Application of the Vortex Theory to High-speed Horizontal-axis Wind Turbines*

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The vortex theory of screw propellers is modified and applied to the high-speed horizontal-axis wind turbines. The turbine blade can be replaced by a bound vortex system, and when the energy loss per unit time is a minimum, the trailing vortices move backwards with a constant velocity and build a helical vortex sheet. A velocity potential function which represents the vortex system is obtained. The circulation distribution along the blade is calculated iteratively until the hydrodynamic pitch of helical vortices converges. This method is applied to two examples, one with a sectional shape of modified Clark-Y type and the other with a sectional shape of NACA 65-series type. Torque, power and resistance are computed for a turbine which has two to six blades, respectively. Numerical values agree well with the sectional shape of experimental ones and it is concluded that the vortex theory is applicable to high-speed horizontal-axis wind turbines.

Key words : Horizontal-axis Wind Turbine, Vortex Theory, Lifting-line Theory, Circulation Distribution, Performance of a Wind Turbine

1. Introduction

With a rise of interest in wind energy, many studies have come to be undertaken on wind turbines. Many experimental studies on high-speed horizontal-axis wind turbines, which are used for the large and middle scale wind power plants, have hitherto been made, while theoretical investigations have not so much progressed since the theory of H. Glauert⁽¹⁾. Even now there are not a few theoretical analyses⁽²⁾ which are based on the momentum theory. However, the theories in the past seem not enough to predict the experimental results, because the flow over the blades of high-speed horizontal-axis wind turbines which have two or three blades is three-dimensional.

In this paper, in which the vortex theory of wind turbines is constructed by the application of the lifting surface theory of screw propellers, the performances of wind turbines are investigated theoretically and experimentally. This paper concerns the case in which a uniform stream flows parallel to the rotating axis of the wind turbine and the fluid motion is in a stationary state. The modeling of the flow field is the same one based on the constant pitch non-linear theory of propellers. Trailing vortices and bound vortices distribute along the helical surface whose pitch is constant both in the radial and helical directions. The calculation of the performances of high-speed horizontal-axis

wind turbines whose aspect ratio is large uses the lifting line approximation.

Nomenclature

x, r, θ : cylindrical coordinates
 x', r', θ' : cylindrical coordinates of helicoid
 τ, σ, μ : helical coordinates
 $\tau = \theta + x/h, \sigma = \theta - x/h, \mu = r/h$
 ρ : density of fluid
 V : uniform flow velocity (in the positive direction of x axis)
 Ω : angular velocity of rotor (in the negative direction of x axis)
 r_0 : tip radius
 r_b : boss radius
 $\mu_0 = r_0/h, \mu_b = r_b/h$
 s : distance measured along the helicoid
 u : peripheral velocity at tip = Ωr_0
 X : tip speed ratio = u/V
 $2\pi h$: pitch of helicoid
 w_z : upwash on lifting line
 $w_\alpha = w_z \cos \epsilon_z, w_t = -w_z \sin \epsilon_z$
 $\bar{w}_\alpha$: mean value of w_α in radius
 $\bar{w}_t/r$: mean value of w_t/r in radius
 $h = (V + \bar{w}_\alpha) / (\Omega + \bar{w}_t/r)$
 Γ : circulation of blade section
 G : dimensionless circulation
 $= \Gamma / 2\pi r_0 V$
 C_o : torque coefficient
 C_R : resistance coefficient
 C_P : power coefficient
 l : number of blades
 $\tan \epsilon_z = 1/\mu$
 $\sin \epsilon_z = 1/\sqrt{1+\mu^2}$
 $\cos \epsilon_z = \mu/\sqrt{1+\mu^2}$
 $V^* = V + \bar{w}_\alpha, W = V^* \sqrt{1+\mu^2}$

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2. Theoretical analysis

2.1 Vortex theory of high-speed horizontal-axis wind turbines

The vortex theory of high-speed horizontal-axis wind turbines which is constructed in this section is based upon the fundamental theory of a screw propeller(3) (4).

2.1.1 Velocity potential of the flow field

As shown in Fig. 1, the axis x is the rotating axis of rotor, a uniform stream flows in the positive direction of x axis with a velocity V and the blades rotate in the negative direction of x axis with a constant angular velocity Ω . Both the bound vortex, whose density of circulation is γ , and free vortices which trail behind

Fig.1 Flow model

the bound vortex distribute on the helical surface whose pitch is constant($2\pi h$), then the velocity field is continuous everywhere except on the helical surface, where a jump in potential of $\int_{s_1}^s \gamma ds$ exists.

The fluid motion is steady for the coordinate system which rotates with the rotor, then the velocity potential Φ can be written as follows(5):

$$\Phi(x, r, \theta) = \frac{1}{4\pi} \sum_{m=0}^{l-1} \int_{r_b}^{r_0} dr \int_{s_1}^{\infty} \frac{\partial}{\partial n''} G(xr\theta | x'r'\theta') ds' \int_{s_1}^{s'} \gamma ds'' \quad (1)$$

,where the rotor has symmetrically spaced blades, l denotes the number of blades, r_0 and r_b are respectively the tip and boss radii, s is the distance along the helicoid, s_1 denotes s coordinate of leading edge, $\partial/\partial n''$ the normal differentiation on the helical surface. While, $G(xr\theta | x'r'\theta')$ expresses the Green function, which is written as follows:

$$G(xr\theta | x'r'\theta') = 1/R = 1/\sqrt{(x-x')^2 + r^2 + r'^2 - 2rr' \cos \theta_m}$$

$$= \begin{cases} \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i\lambda(x-x') + in\theta_m} I_n(|\lambda|r) K_n(|\lambda|r') d\lambda, & r' > r \\ \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i\lambda(x-x') + in\theta_m} I_n(|\lambda|r') K_n(|\lambda|r) d\lambda, & r' < r \end{cases} \quad (2)$$

where I_n and K_n represent modified Bessel functions, θ_m denotes $\theta - \theta' - 2m\pi/l$.

In equation (1), reverse the order of the repeated integral with respect to s' and s'' , and change the cylindrical coordinates for the helical coordinates, then Eq. (1) becomes

$$\Phi(\tau, \sigma, \mu) = \frac{h}{8\pi} \sum_{m=0}^{l-1} \int_{r_b}^{r_0} dr' \int_{s_1}^{s_2} \gamma \sqrt{1+\mu'^2} ds' \int_{\tau'}^{\infty} \frac{\partial}{\partial n''} \frac{1}{R} dT' \quad (3)$$

where s_2 denotes the position of the trailing edge of blade, R is expressed as follows:

$$R = h\sqrt{(\tau - T' - \sigma + \sigma')^2 + 4 + \mu^2 + \mu'^2 - 2\mu\mu' \cos\{(\tau - T' + \sigma - \sigma')/2 - 2m\pi/l\}} \quad (4)$$

$\partial/\partial n''$ in Eq. (3) denotes the differential operator whose derivatives and variable are $\partial/\partial T'$, $\partial/\partial \sigma'$ and μ' by which $\partial/\partial \tau$, $\partial/\partial \sigma$ and μ in the operator $\partial/\partial n$ are replaced respectively.

Using the expression of Eq. (2) as the Green function, the formula is obtained as follows:

$$\sum_{m=0}^{l-1} \frac{1}{R} = \frac{l}{\pi h} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \cos \frac{1}{2} \{(\lambda + lk)(\tau - \tau') - (\lambda - lk)(\sigma - \sigma')\} I_{lk}(|\lambda|\mu) K_{lk}(|\lambda|\mu') d\lambda, \quad \mu' > \mu \quad (5)$$

After the integration, using formula (5), the expression (3) of velocity potential becomes

$$\Phi = \Phi_{II} + \Phi_I \quad (6)$$

$$\Phi_{II} = -\frac{l}{4\pi^2 h} \int_{r_b}^{r_0} dr' \int_{s_1}^{s_2} \gamma ds' \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mu' \lambda - lk/\mu'}{\lambda + lk} \cos \frac{1}{2} \{(\lambda + lk)(\tau - \tau') - (\lambda - lk)(\sigma - \sigma')\} \\ \times I_{lk}(|\lambda|\mu) K_{lk}(|\lambda|\mu') d\lambda \quad (7)$$

$$\Phi_I = -\frac{l^2}{2\pi h} \int_{r_b}^{r_0} dr' \int_{s_1}^{s_2} \gamma ds' \sum_{k=1}^{\infty} k(\mu' + 1/\mu') I_{lk}(\lambda\mu) K_{lk}(\lambda\mu') \sin lk(\sigma - \sigma') \quad (8)$$

where $\mu_>$ and $\mu_<$ are larger and smaller ones between μ and μ' respectively.

$$\lim_{\tau \rightarrow \infty} \int_{-\infty}^{\infty} \frac{e^{iT(\lambda+n)/2}}{\lambda + n} f(\lambda) d\lambda = i\pi f(-n) \quad (9)$$

Using formula (9), Φ_{II} at $\tau - \tau' \rightarrow +\infty$ equals Φ_I . Therefore, the velocity potential far behind becomes

$$\Phi_{\infty} = \lim_{\tau - \tau' \rightarrow +\infty} \Phi_{II} + \Phi_I = 2\Phi_I \quad (10)$$

2.1.2 Upwash and pitch of helical vortex

The method for modeling of the lifting line theory is to concentrate the bound vortices which distribute on the surface of $\sigma=\sigma'$ to a vortex line at the radius of the blade, then the rotor has l pieces of vortex line. The induced velocity along the helical vortex ($\lim_{\sigma \rightarrow \sigma'} \partial \Phi / \partial s$) equals zero at the vortex line ($\tau=\tau'$), and the only one induced velocity normal to the helical surface (upwash) exists. The upwash is expressed as follows:

$$w = \partial \Phi / \partial n|_{\sigma=\sigma'} = w_n + w_t \dots \dots \dots (11)$$

$$w_n = \partial \Phi_n / \partial n|_{\sigma=\sigma'} = \frac{l}{4\pi^2 h^2 \sqrt{1+\mu^2}} \int_{r_0}^{r_0} dr' \int_{s_1}^{s_2} \gamma ds' \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(\mu' \lambda - lk/\mu')(\mu \lambda - lk/\mu)}{\lambda + lk} \times \sin(1/2)(\lambda + lk)(\tau - \tau') I_{1k}(|\lambda| \mu_<) K_{1k}(|\lambda| \mu_>) d\lambda \dots \dots \dots (12)$$

$$w_t = \frac{\partial \Phi_t}{\partial n}|_{\sigma=\sigma'} = \frac{l^3}{2\pi h^2 \sqrt{1+\mu^2}} \int_{r_0}^{r_0} dr' \int_{s_1}^{s_2} \gamma ds' \sum_{k=1}^{\infty} k^2 \left(\mu' + \frac{1}{\mu'} \right) \left(\mu + \frac{1}{\mu} \right) I_{1k}(lk \mu_<) K_{1k}(lk \mu_>) \dots \dots \dots (13)$$

It is clear that $w_n|_{\tau=\tau'}=0$ by Eq. (12), then the upwash reduces to

$$w|_{\tau=\tau'} = w_t = \frac{1}{2} \frac{\partial \Phi_t}{\partial n}|_{\sigma=\sigma'} \dots \dots \dots (14)$$

In the non-linear wing theory, the effect that the free vortices are deformed by themselves is assumed. According to its gist, the amount of trailing vortices is assumed to be determined by the mean in radius of the induced velocity at the bound vortex of the lifting line. Therefore, using the axial component w_a and the circumferential component w_t of the upwash at the bound vortex of lifting line, the pitch of the helical vortex is obtained as follows:

$$h = (V + \bar{w}_a) / (\Omega + \bar{w}_t/r) \dots \dots \dots (15)$$

where $\bar{w}_a$ and $\bar{w}_t/r$ denote the mean values in radius of w_a and w_t/r respectively. This is the assumption of the constant pitch non-linear theory. Figure 2 shows

$$W^* = \sqrt{(V + \bar{w}_a)^2 + r^2 (\Omega + \bar{w}_t/r)^2} \\ = (V + \bar{w}_a) \sqrt{1 + \mu^2} \dots \dots \dots (16)$$

W^* is assumed to be the inflow velocity to the blade section.

2.1.3 Integral equation of lifting line

The upwash angle of the lifting line theory at the blade is obtained by Munk theorem(6) as follows:

$$\alpha_i = -\frac{w_t}{W^*} = -\frac{1}{W^*} \frac{\partial \Phi_t}{\partial n}|_{\sigma=\sigma'} \dots \dots \dots (17)$$

Since the circulation of the lifting line becomes

$$\Gamma(\mu) = \int_{s_1}^{s_2} \gamma ds \dots \dots \dots (18)$$

Fig.2 Velocity components over wing section

, normal differentiation of Eq. (1) reduces Eq. (17) to

$$\alpha_i = \frac{1}{W^*} \int_{r_0}^{r_0} \Gamma(\mu') K(\mu, \mu') d\mu' \dots \dots \dots (19)$$

The kernel is written as follows:

$$K(\mu, \mu') = -\frac{3\sqrt{1+\mu^2}(1+\mu'^2)^{3/2}}{8\pi} \times \sum_{m=0}^{\infty} \int_{-\infty}^0 \frac{t \sin t_m}{(t^2 + \mu^2 + \mu'^2 - 2\mu\mu' \cos t_m)^{5/2}} dt \dots \dots \dots (20)$$

where t_m equals $t - 2m\pi/l$. The kernel has a pole of 2nd order, so that Eq.(19) becomes a singular integral equation. But the integration of Eq. (19) is treated in the sense of Hadamard's finite part of an infinite integral.

ε_0 denotes the angle between the tangent to the mean camber line and the circumference of rotor. Putting $\tan \varepsilon = V/\Omega r$, $\bar{\alpha}_g = \varepsilon - \varepsilon_0$, we obtain

$$\bar{\alpha}_g = \frac{1}{2\pi W^*} \int_{r_1}^{r_2} \frac{\gamma}{r - r'} dr' + \alpha_i + \bar{u} \dots \dots \dots (21)$$

This is the integral equation of the lifting line theory. $\bar{u}$ expresses the compensative factor in the lifting surface theory for the lifting line theory, which is neglected for a high speed rotor blade whose aspect ratio is large.

Using the dimensionless variables

$$\xi = (r - r_0)/\bar{r}, \quad \bar{r} = (r_2 - r_1)/2, \quad \left. \begin{aligned} \tau_0 &= (r_1 + r_2)/2 \\ \tau &= (r_1 + r_2)/2 \end{aligned} \right\} \dots \dots \dots (22)$$

, the solution of integral equation (21) is written as follows:

$$\frac{\gamma}{W^*} = -\frac{2}{\pi} \frac{\sqrt{1-\xi}}{1+\xi} \int_{-1}^1 \frac{\sqrt{1+\xi'}}{1-\xi'} \frac{\bar{\alpha}_g}{(\xi - \xi')} d\xi' - 2\alpha_i \sqrt{\frac{1-\xi}{1+\xi}} \dots \dots \dots (23)$$

The integration with respect to s between s_1 and s_2 reduces Eq. (23) to

$$\frac{\Gamma}{W^*} = 2c \int_{-1}^1 \frac{\sqrt{1+\xi'}}{1-\xi'} \bar{\alpha}_g d\xi' - 2\pi c \alpha_i \dots \dots \dots (24)$$

where $2c$, $2c = \bar{r} h \sqrt{1+\mu^2}$, denotes the chord length of the blade section. The set of Eqs. (24) and (19) is the integral equation of the lifting line theory of wind turbines.

2.1.4 Singularity of kernel

The kernel K is transformed into $\bar{K}$ as

follows:

$$K(\mu, \mu') = \frac{\bar{K}(\mu, \mu')}{(\mu - \mu')^2} \dots\dots\dots (25)$$

The factor $m \neq 0$ among the factors of the sum with respect to m in Eq. (20) has no singularity, so that it is very small in magnitude in the neighborhood of $\mu = \mu'$ in $\bar{K}$. In the neighborhood of $\mu = \mu'$, $\bar{K}$ becomes

$$\begin{aligned} \bar{K}(\mu, \mu') &\cong -\frac{1}{8\pi} \left\{ \sqrt{1+\mu^2} + \frac{(4-\mu^2)}{4(1+\mu^2)} \right. \\ &\times \left. \frac{(\mu-\mu')^2}{2\sqrt{1+\mu^2}} \ln|\mu-\mu'| \right\} \dots\dots\dots (26) \end{aligned}$$

$\bar{K}$ has a logarithmic singularity besides a pole of 2nd order, which is due to the tortuousness of the free vortex⁽³⁾

2.2 Calculating method

The variables μ and μ' are replaced by the variables φ and φ' as follows:

$$\begin{aligned} \bar{\mu} &= (\mu_0 - \mu_b)/2, \quad \mu_1 = (\mu_0 + \mu_b)/2, \\ \cos \varphi &= (\mu - \mu_1)/\bar{\mu}, \\ \cos \varphi' &= (\mu' - \mu_1)/\bar{\mu} \dots\dots\dots (27) \end{aligned}$$

Using the kernel $\bar{K}^*$ which is made by separating the logarithmic singularity from $\bar{K}$ and multiplying by $2/(1+\mu^2)$, the expression of the upwash angle is given as

$$\begin{aligned} \alpha_i &= \frac{1}{\bar{\mu}} H \int_0^\pi \frac{\gamma(\varphi') \bar{K}^*(\varphi, \varphi') \sin \varphi'}{(\cos \varphi - \cos \varphi')^2} d\varphi' \\ &- \frac{\bar{\mu}(4-\mu^2)}{32\pi(1+\mu^2)^{3/2}} \int_0^\pi \gamma(\varphi') \sin \varphi' \\ &\times \ln|\cos \varphi - \cos \varphi'| d\varphi' \dots\dots\dots (28) \end{aligned}$$

where γ denotes $\Gamma(\varphi')/V^*h$.

Using the approximation of the method of least squares, $\gamma(\varphi') \bar{K}^*(\varphi, \varphi')$ and $\gamma(\varphi')$ are approximated as

$$\begin{aligned} \left\{ \gamma(\varphi') \bar{K}^*(\varphi, \varphi') \right\} &= \frac{2}{m+1} \sum_{j=1}^m \left\{ \frac{\gamma(\varphi_j) \bar{K}^*(\varphi, \varphi_j)}{\gamma(\varphi_j)} \right\} \\ &\times \sum_{k=1}^m \sin k\varphi_j \sin k\varphi' \dots\dots\dots (29) \end{aligned}$$

where φ_j is defined as $\varphi_j = j\pi/(m+1)$. Substitute Eq. (29) into Eq. (28), and integrate Eq. (28), then the integral equation is transformed into linear algebraic equations written as

$$\begin{aligned} \alpha_{iv} &= -\frac{2\pi}{\bar{\mu}} b_{vv} \gamma_v \bar{K}_{vv}^* + \frac{2\pi}{\bar{\mu}} \sum_{j=1}^m \gamma_j \bar{K}_{vj}^* b_{vj} \\ &+ \frac{1}{m+1} \sum_{j=1}^m \gamma_j P_{vj} \dots\dots\dots (30) \end{aligned}$$

$$\begin{aligned} P_{vj} &= \frac{\bar{\mu}(4-\mu^2)}{32\pi(1+\mu^2)^{3/2}} \left\{ \ln 2 \cdot \sin \varphi_j \right. \\ &+ \sum_{k=2}^m \frac{\sin k\varphi_j \cos(k-1)\varphi_v}{k-1} \\ &- \left. \sum_{k=1}^m \frac{\sin k\varphi_j \cos(k+1)\varphi_v}{k+1} \right\} \dots\dots\dots (31) \end{aligned}$$

where b_{vv} and b_{vj} are matrix elements of Multhopp (7), which are defined as

$$\begin{aligned} b_{vv} &= (m+1)/4 \sin \varphi_v, \\ b_{vj} &= \frac{1 - (-1)^{j+v} \sin \varphi_j}{2(m+1)(\cos \varphi_j - \cos \varphi_v)^2} \dots\dots\dots (32) \end{aligned}$$

and $\sum_j$ denotes the summation with respect to j except $j=v$, $\varphi_v, \bar{K}_{vj}^*, \gamma_v$ and $\bar{K}_{vv}^*$ are

defined as follows:

$$\left. \begin{aligned} \varphi_v &= \nu\pi/(m+1), \quad \bar{K}_{vj}^* \equiv \bar{K}^*(\varphi_v, \varphi_j), \\ \gamma_v &= \gamma(\varphi_v), \quad \bar{K}_{vv}^* = -1/(4\pi\sqrt{1+\mu_v^2}) \end{aligned} \right\} \dots\dots (33)$$

The expression of the upwash angle can be written in order as follows:

$$\alpha_{iv} = B_{vv} \gamma_v - \sum_{j=1}^m B_{vj} \gamma_j \dots\dots\dots (34)$$

$$B_{vv} = \frac{b_{vv}}{2\bar{\mu}\sqrt{1+\mu_v^2}} + \frac{1}{m+1} P_{vv} \dots\dots\dots (35)$$

$$B_{vj} = -\frac{2\pi}{\bar{\mu}} b_{vj} \bar{K}_{vj}^* - \frac{1}{m+1} P_{vj} \dots\dots\dots (36)$$

P_{vv} in Eq. (35) is defined in Eq. (31) where $\varphi_j = \varphi_v$.

2.3 Successive approximation method of γ_v

α_0 denotes the zero lift angle of the cross section of blade and α_g the geometric angle of attack, then $\tilde{\alpha}_g = \alpha_g - \alpha_0$. Substituting $\tilde{\alpha}_g$ into Eq. (24) and integrating Eq. (24) with respect to ξ' , the circulation of the lifting line becomes

$$\Gamma/W^* = 2\pi c(\alpha_g - \alpha_0 - \alpha_i) \dots\dots\dots (37)$$

The lift coefficient C_l is expressed by $C_l(\alpha)$, because it is a function of the angle of attack, then the circulation becomes, using the Kutta-Joukowski theorem,

$$\Gamma/W^* = cC_l(\alpha_g - \alpha_i) \dots\dots\dots (38)$$

so that γ_v is written as

$$\gamma_v = \frac{c_v}{h} \sqrt{1+\mu_v^2} C_l(\alpha_g - \alpha_i) \dots\dots\dots (39)$$

where c_v denotes the half chord length.

Using the successive approximation method of Multhopp⁽⁷⁾, equation (39) can be solved. The characteristics of wing sections at $\eta_v = \cos\{\nu\pi/(m+1)\}$ are given at the outset. Using the curve of lift vs. angle of attack, the effective angle ($\alpha_{gv} - \alpha_{iv}$) and the lift coefficient C_l are obtained by substituting n -th approximation of $\gamma_v(n)$ into Eq. (34). Substituting C_l into Eq. (39) again, $(n+1)$ -th approximation of $\gamma_v(n+1)$ is obtained. The pitch of helical vortex is calculated at the stage of each approximation, which is used for the calculation of γ_v . γ_v is calculated until the hydrodynamic pitch converges.

The first approximation of γ_v is obtained by the infinite blade vortex theory⁽⁸⁾.

3. Numerical results

3.1 Geometric form of blades

In this paper, numerical calculations are performed for two kinds of rotor blades. Each blade has a chord length which is constant along the radius and a thickness in percent of the chord which decreases linearly from the boss where it is 21 % to the tip where it is 9 %. Pitching angle β is given as $\beta = \tan^{-1}\{(r_0/r) \times \tan \delta^\circ\}$. The cross sectional shape of the first type has a profile of NACA 65-series symmetrical airfoil, NACA 65-0009 at tip (which is called NACA 65-series in this

paper) and that of the second type has a profile of modified Clark-Y whose thickness distribution along the chord is proportional to the maximum thickness distribution along the radius (which is called Clark-Y in this paper).

Reynolds numbers at which the characteristic data of wing sections are used for the calculation are about 6.0×10^6 for the NACA 65-series and about 3.0×10^6 for the Clark-Y(9)(10). The performances were computed for the rotor which has two to six blades.

3.2 Circulation distribution

Figures 3, 4 represent the circulation distributions of two kinds of rotors at tip speed ratio $X=5.0$ and $X=6.0$ respectively. The circulation of Clark-Y type is larger than that of NACA 65-series type at the same tip speed ratio. The circulation distribution of NACA 65-series is rather even. The circulation of each type of rotor increases as the number of blades decreases.

3.3 Performance of rotors

Using lift coefficient C_l and drag coefficient C_d , torque, resistance and power coefficients are calculated as follows:

$$C_Q = \frac{2cl}{\pi r_0^3 V^2} \int_{r_0}^{r_0} (V - w_a)^2 \sqrt{1 + \mu^2} r (C_l - \mu C_d) dr \quad \dots\dots\dots (40)$$

$$C_R = \frac{2cl}{\pi r_0^3 V^2} \int_{r_0}^{r_0} (V - w_a)^2 \sqrt{1 + \mu^2} (\mu C_l + C_d) dr \quad \dots\dots\dots (41)$$

$$C_P = X C_Q \quad \dots\dots\dots (42)$$

Fig.3 Circulation distribution

Fig.4 Circulation distribution

Figures 5, 6 represent the torque and resistance coefficients of NACA 65-series and Clark-Y, respectively.

The torque coefficient of NACA 65-series increases as the number of blades increases, while the torque coefficient of the three bladed rotor is larger than others in the case of Clark-Y type. The resistance coefficient of each type of rotor increases as the number of blades increases. The resistance of Clark-Y is larger than that of NACA 65-series type. The power coefficient is similar to the torque coefficient.

4. Experimental apparatus and procedure

Figure 7 shows a schematic view of experimental apparatus, which has such a feature that torque and resistance can be measured simultaneously. The data on torque, resistance and speed of revolution were analysed statistically by a microcomputer and results were recorded by a printer. A variation of voltage supply to an electromagnetic brake changes the load on the rotor, and then the number of revolutions varies. The geometric form of the rotor employed is the same as that of the calculation ($2\sigma=50$ mm, $r_b=50$ mm, $r_0=300$ mm). The experiment was made with a rotor which had two to six blades. The test section of wind tunnel is 1.2×1.2 m in width. Three kinds of uniform stream speeds were employed for the performance test. The turbulent intensity of wind tunnel is about 3 %.

(a) Torque coefficient

(b) Resistance coefficient

Fig.5

5. Comparison of numerical calculations with experiments

In order to examine the applicability of the vortex theory to wind turbines, the computational results and the experimental results are compared, which are shown in Figures 8, 9. The numerical calculations in this section employ the characteristic data of wing sections at the Reynolds numbers which are close to the experimental ones(11)(12). The computed torque coefficient agrees well with the experimental one. The computed resistance coefficient is similar to that of experiment for each rotor, though the theoretical values are

(a) Torque coefficient

(b) Resistance coefficient

Fig.6

Fig.7 Experimental apparatus

(a) Torque coefficient

(b) Resistance coefficient

Fig.8

(a) Torque coefficient

(b) Resistance coefficient

Fig.9

rather smaller than the experimental ones. The computational results of each rotor show a flow separation near the tip speed ratio of 4.0, while the experimental results of NACA 65-series type seem to show a flow separation at a larger tip speed ratio than 4.0, shown in Fig. 8 (a). The performances are computed until the flow separates partially.

Non-dimensional performances of the experiments which are made for three kinds of uniform stream velocities coincide each other.

6. Conclusions

The present investigation is the first step to establishment of an unsteady lifting surface theory of wind turbines. The present vortex theory, which uses lifting line approximation, has been confirmed to be applicable to the wind turbines.

Furthermore, it is verified numerically and experimentally that the performances of wind turbines depend on the sectional shape of rotor blade and the number of blades.

The computer used for the calcula-

tions is Melcom 800-III of Kagoshima University.

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