Estimation of First Excursion Probability for Mechanical
Appendage System Subjected to Nonstationary
Earthquake Excitation*

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An estimation technique whereby the first excursion probability of the mechanical appendage system subjected to the nonstationary seismic excitation can be conventionally calculated is proposed. The first excursion probability of the appendage system is estimated by using this method and the following results are obtained.

(1) The probability from this technique is more conservative than that from a simulation technique taking artificial time histories compatible to the design spectrum as input excitation.

(2) The first excursion probability is practically independent of the natural period of the appendage system when the tolerable barrier level is normalized by the response amplification factor given by the design spectrum.

(3) The first excursion probability decreases as the damping ratio of the appendage system increases. It also decreases as the mass ratio of the appendage system to the supporting system increases.

(4) For the inelastic appendage system, the first excursion probability is reduced, if an appropriate elongation is permitted.

Key Words: Vibration, Random Vibration, First Excursion Probability,
Nonstationary Artificial Time History, Envelope Function

1. Introduction

The important mechanical appendage systems such as mechanical equipment supported by the primary structure system built in nuclear power plant and other industrial facilities should be designed not to lose their performance even if they are subjected to destructive earthquake motions. Therefore, development of aseismic design method of the appendage system has been considered as an important problem up to the present. In order to contribute to aseismic design of the appendage system, authors have proposed a conventional estimation technique of earthquake response of the appendage system.^{1),2)} From an other view point, results of seismic risk analysis for mechanical facilities in urban areas³⁾ and nuclear power plants^{4),5)} are reported. In these reports, estimation of failure probability of the appendage system has been one of the important studies for assesment of seismic risk.

In general, at least two failure mechanisms of structures can be envisioned, that is 1) first excursion failure and 2) cumulative damage failure. Studies about first excursion failure has been presented

with respect to the response of a single-degree of freedom system. These studies are associated with a system subjected to stationary⁶⁾ and nonstationary random excitations⁷⁾ and a system with nonlinear hysteretic characteristics.⁸⁾ Theoretical estimation method of the first excursion probability for a single-degree of freedom system subjected to some types of artificial time histories was also reported which uses an approximate determination of the distribution of the highest response peak.⁹⁾ Studies concerning the appendage system have been proposed.¹⁰⁾ However, it is considered that effect of the response characteristics of the appendage system and that of the supporting system on the first excursion probability has not been studied enough. If plastic deformation within allowable range is permitted, the response is expected to be reduced by energy absorbing effect. However, reduction of the first excursion probability by energy absorbing effect has not been made clear.

From an other view point, design response spectrum is used for actual aseismic design.¹¹⁾ Envelope functions which represent amplitude nonstationary characteristics of earthquake excitations are proposed.^{12),13)} Therefore, it is important for aseismic design to develop a first excursion probability estimation technique of the appendage system subjected to nonstationary excitation compatible to the design response spectrum.

In this paper, considering the above mentioned points, a theoretical estimation method of the first excursion probability

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of the appendage system subjected to non-stationary earthquake excitation compatible to the design response spectrum is proposed. Next, using the proposed method, effect of the response characteristics of the appendage system and those of the supporting system on the first excursion probability of the appendage system is examined. Next, reduction of the first excursion probability of the appendage system with perfectly-elasto-plastic restoring force-deformation relation is estimated. Finally, the first excursion probability is estimated by simulation method using nonstationary artificial time histories compatible to the design response spectrum in order to examine the proposed theoretical method.

2. Nomenclature

- P_f : first excursion probability of the appendage system
 B : tolerance level
 γ : mass ratio of the appendage system to the supporting system
 h_a : damping ratio of the appendage system
 h_s : damping ratio of the supporting system
 ω_a : natural circular frequency of the appendage system
 ω_s : natural circular frequency of the supporting system
 x_a : absolute displacement of the appendage system
 x_s : absolute displacement of the supporting system
 y : absolute displacement of the ground
 z_a : relative displacement of the appendage system to the supporting system $=x_a - x_s$
 z_s : relative displacement of the supporting system to the ground $=x_s - y$
 z_g : relative displacement of the ground to the base rock
 h_g : damping ratio of the ground model
 ω_g : predominant circular frequency of the ground model
 $I(t)$: envelope function which represent nonstationary amplitude characteristics of earthquake excitation
 $G(\omega)$: power spectral density function of the ground acceleration
 G_0 : power spectral density of the white noise which represents base rock motion
 σz_a^2 : variance of z_a
 $\sigma \dot{z}_a^2$: variance of relative velocity $\dot{z}_a$
 $\kappa z_a \dot{z}_a$: covariance of z_a and $\dot{z}_a$
 B_A : tolerance level with respect to absolute acceleration of the appendage system
 B_D : tolerance level with respect to z_a
 S : response amplification factor given by design response spectrum
 f : restoring force in the appendage system
 F : yielding force
 C_e : equivalent damping coefficient

- ω_e : equivalent natural circular frequency
 z_e : yielding displacement

3. First excursion probability estimation technique

3.1 Analytical model and input ground motion

A mechanical model used for estimation of P_f is shown in Fig.1. In this

Fig.1 First excursion problem

study, the appendage system and the supporting system are simulated by a single-degree of freedom system respectively having the fundamental natural frequency of each system. Here, a two-sided problem is considered. In this problem, failure of the appendage system occurs at the instant absolute response value of the appendage system first reaches B . P_f is described as follows.

$$P_f(t_i) = P(|x(t)|_{\max} > B; 0 < t < t_i) \quad (1)$$

Nonstationary artificial time histories compatible to design response spectrum are used as input ground motion for the following reasons: design response spectrum has been used for aseismic design¹¹⁾; P_f is affected by nonstationary characteristics of input ground motion. The design response spectrum used in this study is the standard response amplification factor for the high pressure gas facilities established by the Japanese Ministry of International Trade and Industry.¹¹⁾ This design response spectrum is shown in Fig.2. The Roman

Fig.2 Target response spectrum for 5% damping ratio of critical

numerals in this figure represent the kind of the ground.11) Envelope function representing nonstationary characteristics of the ground motion is shown in Fig.3. This envelope function is pro-

Fig.3 Envelope function for earthquake

posed by Jennigs et al.¹³⁾ for the significant ground motion such as El Centro and Taft. Artificial time histories used for simulation technique are generated by using the method presented by Vanmarke et al.^{12),14)}

Parameters with respect to the appendage system and the supporting system are selected by referring to the previous measuring results^{15),16)} as follows.

$$\gamma : 0, 0.01, 0.02, 0.05$$

$$h_a : 0.01, 0.02, 0.05$$

$$h_s : 0.05$$

The natural period of the appendage system is selected for the condition of $T_a (=2\pi/\omega_a) = T_s (=2\pi/\omega_s)$ as in the case where the response of the appendage system is large and failure of the appendage system becomes critical. T_a 's are selected in the range from 0.2s to 1.0s. This range includes the natural period which many mechanical facilities in actual plant have.

3.2 Response characteristics of the ground model

In order to derive theoretical estimation equation of P_f , identification of the response characteristics of the ground motion is necessary. Expected value of $G(\omega)$ is shown in Fig.4 with solid line, which is estimated by using 50 artificial time histories compatible to the response spectrum of the first kind of the ground¹¹⁾ which correspond to Tertiary formation. (Estimation results with

Fig.4 Power spectral density function of ground motion

respect to the first kind of the ground are mentioned as follows.) In this figure, if the input to the base rock were white noise, $G(\omega)$ could be expressed by Tajimi's equation¹⁷⁾ as follows.

$$G(\omega) = \frac{(2h_g\omega_g\omega)^2 + \omega_g^4}{(\omega_g^2 - \omega^2)^2 + (2h_g\omega_g\omega)^2} G_0 \quad (2)$$

Using the least square method, when $h_a=0.5$, $T_g (=2\pi/\omega_g)=0.285s$ and $G_0=1.94 \times 10^{-3}$ (1/s), the best fit curve is obtained. It is shown in Fig.4 as dashed line.

3.3 Theoretical equation for estimation of the first excursion probability

P_f is given as

$$P_f(t) = 1 - \exp\left\{-2 \int_0^t v(t) dt\right\} \quad (3)$$

assuming that the distribution of $z_a(t)$ is normal distribution and the instants at which $z_a(t)$ crosses B_D are statistically independent, $v(t)$ is given as

$$v(t) = \frac{1}{2\pi} \frac{\sqrt{D}}{\sigma_{z_a}} \left[\exp\left\{-\frac{B_D^2}{2\sigma_{z_a}^2} \left(1 + \frac{x_{z_a}^2}{D}\right)\right\} + B_D x_{z_a} \sqrt{\frac{\pi}{2D\sigma_{z_a}^2}} \exp\left\{-\frac{B_D^2}{2\sigma_{z_a}^2}\right\} (1 + \text{erf}(C)) \right] \quad (4)$$

$$\text{where } C = \frac{x_{z_a} B_D}{\sqrt{2D\sigma_{z_a}^2}}, D = \sigma_{z_a}^2 \sigma_{z_s}^2 - x_{z_a}^2 x_{z_s}^2, \text{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-y^2} dy$$

Using Eq.(2), Fokker-Planck equation for p with respect to $z_a, \dot{z}_a, z_s, \dot{z}_s, z_g, \dot{z}_g$ is obtained as follows.¹⁴⁾

$$\begin{aligned} \frac{\partial p}{\partial t} = & -\frac{\partial p}{\partial z_a} \dot{z}_a + 2h_a\omega_a p - \frac{\partial p}{\partial \dot{z}_a} (-2h_a\omega_a \dot{z}_a - \omega_a^2 z_a) \\ & + \gamma(2h_a\omega_a \dot{z}_a + f) + I(t)(2h_g\omega_g \dot{z}_g + \omega_g^2 z_g) \\ & - \frac{\partial p}{\partial z_s} \dot{z}_s + 2h_s\omega_s(1+\gamma)p - \frac{\partial p}{\partial \dot{z}_s} (-2h_s\omega_s(1+\gamma)\dot{z}_s \\ & - (1+\gamma)f + 2h_s\omega_s \dot{z}_s + \omega_s^2 z_s) - \frac{\partial p}{\partial z_g} \dot{z}_g + 2h_g\omega_g p \\ & + \frac{\partial p}{\partial \dot{z}_g} (2h_g\omega_g \dot{z}_g + \omega_g^2 z_g) + \frac{\partial^2 p}{\partial z_a^2} \frac{\pi G_0}{2} \end{aligned} \quad (5)$$

The second moments with respect to z_a and $\dot{z}_a$ are to be obtained in order to use Eq.(4). Partial integral method is applied to Eq.(5), then 21 differential equations of the first order are obtained. P_f is estimated by solving these differential equations and using Eq.(3).

3.4 Estimation of the first excursion probability of the inelastic appendage system

Estimation technique of P_f for the appendage system with perfectly-elasto-plastic restoring force-deformation relation shown in Fig.5 is presented in this section. f in Eq.(5) is equivalently linearized as follows.

$$f = C_e \dot{z}_a + \omega_e^2 z_a \quad (6)$$

When the yielding effect is not so great, it is assumed that the response of the restoring force produces a yielding force after the response increases and it approaches the stationary random process. C_e and ω_e^2 are obtained approximately from stationary random theory as follows.¹⁴⁾

Fig. 5 Perfectly-elasto-plastic model

$$\left. \begin{aligned} C_e &= \frac{2\omega_a^2 \operatorname{erf} c(-\eta^{-1})}{\sqrt{\pi} \omega_e \eta} \\ \omega_e^2 &= \omega_a^2 - \omega_a^2 \exp(-\eta^{-2}) \\ &\quad - \eta^{-1} \sqrt{\pi} \operatorname{erf} c(-\eta^{-1}) \end{aligned} \right\} \quad (7)$$

$$\text{where } \operatorname{erf} c(u) = 1 - \frac{2}{\sqrt{\pi}} \int_0^u e^{-y^2} dy, \quad \eta = \sqrt{2} \sigma_{se} / Z_e$$

F is a parameter for a perfectly-elasto-plastic model. F is determined by the following equation.

$$F = \alpha \times S \quad (8)$$

where α is a parameter which represents yielding effect.

In order to examine the equivalent linearization technique, variance of absolute acceleration response of the appendage system is estimated by Eq.(7) and by simulation technique using 50 artificial time histories. The results for $\alpha=10$ are shown in Fig.6. Dots are estimation values obtained by simulation technique and solid line is obtained by using Eq.(7). From this figure, equivalent linearization method using Eq.(7) is found effective for the case where the yielding effect is not so great.

4. Estimation results of the first excursion probability

P_f is a function of time as expressed in Eq.(1). However, P_f becomes constant value when enough time passes after main

Fig. 6 Mean square acceleration response of the appendage system ($\gamma=0, h_a=0.01, h_s=0.05, T_a=T_s=0.3s, \alpha=10$)

shock.18),19) The appendage system must be so designed as to be able to survive earthquake excitation. Therefore, attention is focused on P_f at the time when enough time passes after main shock.

4.1 Estimation results for linear system

The natural period of the appendage system dealt with in this study is relatively short. Since failure of such appendage system is caused by force, attention is focused on the absolute acceleration response $\ddot{x}_a$. In this case, Eq.(4) can not be directly applied. However, since $\ddot{x}_a$ is nearly equal to $\omega_a^2 z_a$, P_f can be estimated by substituting B_A/ω_a^2 into B_D . In this study, the tolerance level is represented by non-dimensional value normalized by the response spectrum as follows.

$$\lambda_t = B_A/S \quad (9)$$

P_f is shown in Fig.7 taking natural period as a parameter for $\gamma=0$ and $h_a=0.01$. From this figure, variation of P_f is found not so great for the same values of λ_t , so P_f is not so dependent on natural period. Same characteristics can be recognized for other parameter values. Therefore, P_f can be estimated conventionally when the tolerance level is normalized as Eq.(9).

Fig. 7 Probability of failure of the appendage system (Theory) ($\gamma=0, h_a=0.01, h_s=0.05, T_a=T_s$)

Next, in order to examine the effect of h_a and γ on P_f , P_f is shown in Fig.8(a) taking h_a as a parameter for $\gamma=0$ and in Fig.8(b) taking γ as a parameter for $h_a=0.01$. From these figures, P_f for the same λ_t decreases as h_a or γ increases. In this figure, $T_a=T_s=0.3s$, however, almost the same value of P_f can be obtained for other values of natural period, so P_f for other values of natural period can be estimated from Fig.8.

4.2 Estimation results for inelastic system

P_f of the appendage system with perfectly-elasto-plastic restoring force-deformation relation is very small for λ_t which is greater than acceleration corres-

Fig.8 Probability of failure of the appendage system (Theory) ($h_s=0.05, T_a=T_s=0.3s$)

ponding to F.18) On the other hand, failure could be caused by elongation so μ_t is used for tolerance level. μ_t is expressed as follows.

$$\mu_t = \frac{B_0}{Z_s} \quad (10)$$

In Fig.9, P_f is shown for $\alpha=10$ and $\alpha=5$. Comparing the above result with that of linear system, for example, when $\lambda_t=20$, from Fig.7, P_f is about 90% in the case of $T_a=T_s=0.3s$. For inelastic system, P_f is almost zero when $\lambda_t=20$ for both values of α .¹⁸⁾ On the other hand, for z_a , from Fig.9, P_f is less than that for linear system if allowable elongation is more than 1.7 times and 2.3 times z_e for the case of $\alpha=10$ and $\alpha=5$, respectively.

4.3 Comparison with the results of simulation technique

In order to examine the theoretical method, P_f is estimated by simulation technique. P_f is estimated by using 50 artificial time histories and using the following equation.

$$P_f = N_f / N \quad (11)$$

where N is the number of artificial time histories, N_f is the number of artificial time histories in which $|x(t)|$ crosses the tolerance level until enough time passes after main shock.

In Fig.10, P_f for $\gamma=0$ and $h_a=0.01$ is shown taking the natural period as a parameter. In order to distinguish from tolerance level for theoretical method, symbol λ_s is used for the tolerance level

Fig.9 Probability of failure of the appendage system (Theory) ($\gamma=0, h_a=0.01, h_s=0.05, T_a=T_s=0.3s$)

Fig.10 Probability of failure of the appendage system (Simulation) ($\gamma=0, h_a=0.01, h_s=0.05, T_a=T_s$)

expressed by Eq.(9). Just as in Fig.7, it is recognized that P_f is not so dependent on the natural period for the same values of λ_s .

In Fig.11(a), P_f for $\gamma=0$ is shown taking h_a as a parameter and in Fig.11(b), P_f for $h_a=0.01$ taking γ as a parameter. These figures correspond to Fig.8. Just as the theoretical results, it is recognized that P_f decreases as h_a or γ increases.

Comparing theoretical results with simulation results, P_f estimated by theoretical method is found greater than one by simulation method for the same tolerance level. The reason is that Eq. (4) for estimation of $v(t)$ is derived by assuming that instants at which $z_a(t)$ crosses B_D are statistically independent. However, since h_a used in this paper is less than 0.05, $z_a(t)$ is a narrow band random process. Therefore, once $z_a(t)$ crosses B_D , $z_a(t)$ does again with high probability after infinitesimal time. Then the assumption that instants at which $z_a(t)$ crosses B_D are statistically independent is strictly not appropriate.

Comparing Fig.10 with Fig.7 or Fig.11 with Fig.8, the relation can be seen between λ_t and λ_s as follows.

$$\lambda_s \approx 0.8\lambda_t \quad (12)$$

That is, when λ_t is multiplied by 0.8, theoretical results coincide with simulation results quantitatively.

Fig. 11 Probability of failure of the appendage system (Simulation) ($h_s = 0.05$, $T_a = T_s = 0.3s$)

Fig. 12 Probability of failure of the appendage system (Simulation) ($\gamma = 0$, $h_a = 0.01$, $h_s = 0.05$, $T_a = T_s = 0.3s$)

Next, P_f for the appendage system of which restoring force-deformation relation is represented by a perfectly-elasto-plastic model is estimated by simulation method. Estimated P_f is shown in Fig. 12. This figure corresponds to Fig. 9. In order to distinguish the tolerance level for simulation method from that for theoretical method expressed by Eq. (10), μ_s is used. P_f estimated by theoretical method is greater than that by simulation method for the same value of tolerance level as in the case of linear system. In this case, there is the following relation between μ_t and μ_s .

$$\mu_t \approx 0.5\mu_s \quad (13)$$

As mentioned in this section, it is clear that theoretical method gives more conservative values of P_f than simulation method and that relation between the tolerance level of theoretical method and that of simulation method is given by Eq. (12) for linear system and by Eq. (13) for inelastic system.

5. Conclusions

A theoretical estimation method of the first excursion probability P_f of the appendage system subjected to nonstationary earthquake excitation compatible to the design response spectrum is proposed. Using this method, P_f is estimated under the condition that the natural period of the appendage system coincides with that of the supporting system. Obtained results are summarized as follows:

- (1) Theoretical method gives more conservative values of P_f than simulation method. There are relation between tolerance level of the theoretical method and that of simulation method given by Eq. (12) for linear system and by Eq. (13) for inelastic system.
- (2) When the tolerance level is normalized by response amplification factor given by the design response spectrum, P_f is conventionally estimated only from mass ratio of the appendage system to the supporting system γ , the damping ratio of the appendage system h_a and normalized tolerance level.
- (3) P_f for the same tolerance level decreases as γ or h_a increases.
- (4) It is quantitatively shown that P_f of the appendage system with perfectly-elasto-plastic restoring force-deformation relation can be reduced by energy absorbing effect.

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