Curving Simulation of Four Axle Railway Vehicles

with Conventional Two Axle Bogies*

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We developed a dynamic model for curving of four axle rail vehicles with conventional two axle bogies, and derived equations of their motion. Then some digital simulations were made to obtain time history of rail-wheel vertical and lateral forces during curving. In the case of an ordinary truck, it is shown that it runs on curved tracks with its outer wheel of the front axle slipping between the flange and the rail, as is well known. However, slipless curving is achieved by a proper selection of parameters. This simulation was also applied to those cases where dynamic coupler forces were applied, and there was a lateral rail irregularity.

Key Words : Transport Vehicles, Railways, Curving Performance Simulation, Rail/Wheel Vertical and Lateral Forces, Coupler Forces, Rail Irregularity

1. Introduction

Important characteristic values in evaluating the curving performance of railway vehicles include wheel load (vertical loads upon track surface) and lateral forces (horizontal and transverse forces on track surface). It is thought that, in general, the smaller the lateral forces, the less the wear experienced in the wheel flanges and the less likely is squealing of the wheels on the rails. Also, a smaller derailment coefficient (ratio of lateral forces to wheel load) means a smaller risk of climb-on-rail derailment. Many investigations, not all of which are recent, have been conducted about the wheel load and lateral forces at work when a railway vehicle negotiates a curve. In some studies, the steady-state values have been determined by using static equilibrium equations, while in others, simulations have been conducted of dynamic phenomena. In Japan, much research has been carried out on two-axle freight cars^{(1),(2)}, but little has been done on four-axle railway vehicles. In contrast, studies have been reported abroad on two-axle and four-axle railway vehicles^{(3),(4)}. These studies closely simulate creep forces taking into account the wheel-rail contact geometry and the spin creepage, but it seems that they

do not consider nonlinear elements in various parts of the vehicle.

In the present study, the authors treat the wheel-rail contact forces in about the same way as in the study by Arai and Yokose (2), and construct a dynamic model of four-axle railway vehicles with conventional two-axle bogies taking into account non-linear elements such as a truck yaw restraining spring with friction in series, and derive equation of motion which they apply to carry out computer simulations of curving railway vehicles.

They also discuss an application of this model to the cases in which there exist some disturbances, such as the dynamic coupler forces that act when braking forces are applied on a curve, or the presence of lateral irregularities in the rails.

2. Dynamic Model and Equation of Motion

2.1. Preconditions

It is assumed that :

- (1) the wheel is in contact with the rail at one point,
- (2) the creep force-creepage relationship is expressed by Chartet's equation⁽⁵⁾, and
- (3) the radius of the curve is sufficiently large with respect to the length of the vehicle.

2.2. Notations

Figure 1 gives a schematic view of the dynamic model of a railway vehicle that was chosen as the subject of the study. The following coordinates and nomenclature have been adopted. Hereafter, the z-axis direction will be referred to as the vertical direction, the x-axis direction as the longitudinal direction, the y-axis direction as the lateral direction, angular displacement about the z axis as

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yaw, angular displacement about the x axis as roll, and angular displacement about the y axis as pitch. The meanings of the most important symbols used in this paper are as follows:

y_b, y_{ti} : lateral displacements of center of gravity of body and truck frame
 y_j, y_{oj} : lateral displacements of center of gravity of wheelset and track mass
 z_b, z_{ti} : vertical displacements of center of gravity of body and truck frame
 z_j : vertical displacement of center of gravity of wheelset
 $\phi_b, \phi_{ti}, \phi_j$: yaw angles of body, truck frame, and wheelset respectively
 $\theta_{bi}, \theta_{ti}, \theta_j$: roll angles of body, truck frame, and wheelset respectively
 ψ_b, ψ_{ti} : pitch angles of body and truck frame
 $2m_b, m_t$: masses of body and truck frame
 m_w, m_r : masses of wheelset and truck
 i_{bx}, i_{tx} : radii of gyration of body and truck frame in yaw
 i_w : radius of gyration of wheelset in yaw
 i_{bx}, i_{tx} : radii of gyration of body and truck frame in roll
 i_{by}, i_{ty} : radii of gyration of body and truck frame in pitch
 k_1, c_1 : vertical stiffness and damping of axle spring
 k_2, c_2 : vertical stiffness and damping of bolster spring
 k'_2 : lateral stiffness of bolster spring
 c_y : lateral damping between body and truck
 k_0 : stiffness of bolster anchor
 k'_0 : longitudinal stiffness of bolster spring converted to bolster anchor position
 k_3, k'_3 : lateral stiffness between body and truck at center pin
 k_{wx}, k'_{wx} : lateral supporting stiffnesses of wheelset

k_{wx}, k'_{wx} : longitudinal supporting stiffnesses of wheelset
 k_r, c_r : lateral stiffness and damping of track
 k_b, c_b : torsional stiffness and damping of body
 e_1, e_2, e_3, e_4 : clearances and spring deflection strokes in wheelset suspension
 e_5, e_6 : clearances at center pin and bolster anchor
 $a, b, b_1, b_2, b_0, b'_0, l, h_t, h_1, h_2, h_3, h'_3, h_4, h'_4$: distances and heights shown in Figs. 1, 6, and 7
 r : rolling radius of wheel at neutral position
 r_b : inner race radius of journal bearing
 $\Delta r, \Delta r_L, \Delta r_R$: rolling radius variations from nominal value r
 α_{Lj}, α_{Rj} : contact angles between wheel and rail
 P, P_L, P_R : wheel loads
 Q, Q_L, Q_R : lateral forces
 $\epsilon_{Lj}, \epsilon_{Rj}$: nonlinear coefficients of creeping forces [Eq.15]
 μ : adhesion coefficient between wheel and rail
 p_0 : static wheel load
 f_0 : creep coefficient between wheel and rail
 f_b : lateral creep coefficient of journal bearing
 F_s : friction force at side bearer
 X_t : length of leading curve
 C_{aj}, R_j : cant and curve radius
 C_{a0}, R_0 : cant and curve radius on circular curve
 ϕ_{aj} : angle of attack due to curve
 ϕ_{si} : static relative yaw angle between body and truck frame
 ΔY_{rj} : relative lateral displacement between wheel and rail
 F_f, F_r : front and rear coupler forces
 y_{rr} : irregularity of rail
 v : travelling speed of vehicle
 The subscripts have the following meanings:
 $j = 1, 2, 3, 4$: 1st, 2nd, 3rd, and 4th

Fig. 1 A model of vehicle

axles, from the front to the back of the vehicle respectively
 $i = 1, 2$: front and rear trucks respectively
 R, L : right and left sides in the travelling direction respectively

2.3 Dynamic model and coordinates of vehicle

As shown in Fig. 1, the roll of the vehicle body is considered by a model in which the lumped roll moments of inertia at the front and rear truck positions are coupled with the torsional stiffness k_b ; thus the torsional elasticity is taken into account.

Figure 2(a) shows the coordinates for a vehicle passing a curve. The direction of the curve is set as shown in this figure, that is, the R side is the outer side of the rail. Assume the point on each axle lying on the track center line as the origin of y_i when the trucks are at rest with the lateral center line of each truck facing the center of the curve, the positions of the center of gravity of the truck frame and the car body in that state respectively as the origins of y_{i1} and y_b , and the positive directions of y_i , y_{i1} , and y_b as those denoted by the arrows in the figure. The origin and positive direction of the yaw angle will also be as denoted. In this fashion, the lateral displacement and yaw angle shall be the displacements in the coordinate system that moves with the truck along the curve. The vertical displacements, and the angles of roll and pitch shall be positive in the directions of the arrows in Fig. 1, zero displacement being the state in which the vehicle is at a standstill on a level track. The track surface banks due to canting, the vertical and the lateral directions being taken to be perpendicular and horizontal to the track surface respectively, as shown in Fig. 2(b).

The directions of the coordinate axes thus differ for each mass, but working on the assumption that the radius of the curve is sufficiently large in comparison with the length of the vehicle body, with the exception of the static relative yaw angle between body and truck frame, the disparity of the directions of the coordinate axes has little effect and therefore is ignored.

2.4 Effect of a curve

(a) Centrifugal force

From Fig. 2(b), the lateral effective acceleration α_{ej} at each axle position is

$$\alpha_{ej} = -\frac{v^2}{R_j} \cos \theta_{cj} + g \sin \theta_{cj} \quad (1)$$

and the variation β_{ej} in the vertical effective acceleration is

$$\beta_{ej} = -\frac{v^2}{R_j} \sin \theta_{cj} + g(1 - \cos \theta_{cj}) \quad (2)$$

where θ_{cj} is the angle of banking of the track surface, and β_{ej} is the variation from the value $-g$ in a stationary state on a level track.

Take the effective acceleration at

Fig. 2 Coordinates for curving

Fig. 3 Angle of attack due to curve

Fig. 4 Body-truck frame static relative yaw angle

the center of gravity of the truck frame as the mean of the values at the front and rear axle, and the effective acceleration value at the center of gravity of the body as the mean value for all axles. The forces that act owing to this effective acceleration are given by the product of the effective acceleration and the masses that move with this acceleration.

(b) Wheel-rail longitudinal creepage

The speed of the vehicle on the outer rail side is by bv/R_j larger, and that on the inner rail side, by bv/R_j smaller, than the travelling speed v at the center of the track. This gives rise to a creepage on the inner rail side of b/R_j , and $-b/R_j$ on the outer rail side.

(c) Wheel-rail lateral creepage

As can be seen in Fig. 3, because the gradient φ_{Pj} of the tangent to the curve at the truck center position differs from the gradient φ_{Pj+1} of the tangent to the curve at the front axle position, this gives rise to

an attack angle $-\phi_{aj}[-(\phi_{pj}-\phi_{pi})]$ at the front axle, and an attack angle $\phi_{aj+1}(=\phi_{pi}-\phi_{pj+1})$ at the rear axle. These attack angles give rise to the lateral creepage.

(d) Static relative yaw angle between body and truck frame

As shown in Fig. 4, since the gradient ϕ_{p1} of the tangent to the curve at the center position of the front truck and the gradient ϕ_{p12} of the straight line joining the center points of the front and the rear trucks differ, static relative yaw angles $\phi_{s1}(=\phi_{p1}-\phi_{p12})$ and $-\phi_{s2}[-(\phi_{p12}-\phi_{p2})]$ can be determined for the front and the rear truck respectively.

The cant C_{aj} , the radius R_j , and the curve function (relationship between X_j and y_{ij} in Fig. 3) of the leading curve are given as follows:

$$C_{aj} = C_{a0} \{1 - \cos(\pi X_j / X_i)\} / 2 \quad (3)$$

$$R_j = 2R_0 / \{1 - \cos(\pi X_j / X_i)\} \quad (4)$$

$$y_{ij} = \frac{X_j^2}{4R_0} - \frac{X_j^2}{2\pi^2 R_0} \left\{1 - \cos\left(\frac{\pi X_j}{X_i}\right)\right\} \quad (5)$$

θ_{aj} is given by $C_{aj}/2b$, and the gradient of the tangent to the curve is obtained from dy_{ij}/dX_j .

When the front and the rear axles of the trucks lie on a circular curve, $\phi_{aj} = \phi_{aj+1} = a/R_0$, and when the centers of the front and the rear trucks have both entered a circular curve, $\phi_{s1} = \phi_{s2} = l/R_0$.

2.5 Dynamic model and characteristics of non-linear elements

(a) Lateral force of wheelset lateral suspension

Figure 5 (a) shows a dynamic model of the wheelset. The lateral restoring force F_{yj} with respect to the lateral relative displacement $\Delta Y_j = y_j - y_{ij} \mp a\phi_{pi} - h_i\theta_{ij}$ (the minus/plus sign means that to the front axle is assigned the negative sign, and to the rear axle the positive) is shown in Fig. 5 (b). The sum of F'_{yj} and the creep force F_{ij} of the journal bearing gives the lateral force F_{yj} . Putting the angular velocity of rotation of the axle as ω , F_{ij} is given by

$$F_{ij} = 2f_b \Delta Y_j / (r_b \omega) \quad (6)$$

where ΔY_j is a differential value of ΔY_j , and f_b is the following value obtained from experiments made by the present authors.

$$f_b = 2.59 \times 10^4 P_b^{2/3} \quad \text{N} \quad (7)$$

Here, P_b is the radial load (KN) on the journal bearing.

(b) Restoring torque T_{xj} by wheelset longitudinal suspension

The restoring torque T_{xj} for the lateral relative displacement $\Delta X_j = b_1(\phi_j - \phi_{ij})$ is given by Fig. 5 (c).

(c) Lateral restoring force F_{ij} at center pin

Figure 6 (a) gives a dynamic model of the center pin, while Fig. 6 (b) shows the lateral restoring force F_{ij} with respect to the lateral relative displacement $\Delta Y_{ij} = y_{ij} \pm l\phi_b + k_3\theta_{bi} - y_{ij} - h_3\theta_{ij}$, where the plus/minus sign means that to the front truck is assigned the positive sign, and to the rear truck the negative.

Fig. 5 Model and characteristics of wheelset suspension

Fig. 6 Model and characteristic of body/truck frame lateral connection at center pin

Fig. 7 Model and characteristic of truck yaw restraining elements

Fig. 8 Wheel-rail contact forces

(d) Restoring torque T_{xi} by truck yaw restraining elements

Figure 7 (a) gives a dynamic model of truck yaw restraining elements in which a bolster anchor spring and side bearer friction are coupled in series (see also Fig. 1). Figure 7 (b) gives the restoring torque T_{xi} with respect to the relative displacement $\Delta X_{xi} = b_0(\phi_{ii} - \phi_b \pm \phi_{si})$ where the plus/minus sign means that to the front truck is assigned the positive sign, and to the rear truck the negative.

2.6 Wheel-rail contact forces

Figure 8 shows the wheel-rail contact forces. Putting the resultant creepage as ν and the normal force as N , the resultant creep force F on the wheel-rail contact plane can be expressed from Chartet's equation as follows:

$$\left. \begin{aligned} \frac{1}{F^2} &= \frac{1}{(\mu N)^2} + \frac{1}{(f_0 \nu)^2} \} \dots\dots\dots (8) \\ f_0 &= E \bar{a} \bar{b} \end{aligned} \right\}$$

Here, E is Young's modulus of the wheel and the rail materials, and $\bar{a}$ and $\bar{b}$ are half of the major and the minor axes of the wheel-rail contact ellipse respectively. Modifying Eq. (8) yields :

$$\left. \begin{aligned} F &= \epsilon f_0 \nu \\ \epsilon &= 1 / \sqrt{1 + \{f_0 \nu / (\mu N)\}^2} \end{aligned} \right\} \dots\dots\dots (9)$$

The longitudinal creepages ν_{L1} and ν_{R1} of the left and right wheels are respectively as follows :

$$\left. \begin{aligned} \nu_{L1} &= \frac{b \phi_j}{v} + \frac{\Delta r_{Lj}}{r} + \frac{b}{R_j} \\ \nu_{R1} &= -\frac{b \phi_j}{v} + \frac{\Delta r_{Rj}}{r} - \frac{b}{R_j} \end{aligned} \right\} \dots\dots\dots (10)$$

The lateral creepages ν_{L2} and ν_{R2} of the left and right wheels are respectively as follows :

$$\left. \begin{aligned} \nu_{L2} &= \left(-\frac{\dot{y}_j + r \dot{\theta}_j - \dot{y}_{0j}}{v} + \phi_j \mp \Phi_{aj} \right) / \cos \alpha_{Lj} \\ \nu_{R2} &= \left(-\frac{\dot{y}_j + r \dot{\theta}_j - \dot{y}_{0j}}{v} + \phi_j \mp \Phi_{aj} \right) / \cos \alpha_{Rj} \end{aligned} \right\} \dots\dots\dots (11)$$

Here, the minus/plus signs mean that to the front axle on each truck is assigned the negative sign, and to the rear axle the positive. The resultant creepages ν_{Lj} and ν_{Rj} of the left and right wheels are respectively

$$\left. \begin{aligned} \nu_{Lj} &= \sqrt{\nu_{L1}^2 + \nu_{L2}^2} \\ \nu_{Rj} &= \sqrt{\nu_{R1}^2 + \nu_{R2}^2} \end{aligned} \right\} \dots\dots\dots (12)$$

The lateral components f_{Lj} and f_{Rj} of the creep forces are respectively

$$\left. \begin{aligned} f_{Lj} &= \epsilon_{Lj} f_0 \nu_{Lj} \nu_{L1} / \nu_{Lj} = \epsilon_{Lj} f_0 \nu_{Lj} \nu_{L1} \\ f_{Rj} &= \epsilon_{Rj} f_0 \nu_{Rj} \nu_{R1} / \nu_{Rj} = \epsilon_{Rj} f_0 \nu_{Rj} \nu_{R1} \end{aligned} \right\} \dots\dots\dots (13)$$

Similarly, the longitudinal components Q_{1Lj} and Q_{1Rj} of the creep forces are respectively

$$\left. \begin{aligned} Q_{1Lj} &= \epsilon_{Lj} f_0 \nu_{Lj} \nu_{L2} \\ Q_{1Rj} &= \epsilon_{Rj} f_0 \nu_{Rj} \nu_{R2} \end{aligned} \right\} \dots\dots\dots (14)$$

where

$$\left. \begin{aligned} \epsilon_{Lj} &= 1 / \sqrt{1 + \{f_0 \nu_{Lj} / (\mu N_{Lj})\}^2} \\ \epsilon_{Rj} &= 1 / \sqrt{1 + \{f_0 \nu_{Rj} / (\mu N_{Rj})\}^2} \\ f_0 \nu_{Lj} &= f_0 (N_{Lj} / N_0)^{2/3} \\ f_0 \nu_{Rj} &= f_0 (N_{Rj} / N_0)^{2/3} \end{aligned} \right\} \dots\dots\dots (15)$$

Here, N_0 is the normal force corresponding to P_0 , and this takes into account that $\bar{a} \cdot \bar{b}$ is proportional to the normal force to the 2/3 power.

The wheel loads P_{Lj} and P_{Rj} are determined by solving simultaneous equilibrium equations of the vertical force and moment around point 0.

Fig. 9 Forces applied to wheelset

Once f_{Lj} , f_{Rj} , P_{Lj} , and P_{Rj} have been determined, Q_{Lj} , Q_{Rj} , N_{Lj} , and N_{Rj} can be obtained from the following set of equations.

$$\left. \begin{aligned} Q_{Lj} &= -P_{Lj} \tan \alpha_{Lj} + f_{Lj} / \cos \alpha_{Lj} \\ N_{Lj} &= P_{Lj} / \cos \alpha_{Lj} - f_{Lj} \tan \alpha_{Lj} \\ Q_{Rj} &= P_{Rj} \tan \alpha_{Rj} + f_{Rj} / \cos \alpha_{Rj} \\ N_{Rj} &= P_{Rj} / \cos \alpha_{Rj} + f_{Rj} \tan \alpha_{Rj} \end{aligned} \right\} \dots\dots\dots (16)$$

2.7 Equations of motion

The equations of motion are as follows :

For the track :

$$m_r \ddot{y}_{0j} = -k_r y_{0j} - c_r \dot{y}_{0j} - Q_{Rj} - Q_{Lj} \dots\dots\dots (17)$$

for the axles :

$$\left. \begin{aligned} m_w \ddot{y}_j &= -F_{y1} + Q_{Rj} + Q_{Lj} + m_w a_{ej} \\ m_w i_w^2 \ddot{\phi}_j &= -T_{x1} + b(Q_{1Rj} - Q_{1Lj}) \\ z_j &= C_{aj} / 2 \\ \theta_j &= -C_{aj} / (2b) \end{aligned} \right\} \dots\dots\dots (18)$$

For the truck frame :

$$\left. \begin{aligned} m_t \ddot{y}_{t1} &= K2D_1 + CY_1 + F_{y1} + F_{y2} + F_{t1} \\ &\quad + m_t \frac{a_{e1} + a_{e2}}{2} \\ m_t \ddot{y}_{t2} &= K2D_2 + CY_2 + F_{y3} + F_{y4} + F_{t2} \\ &\quad + m_t \frac{a_{e3} + a_{e4}}{2} \\ m_t i_{t2}^2 \ddot{\phi}_{t1} &= T_{x1} + T_{x2} + a(F_{y1} - F_{y2}) - T_{s1} \\ m_t i_{t2}^2 \ddot{\phi}_{t2} &= T_{x3} + T_{x4} + a(F_{y3} - F_{y4}) - T_{s2} \\ m_t i_{t2}^2 \ddot{\theta}_{t1} &= b_1(K1_{R1} + K1_{R2} - K1_{L1} - K1_{L2}) \\ &\quad + b_2(K2_{L1} - K2_{R1}) - h_1 K2D_1 + h_3 F_{t1} \\ &\quad + h_4 CY_1 + h_t(F_{y1} + F_{y2}) \\ m_t i_{t2}^2 \ddot{\theta}_{t2} &= b_1(K1_{R3} + K1_{R4} - K1_{L3} - K1_{L4}) \\ &\quad + b_2(K2_{L2} - K2_{R2}) - h_1 K2D_2 + h_3 F_{t2} \\ &\quad + h_4 CY_2 + h_t(F_{y3} + F_{y4}) \\ m_t \ddot{z}_{t1} &= -K1_{R1} - K1_{L1} - K1_{R2} - K1_{L2} \\ &\quad + K2_{R1} + K2_{L1} + m_t(\beta_{e1} + \beta_{e2}) / 2 \\ m_t \ddot{z}_{t2} &= -K1_{R3} - K1_{L3} - K1_{R4} - K1_{L4} \\ &\quad + K2_{R2} + K2_{L2} + m_t(\beta_{e3} + \beta_{e4}) / 2 \\ m_t i_{t2}^2 \ddot{\psi}_{t1} &= a(K1_{R1} + K1_{L1} - K1_{R2} - K1_{L2}) \\ m_t i_{t2}^2 \ddot{\psi}_{t2} &= a(K1_{R3} + K1_{L3} - K1_{R4} - K1_{L4}) \end{aligned} \right\} \dots\dots\dots (19)$$

For the vehicle body :

$$\begin{aligned}
 2m_b \ddot{y}_b &= -K_2 D_1 - K_2 D_2 - C Y_1 - C Y_2 \\
 &\quad - F_{11} - F_{12} + m_b(a_{e1} + a_{e2} + a_{e3} + a_{e4})/2 \\
 2m_b i_{bx}^2 \ddot{\phi}_b &= l(K_2 D_2 - K_2 D_1 + C Y_2 \\
 &\quad - C Y_1 + F_{12} - F_{11}) + T_{s1} + T_{s2} \\
 &\quad + m_b l(a_{e1} + a_{e2} - a_{e3} - a_{e4})/2 \\
 m_b i_{bx}^2 \ddot{\theta}_{b1} &= b_2(K_2 R_1 - K_2 L_1) \\
 &\quad - h_2 K_2 D_1 - h_3' F_{11} - h_4' C Y_1 - T K B \\
 m_b i_{bx}^2 \ddot{\theta}_{b2} &= b_2(K_2 R_2 - K_2 L_2) \\
 &\quad - h_2 K_2 D_2 - h_3' F_{12} - h_4' C Y_2 + T K B \\
 2m_b \ddot{z}_b &= -K_2 R_1 - K_2 L_1 - K_2 R_2 - K_2 L_2 \\
 &\quad + m_b(\beta_{e1} + \beta_{e2} + \beta_{e3} + \beta_{e4})/2 \\
 2m_b i_{bz}^2 \ddot{\psi}_b &= l(K_2 R_1 + K_2 L_1 - K_2 R_2 - K_2 L_2) \\
 &\quad \dots\dots\dots (20)
 \end{aligned}$$

Here,

$$\begin{aligned}
 K1_{Lj} &= k_1(z_{ti} \mp a\psi_{ti} + b_1\theta_{ti} - z_j - b_1\theta_j) \\
 &\quad + c_1(\dot{z}_{ti} \mp a\dot{\psi}_{ti} + b_1\dot{\theta}_{ti} - \dot{z}_j - b_1\dot{\theta}_j) \\
 K1_{Rj} &= k_1(z_{ti} \mp a\psi_{ti} - b_1\theta_{ti} - z_j + b_1\theta_j) \\
 &\quad + c_1(\dot{z}_{ti} \mp a\dot{\psi}_{ti} - b_1\dot{\theta}_{ti} - \dot{z}_j + b_1\dot{\theta}_j) \\
 &\quad \dots\dots\dots (21 \cdot a)
 \end{aligned}$$

where the minus/plus sign means that to the front axles on each truck are assigned the negative sign, and to the rear axles the positive.

$$\begin{aligned}
 K2_{Li} &= k_2(z_b \mp l\psi_b + b_2\theta_{bi} - z_{ti} - b_2\theta_{ti}) \\
 &\quad + c_2(\dot{z}_b \mp l\dot{\psi}_b + b_2\dot{\theta}_{bi} - \dot{z}_{ti} - b_2\dot{\theta}_{ti}) \\
 K2_{Ri} &= k_2(z_b \mp l\psi_b - b_2\theta_{bi} - z_{ti} + b_2\theta_{ti}) \\
 &\quad + c_2(\dot{z}_b \mp l\dot{\psi}_b - b_2\dot{\theta}_{bi} - \dot{z}_{ti} + b_2\dot{\theta}_{ti}) \\
 &\quad \dots\dots\dots (21 \cdot b)
 \end{aligned}$$

where the minus/plus sign means that to the front truck is assigned the negative sign, and to the rear truck the positive.

$$\begin{aligned}
 K2D_i &= 2k_2'(y_b \pm l\phi_b + h_2\theta_{bi} \\
 &\quad - y_{ti} + h_1\theta_{ti}) \\
 C Y_i &= c_y(\dot{y}_b \pm l\dot{\phi}_b + h_2'\dot{\theta}_{bi} \\
 &\quad - \dot{y}_{ti} - h_1\dot{\theta}_{ti}) \\
 &\quad \dots\dots\dots (21 \cdot c)
 \end{aligned}$$

where the plus/minus sign means that to the front truck is assigned the positive sign and to the rear truck the negative.

$$TKB = k_b(\theta_{b1} - \theta_{b2}) + c_b(\dot{\theta}_{b1} - \dot{\theta}_{b2}) \dots\dots\dots (21 \cdot d)$$

These equations of motion are simultaneous nonlinear ordinary differential equations. A time history is determined by these equations in step-by-step integration including the Runge-Kutta-Gill method. It is assumed, as the initial conditions that the foremost axle is situated at the origin of the leading curve, and at time $t=0$, all displacements and velocities are zero.

3. Computer Simulation

3.1 Conventional truck and steering truck

Table 1 gives the numerical values used in the simulation, Cases 1 and 2 being

examples of the values for an electric car and a passenger car respectively.

Figure 10 gives plots of the relationship between the wheel-rail lateral relative displacement ΔY_{rj} and the values Δr_{Lj} and Δr_{Rj} . The values Δr in Fig. 10 are taken as equally spaced discrete values ΔY_{rj} , and values between the marked points are determined by linear interpolation.

As for α_{Lj} and α_{Rj} , these can be calculated from the following equations by putting the wheel tread conicity respectively as α_{0Lj} and α_{0Rj} , (The flanges are treated in the same manner as the treads.)

$$\begin{aligned}
 \alpha_{Lj} &= \alpha_{0Lj} - (\Delta r_{Rj} - \Delta r_{Lj})/(2b) \\
 \alpha_{Rj} &= \alpha_{0Rj} + (\Delta r_{Rj} - \Delta r_{Lj})/(2b)
 \end{aligned} \dots\dots\dots (22)$$

ΔY_{rj} can be expressed as follows :

$$\Delta Y_{rj} = y_j + r\theta_j - y_{0j} \dots\dots\dots (23)$$

Case 1 is an example of an electric car with conventional trucks. The results of the simulation are shown in Fig. 11. This shows the results obtained when a truck passes a leading curve and enters a circular curve up until the wheel loads, lateral forces, etc. reach steady values. Numerical integration was carried out by the Runge-Kutta-Gill method in this simulation at time intervals of 0.5 ms.

From Fig. 11, it can be seen that, on circular curves where $t > 9s$, the outer wheel (R side) on the front axle of each truck (axles 1 and 3) has a large lateral force,

Table 1 Numerical values

Item	Unit	Case 1	Case 2	Item	Unit	Case 1	Case 2
mb	kg	6960	1750	l _{tx}	cm	80	107
mr	"	1250	968	l _{ty}	"	80	46
mw	"	1130	575	l _w	"	58	54
mr	"	490	110	a	"	90	65
kwy	kN/cm	16	200	b	"	75	56
kwy'	"	1.0x10 ³	200	b _i	"	97	74
kwx	"	15.7	1.0x10 ³	b _z	"	80	74
kwx'	"	1.0x10 ³	1.0x10 ³	b _a	"	119	74
k _o	"	52	5.9x10 ²	b _{o'}	"	40	7
k _{o'}	"	2.2	26	l	"	500	350
k _i	"	12	20	h _t	"	0	0
k ₂	"	3.1	20	h _i	"	37	0
k _{2'}	"	2.5	3.4	h _z	"	75	92
k ₃	"	0	0	h _s	"	-2	-25
k _{3'}	"	25	100	h _{s'}	"	110	67
k _r	"	245	100	h _a	"	-30	-25
k _b	kN/cm	2.0x10 ⁴	1.0x10 ⁴	h _{a'}	"	82	67
C _r	kN-s/cm	0.049	0.39	r	"	38	28
C ₂	"	0.10	0.21	r _b	"	6	—
C ₃	"	0	0	e _i	"	0.006	0
C _y	"	0.69	0.39	e _z	"	0.1	0.01
C _r	"	0.49	0.29	e _s	"	0.3	—
C _b	kN-cm-s	1.0x10 ³	1.0x10 ⁴	e _{s'}	"	0.16	—
f _o	kN	2.9x10 ³	2.2x10 ³	e _s	"	1.0	0.5
f _b	"	186	0	e _s	"	0.02	0.1
F _s	"	4.5	1.3	μ	"	0.3	0.3
P _o	"	25.5	12.9	X _z	m	100	8
ib _z	cm	450	305	C _{ao}	cm	10	1.7
ib _x	"	140	75	R _o	m	100	100
ib _y	"	450	305	U'	km/h	40	15
itz	"	80	120				

IN=0.102 kgf

Fig. 10 Relation between Δr and ΔY_{rj}

the lateral forces on the front axle acting to spread the track gauge. It is also apparent that, from $\Delta Y_{r1} < -1\text{cm}$ and $\Delta Y_{r3} < -1\text{cm}$, the flange on the outer wheel of the front axle is in contact with the rail, and that from $b\phi_1 < 0$ and $b\phi_3 < 0$, the front axle undergoes a yaw angular displacement in a direction opposite to that of the curve. Also, ϵ_{R1} decreases significantly compared to unity (creepage = 0), and approaches 0 (creepage = ∞). This means that a large slippage arises at the wheel flange. This agrees with the well-known behavior of conventional trucks on curves.

Next, to improve the curving performance, it is tried to reduce k_{wx} , k'_{wx} , and r , and to increase the wheel tread concicity and the slack (clearance between the wheel flange and the rail). Fig. 12 shows the results of simulations carried out when $k_{wx} = k'_{wx} = 6\text{ kN/cm}$, $r = 26\text{ cm}$ and when the wheel tread concicity is set at $1/10$, or double that in Fig. 10, and the slack S is set at 2.5 cm . From Fig. 12, it is clear that, because $|\Delta Y_{r1}|$, $|\Delta Y_{r3}|$ is smaller than the slack 2.5 cm on a circular curve where $t > 9\text{ s}$, the flange is not in contact with the rail and because $b\phi_1 > 0$ and $b\phi_3 > 0$, the front axle of each truck makes a yaw angular displacement in the same direction as the curve. It can be also seen that because there is almost no increase in lateral forces due to the curve, and ϵ_{R1} has a value close to unity, there is little slippage between the wheels and the rails. This is because, due to the rolling radii difference between the left and the right wheel at the point of contact with the rails, each axle self-steers and shifts toward the radial position. As a result, the longitudinal and the lateral creepage values obtained from Eqs. (10) and (11) are very small.

3.2 Effect of dynamic coupler forces

Computer simulations can also be used to determine the behavior that occurs when various disturbances act upon a railroad car. Here is taken up a typical case in which dynamic coupler forces are applied on a curve.

When a train is braked while negotiating a curve, dynamic coupling forces arise from disparity in the braking forces of each car. The effects of these forces must also be considered.

Figure 13 (a) shows a number of coupled cars on a curve. Taking any one of these cars, it can be considered in terms of a dynamic model in which, as shown in Fig. 13(b), dynamic forces F_r and F_f act on the center line of the vehicle body in the direction of the angle ϕ , at a height by under the center of gravity of the body, and at a position away from the body center by l' . These dynamic coupling forces cause the following forces and moments to act upon the vehicle body.

- (a) lateral force: $-(F_r + F_f) \sin \phi$
- (b) yaw moment: $(F_r - F_f) l' \sin \phi$
- (c) roll moment: $-h_s F_r \sin \phi$ (front half of body), $-h_s F_f \sin \phi$ (rear half of body)
- (d) pitch moment: $(F_r - F_f) h_s \cos \phi$

Fig. 14 shows the results of a simulation of Case 2 in Table 1 in which $l' = 500$

Fig. 11 Results of simulation when conventional trucks were used

Fig. 12 Results of simulation when curving performance was improved

Fig. 13 Vehicle model when dynamic coupler forces were applied

cm, $h_s=56$ cm, and $\phi=0.055$ rad. In this simulation, the time intervals in the Runge-Kutta-Gill method were set at 0.2 ms. The dynamic coupling forces were put as shown in Fig. 14. When the dynamic coupling forces are added to the model, the lateral force Q and the wheel load P fluctuate. The derailment quotient Q/P can be calculated from this, and the safety against derailment can be assessed.

3.3 Effect of rail irregularity

Dynamic fluctuations in the wheel loads and the lateral forces arise in the presence of rail irregularities on a curve. Take one example in which, rail joints are considered in the model and the presence of sinusoidal troughs in the outer rail is assumed. The rail irregularities y_{rr} can be expressed as follows (see Fig. 15) :

$$y_{rr} = \begin{cases} -\frac{H}{2} \left[1 - \cos \left\{ \frac{2\pi}{l_0} v(t-t_r) \right\} \right] & (t_r < t < t_r + l_0/v) \\ 0 & (t \leq t_r \text{ and } t \geq t_r + l_0/v) \end{cases} \quad \dots (24)$$

When there are rail irregularities, ΔY_{rj} when calculating Δr_{Lj} , Δr_{Rj} , a_{Lj} , and a_{Rj} is given by

$$\Delta Y_{rj} = y_j + r\theta_j - (y_{oj} + y_{rr}) \quad \dots (25)$$

Fig.16 shows the results of a simulation using the values $l_0=50$ cm, $H=1$ cm, and $t_r=5$ s. This figure shows the phenomena during the interval 4~6.5 seconds after the first axle has left the origin of the leading curve. Clearly, rail irregularities cannot be disregarded.

4. Conclusions

The following results were obtained from a computer simulation study of railway vehicles negotiating a curve.

- (1) A dynamic model of four-axle railway vehicles with conventional two-axle bogies negotiating a curve was constructed, and used to derive a set of equation of motion.
- (2) Computer simulations were performed for numerical examples using conventional bogies, and the results obtained were agreed with the well-known phenomenon where the flange on the outer wheel of the front axle on each truck comes into contact with the rail and rounds the curve while undergoing slippage. It was found that by selecting suitable parameters, the vehicle can be made to steer along the curve with almost no slippage.
- (3) To investigate what happens in the presence of external disturbances, simulations were made in which dynamic coupling forces act for example due to braking on a curve, and also cases in which rail irregularities are present were investigated.

Fig. 14 Results of simulation when dynamic coupler forces were applied

Fig. 15
Rail irregularity

Fig. 16
Results of simulation when there was a rail irregularity

These simulations show that the effects of such disturbances cannot be disregarded.

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