VIBRATION OF PSEUDO-SHOCK IN STRAIGHT DUCT*

(2ND REPORT, CORRELATION OF STATIC PRESSURE FLUCTUATION)

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In the 1st report pressure fluctuation at each point in the duct was measured and analyzed. In the present study pressure fluctuations at two points along the duct were simultaneously measured and correlated statistically in the forms of correlation factors and coherences. Thereby the propagation of the pressure fluctuation was investigated. The variation of a pseudo-shock had two main frequencies, high and low ones. The former, the vibration of the air column in the divergent passage, propagated upstream as a pressure wave, while the latter propagated convectionally with the main flow.

Key Words: Fluid Mechanics, Compressible Flow, Shock Wave, Fluid Vibration

1. Introduction

In the previous report(1) it was shown that the pseudo-shock oscillation was a kind of self-excited oscillation caused by the duct configuration. The static pressure fluctuation at the duct walls were measured and statistically analyzed to obtain spectrum and auto-correlation. As the result the pressure fluctuations were found to have two main components, a higher frequency vibration which was due to the oscillation of the air column in the divergent passage and the lower frequency one due to the first shock oscillation, each travelling in the opposite direction and exciting each other.

In the present study the pressure fluctuations at two different points were simultaneously measured and statistically analyzed to obtain the cross-correlation coefficients, the coherence and so on, so as to investigate the propagation of the pressure fluctuations in the pseudo-shock and the downstream duct.

2. Nomenclature

$C_x(\tau), C_y(\tau)$: auto-correlation function
 $C_{xy}(\tau)$: cross-correlation function
 $Con(f)$: coherence
 D : hydraulic diameter of straight duct = 34.3 mm
 f : frequency

P : pressure
 $\bar{P}$: time-mean pressure
 $\hat{P}$: RMS value of pressure fluctuation
 P_{os} : stagnation pressure in stagnation chamber
 p : static pressure fluctuation
 $R_{xy}(\tau)$: cross-correlation coefficient
 $\tilde{S}_i(f)$: imaginary part of cross-spectrum (quadrature spectrum)
 $\tilde{S}_r(f)$: real part of cross-spectrum (cospectrum)
 $\tilde{S}_x(f), \tilde{S}_y(f)$: power spectrum
 $\tilde{S}_{xy}(f)$: cross-spectrum
 t : time
 x : position of reference point, mm
 y : position of correlated point, mm
 $\theta(f)$: phase
 τ : time lag

3. Experiment

3.1 Apparatus and measurements

The experimental setup is shown in ref.(1). The tested ducts were I and IV of lengths 400 and 1000 mm respectively. The duct IV was long enough to cause a choking at the duct exit for a low back pressure. The stagnation pressure P_{os} in the stagnation chamber was 0.366 MPa, and the shock position was fixed at 200 mm from the duct inlet. Test conditions were three; for the duct I and for the duct IV, with and without a choking. The duct I was mainly used for the investigation of the propagation of the pressure fluctuation inside the pseudo-shock, while the duct IV was for the investigation of the propagation in the downstream duct and the effect of choking.

The pressure fluctuations at two different points were measured simultaneously with two pressure transducers and recorded in the data recorder and then analyzed with an electronic computer. As shown in Fig.1

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the four reference points were located at the foot of the first shock, $x = 205$ mm ($x/D = 5.98$), near the second shock, $x = 245$ mm ($x/D = 7.14$), in the middle of the pseudo-shock, $x = 315$ mm ($x/D = 9.18$) and at the exit of the duct, $x = 935$ mm ($x/D = 27.26$). The correlated points were selected upstream and downstream of each reference point.

3.2 Statistical analysis of pressure fluctuations

Pressure fluctuations stored in the data recorder were analyzed by the electronic computer HITAC M-200H after the AD-transformation by the minicomputer NOVA IV. Pressure $P(x, t)$ at a reference point x is represented as

$$P(x, t) = \bar{p}(x) + p(x, t) \quad (1)$$

where $\bar{P}(x)$ and $p(x, t)$ are time-mean pressure and fluctuation, respectively. Finite discrete Fourier transformation $X(f)$ of $p(x, t)$ is

$$X(f) = \sum_{n=0}^{N-1} p(x, n\Delta t) \exp\left(-i \frac{2\pi k}{T} n\Delta t\right) \Delta t \quad (k=0, 1, \dots, \frac{N}{2}-1) \quad (2)$$

where f is frequency, Δt the sampling time of data, N the sample number, $f = k/T$ and $T = N\Delta t$. Similarly for the fluctuation $p(y, t)$ at the correlated point y , we have

$$Y(f) = \sum_{n=0}^{N-1} p(y, n\Delta t) \exp\left(-i \frac{2\pi k}{T} n\Delta t\right) \Delta t \quad (3)$$

The conjugate $X^*(f)$ of $X(f)$ is

$$X^*(f) = \sum_{n=0}^{N-1} p(x, n\Delta t) \exp\left(i \frac{2\pi k}{T} n\Delta t\right) \Delta t \quad (4)$$

Then the power spectra $\tilde{S}_x(f)$ and $\tilde{S}_y(f)$ at the reference and the correlated points are

$$\tilde{S}_x(f) = \frac{1}{T} |X(f)|^2 \quad (5)$$

$$\tilde{S}_y(f) = \frac{1}{T} |Y(f)|^2 \quad (6)$$

The cross-spectrum is

$$\tilde{S}_{xy}(f) = \frac{1}{T} \{X^*(f)Y(f)\} \quad (7)$$

Fig.1 Measuring positions of pressure fluctuation

and the coherence is

$$Coh(f) = \frac{\tilde{S}_{xy}(f)}{\sqrt{\tilde{S}_x(f)\tilde{S}_y(f)}} \quad (8)$$

This value shows the predominant frequencies of the fluctuation propagation. $\tilde{S}_{xy}(f)$ is a complex value. The real and imaginary parts $\tilde{S}_r(f)$ and $\tilde{S}_i(f)$, called cospectrum and quadrature spectrum respectively give the phase $\theta(f)$ as

$$\theta(f) = \tan^{-1} \frac{\tilde{S}_i(f)}{\tilde{S}_r(f)} \quad (9)$$

The sign of this value indicates the direction of propagation, and the time lag τ and the velocity are calculated as $\theta(f)/2\pi f$ and $2\pi|x - y|/\theta(f)$, respectively.

The cross-correlation coefficient is given as a reverse Fourier transformation of the cross-spectrum as follows. Functions $A(\tau)$ and $B(\tau)$ are defined using $\tilde{S}_r(f)$ and $\tilde{S}_i(f)$ as

$$A(\tau) = 2 \sum_{n=0}^{N'-1} \tilde{S}_r\left(\frac{n}{T}\right) \cos\left(2\pi \frac{n\tau}{N'}\right) \frac{1}{T} \quad (10)$$

$$B(\tau) = 2 \sum_{n=0}^{N'-1} \tilde{S}_i\left(\frac{n}{T}\right) \sin\left(2\pi \frac{n\tau}{N'}\right) \frac{1}{T} \quad (k'=0, 1, \dots, \frac{N'}{2}-1) \quad (11)$$

$$\tau = 2k'\Delta t, N' = N/2$$

The cross-correlation function $C_{xy}(\tau)$ is

$$C_{xy}(\tau) = \begin{cases} A(\tau) - B(\tau) & (\tau \geq 0) \\ A(\tau) + B(\tau) & (\tau < 0) \end{cases} \quad (12)$$

Then the cross-correlation coefficient $R_{xy}(\tau)$ is

$$R_{xy}(\tau) = \frac{C_{xy}(\tau)}{\sqrt{C_x(0)C_y(0)}} \quad (13)$$

where $C_x(\tau)$ and $C_y(\tau)$ are the auto-correlation coefficients at the reference and the correlated points, respectively which are represented using the power spectra $\tilde{S}_x(f)$ and $\tilde{S}_y(f)$ as

$$C_x(\tau) = 2 \sum_{n=0}^{N'-1} \tilde{S}_x\left(\frac{n}{T}\right) \cos\left(2\pi \frac{n\tau}{N'}\right) \frac{1}{T} \quad (14)$$

$$C_y(\tau) = 2 \sum_{n=0}^{N'-1} \tilde{S}_y\left(\frac{n}{T}\right) \cos\left(2\pi \frac{n\tau}{N'}\right) \frac{1}{T} \quad (15)$$

In the actual calculation three sets of 4096 samples were AD-transformed with 40 μ s sampling time, and the power spectra of each data set were smoothed through the hanning, and the mean values $\tilde{S}(f)$ were obtained for each frequency. The cross-correlation coefficient was numerically calculated as a reverse Fourier transformation of the spectrum.

4. Results

4.1 Propagation of pressure fluctuations inside of pseudo-shock

The duct I was used for the investigation of the propagation inside of the pseudo-shock. The reference points were located at the foot of the first shock ($x = 205$), near the second shock ($x = 245$) and in the middle of the pseudo-

shock ($x = 315$). The correlated points were upstream and downstream of these points.

(1) **Reference point at foot of first shock ($x = 205$)** Figure 2 shows the cross-correlation coefficient for the correlated points located near the reference points. The correlation with the upstream point ($y = 195$) was very weak, suggesting little influence of upstream disturbance. For the downstream correlated points, the correlation was slightly stronger at the negative side of τ , which showed a propagation in the upstream direction. For further downstream points (Fig.3) high peaks appeared at the negative side of τ , and the peak position shifted to the left with the distance of the correlated points. These showed a propagation of the fluctuation in the upstream direction.

(2) **Reference points inside of pseudo-shock ($x = 245$ and 315)** Figures 4 - 8 shows the cross-correlation coefficients for the reference points at 245 and 315 mm. Clear peaks were seen on both sides of τ and they shifted outside with an increased distance between the points, which showed a fluctuation propagation in both upstream and downstream directions. The correlation for the upstream propagating fluctuations decreased only slightly with an increased distance of the measuring points and clear peaks were observed throughout the duct, suggesting that the fluctuation propagated from the divergent passage to the first shock with little damping. On the other hand for the downstream propagation although the correlation was stronger than that of the upstream propagation over a short distance, it decreased rapidly with the distance and vanished at $70 - 80 \text{ mm } ((2.0 - 2.3)D)$. This suggested a different propagation mechanism between the two fluctuations.

(3) **Direction and velocity of pressure fluctuation propagation** Figure 9 shows the relation between the position τ of the peaks in Figs.2 - 8 and the location of correlated points approximated by the straight lines. The positive gradient corresponds to the downstream propagation, and the negative one to the upstream propagation. The mean propagation velocities were found from the gradients

to be about 230 m/s for the downstream propagation and about 100 m/s for the upstream one.

At the reference point $x = 315$ corresponding to about 30 % of the pseudo-shock length, successive shock waves consisting of the λ -type pseudo-shock might disappear and the main flow might be decelerated to a nearly sonic one(2). Then the main flow velocity was about 250 - 310 m/s. The mean velocity was supposed to be less due to the unrecovered velocity profile of the low speed layer, and be comparable to that of the downstream propagation. This suggests that the upstream fluctuation propagated convectionally in the downstream direction. The convectionally propagating fluctuation was easily disturbed and diffused in a turbulent flow, and reduced rapidly with an increased distance. The velocity of the upstream propagation was almost equal to that of the sound travelling against the flow. This shows that the fluctuation propagated in the upstream direction as a pressure wave, whose correlation decreased little with the distance and had high peaks. Chen et al.(3) investigated the stability of a weak normal shock wave near the throat of the transonic diffuser and found a fluctuation convected downstream and a pressure wave travelling upstream behind the shock wave as also observed in the pseudo-shock in the duct.

Now the propagation of the pressure fluctuation is discussed using the coherence and the phase. Figure 10 shows the coherence of the reference point $x = 315$. High peaks of about 150 Hz were observed in the divergent passage as well as in the duct, which corresponded to the natural frequency(1) of the air column in the divergent passage. The fluctuation with this frequency was seen to propagate in the whole duct. For the closer correlated points the lower frequency components were relatively high, which corresponded to the first shock oscillation. The coherence and the phase for $x = 245$ and $y = 275$ are shown in Fig.11. The coherence was similar to that in Fig.10. The phase was negative near 150 Hz, suggesting an upstream propagation, and positive for the lower frequency mentioned above, showing a

Fig.2 Cross-correlation coefficients

Fig.3 Cross-correlation coefficients

downstream propagation.

To sum up, the higher frequency fluctuation caused by the oscillation of the air column in the divergent passage propagated in the upstream direction to the first shock as a pressure wave, while the lower frequency one due to the

oscillation of the first shock was convected by the flow in the downstream direction. This confirms the previous results(1) obtained without using the cross-correlation.

4.2 Propagation of pressure fluctuation downstream of pseudo-shock and

Fig.4 Cross-correlation coefficients

Fig.5 Cross-correlation coefficients

Fig.6 Cross-correlation coefficients

Fig.7 Cross-correlation coefficients

Fig.8 Cross-correlation coefficients

Fig.9 Propagation of pressure fluctuation

Fig.10 Coherence

Fig.11 Coherence and phase

effect of choke

Experiment was conducted for the longer duct IV to investigate the upstream propagation of the pressure wave.

Figure 12 shows the cross-correlation coefficient without a choke for the reference point near the duct exit ($x = 935$). The correlation was smooth wavy curves with high peaks, suggesting an orderly oscillation. Figure 13 shows the positions of peaks and locations of the correlated points. The plotted points fit a single straight line, whose gradient suggests an upstream propagation with the velocity of about 56 m/s. This was comparable to the velocity of a pressure wave moving against the flow which was nearly sonic at the almost choked duct exit. The fluctuation was suggested to be a pressure wave caused by the air column oscillation in the divergent passage from

the facts that the plotted point for the divergent passage was also on the same line in Fig.13 and that the coherence had a predominant frequency of about 200 Hz corresponding to the oscillation in the

Fig.15 Cross-correlation coefficients (with choke)

Fig.12 Cross-correlation coefficients (without choke)

Fig.16 Coherence (with choke)

Fig.13 Propagation of pressure fluctuation

Fig.17 Cross-correlation coefficients (without choke)

Fig.14 Coherence (without choke)

Fig.18 Cross-correlation coefficients (with choke)

divergent passage in Fig.14.

Figures 15 and 16 show the cross-correlation coefficients and the coherence with a choking at the duct exit. The correlation had no appreciable peaks and showed different trends for the correlated point in the duct and for the one in the divergent passage, suggesting an insulation by choking. The insulation was also observed from the lower coherence than that in Fig.14.

This evidence indicates that the pressure fluctuation due to the oscillation of the air column in the divergent passage propagated as a pressure wave in the upstream direction in the downstream part of the duct.

Figures 17 and 18 show the cross-correlation coefficients for the correlated points inside of the pseudo-shock without and with a choking, respectively. In the case of no choke no appreciable correlation was observed in the present time-lag range. But in the case with a choke a slight but clear correlation was seen throughout the duct. Similar analysis as mentioned above revealed a high frequency fluctuation of 300 - 350 Hz propagating downstream. The fluctuation seemed to be concealed unless the fluctuation in the divergent passage and shock oscillation were suppressed by a choking.

4.3 Another pressure fluctuation

As reported so far the first shock oscillation was a traslational one of the separation shock and was caused by a thickness change of the low speed layer at the virtual throat. The pressure fluctuations propagated convectionally or as a pressure wave. The combination of these mechanisms suggests a new oscillation system. The distortion of the boundary of the low speed layer caused by the separation shock vibration is convected to the virtual throat and changes the throat area. In turn the disturbance is transmitted upstream through the low speed layer as a pressure wave and excites the separation shock oscillation. Thus a closed loop is formed to maintain the oscillation. The

frequency is roughly estimated as

$$f = \frac{1}{l \left(\frac{1}{c_1} + \frac{1}{c_2} \right)} \quad (16)$$

where c_1 is the convection velocity, c_2 the sound velocity and l the distance between the separation point and the virtual throat. For $l \approx 3D(1)$, $c_1 = 230$ m/s, $c_2 = 100$ m/s referring to Sec.4.2, f was 677.3 Hz, which is higher than the natural frequency in the divergent passage. Although this oscillation was not observed due to the restriction in measurements and data analysis, it was supposed to be superposed on the low frequency oscillation of the first shock.

5. Conclusions

The pressure fluctuations measured simultaneously at different points were statistically correlated, and the previous results(1) obtained by the other methods were confirmed and supported as follows:

- (1) The pressure fluctuation had two main components, lower and higher ones.
- (2) The former was caused by the first shock oscillation and was convected in the downstream direction.
- (3) The latter was due to the oscillation of the air column in the divergent passage and propagated in the upstream direction as a pressure wave.
- (4) The former was reduced with the distance but the latter travelled with little damping.
- (5) The latter was predominant at the down stream part of the duct.
- (6) The latter was significantly reduced by a choking at the duct exit.
- (7) A weak fluctuation propagating in the downstream direction was observed in the duct with a choking.

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