . At the tip of the blade, $A=39.2 \text{ cm}^2$, $I_y=180 \text{ cm}^4$, $I_z=91 \text{ cm}^4$,

$I_{yz}=0$. The length of this part is 0.48 cm . A mass of 63 g was added to the blade tip. The number of blades around the disk is 60 .

3.2 The calculated result of natural frequencies

Figure 2 shows the calculated and measured values of natural frequencies. The calculated values are the results in which the data in the previous section is used for the calculation of the natural frequencies. The measured values are the natural frequencies with regard to the blades of an actual turbine. The natural frequencies and mode shapes were obtained from a response that was produced when the blade was excited by an alternating current magnet. In calculation, two natural frequencies were obtained with regard to the same number of nodal diameters. Only the lower natural frequency was measured. From these results, the calculated and measured values of the natural frequencies are found to agree within an error of 10 per cent. It is seen that natural frequencies can be accurately predicted within this order of error even when the disk is thick.

In Fig.2, two natural frequencies appear for the same number of nodal diameters. Also, the relationship between the number of nodal diameters and the natural frequencies yields two curves. The lower frequencies of these two curves can be called the lower order natural frequencies and the higher ones the higher order natural frequencies. The lower order natural frequencies show such a trend that the natural frequency nears a constant value with the increase in the number of nodal diameters. This is because the axial natural frequency of the blade only exists at 6634 Hz , causing a coupled vibration of the blades and the disk to appear.

Figure 3 shows the calculated results of the mode shapes at the natural frequencies. From this picture, the following can be surmised.

(1) With regard to the lower order natural frequencies, the vibration phases of the disk and the blade are the same, and the amplitude of the blade tip becomes large relative to the other parts of the blade with an increase in the number of nodal diameters..

Fig.2 Natural frequency

Fig.3 Calculated mode shapes

Fig.4 The relationship between the number of divisions of the disk and natural frequency

(2) With regard to the higher order natural frequencies, the vibrations of the disk and the blade tip get out of phase, and the amplitude of the blade tip is about two times those of the middle of the disk root and the blade tip. A nodal point exists on the blade; therefore, on the mode shape around disk, the nodal points appear in a circle on the blade. That is, the mode shape at the higher order natural frequency has one nodal point.

(a) Rotary inertia and shear deformation of the blade not considered

(b) Rotary inertia and shear deformation of the blade considered

Fig.5 The effect of rotary inertia and shear deformation on natural frequency

3.3 The effect of the number of divisions

When the natural frequencies of the blade are calculated by the method in which the blade is treated as a concentrated mass and a massless beam, the number of divisions of the blade becomes an important factor. If the number of divisions is small, the cost declines and the calculation is economical, but there is a drawback that the accuracy of the calculation deteriorates. Accordingly, as the number of divisions increases, the accuracy improves, but the cost of calculation increases. Thus, a reciprocity law exists between accuracy and cost of calculation. Therefore, it is important to calculate natural frequencies by finding a proper number of divisions needed to obtain necessary accuracy at an acceptable cost.

The variation of natural frequencies was calculated by varying the number of divisions of the blade and the disk. When the number of divisions of the blade is 16, 23 or 37, it is found that the variation of the natural frequencies is small. When the number of divisions of the disk is 3, 6, 12, 23 or 46, it is found that the natural frequencies vary according to the number of divisions shown in Fig. 4. When the number of divisions is 12, 23 or 46, the values of the variation of the natural frequencies are small, and when the number of divisions is 3 or 6, they are large. When the number of divisions is small, the calculation volume decreases. On the condition that both the error and the number of divisions are small, the number of divisions in the calculation shown in Fig. 2 have been determined to be 12 for the disk, and 16 for the blade.

3.4 The effect of rotary inertia and shear deformation

This calculation method, which differs from methods used in the past is characterized in that rotary inertia and shear deformation are taken into consideration. Therefore, variation in natural frequencies in the case of neglecting rotary inertia and shear deformation, was calculated with regard to lower order natural frequencies.

Figure 5 shows the results of the calculation of the effect of rotary inertia and shear deformation on natural frequencies. (a) is the case of neglecting rotary inertia and shear deformation of the blades, and (b) is the case of taking such characteristics into consideration. When rotary inertia and shear deformation of both the blades and disk are omitted, natural frequencies are about 30 per cent higher than those when they are considered.

Figure 5 (a) shows the following.

(1) The effect of rotary inertia of the disk is 1~2 per cent, which is very small.
 (2) When considering shear deformation of the disk, the calculated results on natural frequencies do not reveal large errors in the low range of the numbers of nodal diameters k . When k becomes large, the natural frequencies obtained by taking shear deformation of the disk into consideration, do not agree with the correct values obtained by taking both rotary inertia and shear deformation into consideration. It is the omission of rotary inertia and shear deformation

mation of the blades that causes a large error when k is large. That is, when the natural frequencies of the axial vibration of the blade alone become inexact, the natural frequency error increases with an increase in k .

From Fig. 5(b), it is found that, due to considerations of blade rotary inertia and shear deformation, the error decreases in the large k range. This is because the natural frequency of the axial vibration of the blade alone is correct and the natural frequency of the blades and disk system comes close to this value with an increase in k , thereby reducing the error. As the effect of rotary inertia is small in the case shown in Fig. 5(b), when taking only shear deformation of the disk into consideration, the results become close to the correct values.

The effects of rotary inertia and shear deformation on natural frequency are as follows:

- (1) With the omission of rotary inertia and shear deformation, natural frequencies increase by about 30 per cent.
- (2) The effect of rotary inertia on the disk is 1-2 per cent, which is very small. Thus, by taking shear deformation into consideration, large errors can be avoided.
- (3) By taking rotary inertia and shear deformation of the disk into consideration, correct results in the small k range can be obtained. In the range where k is large, the correct results can be obtained by considering rotary inertia and shear deformation of the blades.

4. Conclusions

To examine the effect of the disk on natural frequencies of axial blade vibration, a natural frequency equation considering the vibration of the disk is introduced. The calculation and measurement of the natural frequencies of a turbine blade are done with the following results:

- (1) The calculated and measured values of the natural frequencies agree with each

other within an error of 10 per cent. It is found that natural frequencies can be predicted within this order of error in cases where the disk is thick.

(2) Two natural frequencies appear for the same number of nodal diameters. The lower one of these two frequencies can be called the lower order natural frequency and the higher one the higher order natural frequency. With regard to lower order natural frequencies, the vibrations of the disk and of the blades are in the same phase, while with the higher order natural frequencies, a nodal point occurs on the blade. When rotary inertia and shear deformation are omitted with regard to the lower order natural frequencies, the natural frequencies increase by about 30 per cent over the case in which they are considered.

(3) With regard to the lower order natural frequencies, correct results can be obtained by taking rotary inertia and shear deformation of the disk into consideration in the small range of the numbers of nodal diameters, and in the large range, by taking these effects of the blade into consideration. The effect of rotary inertia on the disk is small, therefore, large errors can be avoided by taking the effect of shear deformation into consideration.

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