Dynamic Stability of a Passive Magnetic Suspension with an Eight-pole Stator*

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A passive magnetic radial bearing with an eight-pole stator is statically stable. But its inherent damping characteristics are not sufficient to prevent dynamic instability. The dynamic stability of the magnetic bearing is commonly attained by introducing the friction of a mechanical viscous fluid. Though the static performance in this type of magnetic bearing has been theoretically analyzed by some investigators, its dynamic stability has not been fully determined.

This study presents an analysis of the dynamic characteristics of a passive magnetic radial bearing with an eight-pole stator. In the analysis the perturbation method has been used to obtain a transfer function for the magnetic bearing system. Thus the dynamically as well as statically stable region has been calculated. The analytical results are confirmed by experimental observation.

Key Words: Machine Element, Control Devices, Magnetic Bearing, Radial Bearing, Alternate Current Resonant Circuit, Dynamic Stability

1. Introduction

A passive magnetic suspension system using LCR tuned-circuit has a particular advantage in basic simplicity since a servo control system is not always necessary. A passive magnetic suspension system with an eight-pole stator which suspends the load of the rotor in the radial direction⁽⁵⁾ has been investigated by Frazier et al.⁽⁵⁾ and Takizawa et al.⁽⁶⁾, through the development of a high precision inertial sensor where the rotor is floated in oil. Frazier et al. calculated the static characteristics such as the restoring force, stiffness, and stability criterion of the magnetic suspension system, assuming a simple magnetic circuit. Takizawa et al. calculated the magnetic circuit more exactly, demonstrating the presence of a nonrestorative force which makes the rotor stick to the stator. They also discussed the effects of the system parameter on the static characteristics of the suspension system such as the supply frequency, the mean radial clearance and the number of coil windings. The above-mentioned investigations, however, were made only on the static characteristics, not on the dynamic ones, of the suspension system.

In general, although an ac-tuned circuit passive magnetic suspension system is statically stable, its inherent damping characteristics are not sufficient to prevent dynamic instability. Then the dynamic stability of the magnetic suspension is commonly attained by introducing the friction of a mechanical viscous fluid.^{(1)~(6)} For a passive magnetic suspension which has an

LCR tuned-circuit and suspends an object vertically, some dynamic characteristics^{(1)~(4)} have been studied by several investigators. However, dynamic characteristics of the passive magnetic suspension with an eight-pole stator have not been sufficiently elucidated.

This study presents an analysis of the dynamic characteristics of the ac tuned-circuit passive magnetic suspension with an eight-pole stator. In the analysis, the perturbation method has been used to obtain a transfer function for the magnetic suspension system. As a result, the dynamically as well as statically stable region has been calculated. The analytical results are also confirmed by experimental observation.

Nomenclature

A:	cross-sectional area of each pole	m ²
C:	tuning capacitance	F
D:	damping coefficient of floating oil	Ns/m
Es:	instantaneous value of source voltage	V
Fi:	attractive force at each pole	N
Fx, Fy:	restoring forces in x, y directions	N
Fb:	buoyant force	N
g ₀ :	mean radial clearance	m
h:	length of pole portion of non-winding	m
Ii:	amplitude of tuning coil current	A
Ji:	instantaneous value of coil current	A
Li:	instantaneous inductance of coil at air gap g _i	H
M:	mass of rotor	Kg
m:	radius of circumscribed circle to pole piece cross-section	m
N:	number of coil windings	
R:	resistance of coil	Ω
S=s/ω		
s:	Laplace operator	1/s
V _{ti} :	amplitude of terminal voltage of each coil	V
Vs:	amplitude of source voltage	V
W:	load of rotor	N
X, Y:	vertical and horizontal displacements	

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of rotor
 μ_0 : permeability of air
 ω : angular frequency of source
 Subscripts
 0: equilibrium state
 n: nondimensional value
 rms: root mean square value
 i: each coil number shown in Fig. 1

2. Theoretical Analysis

The ac tuned-circuit passive magnetic suspension with an eight-pole stator is schematically shown in Fig. 1. The suspension consists of four pole pairs from 1 to 4; the radial load of the rotor is suspended by four thrust bearings with a tuned-circuit and a two-pole stator. Its equivalent circuit is shown in Fig. 2. The load of the rotor including the rotor weight itself is supported by the restoring force of the ac tuned-magnets and by the buoyant force of the floating oil. Since the passive magnetic suspension with an ac tuned-circuit does not always require a servo control system, the system can be extremely simple in comparison with a dc active magnetic suspension system. As shown in Fig. 1 and Fig. 2, this suspension system utilizes the resonance in series LCR network. The resonance occurs when the angular frequency of the source approaches the value of $1/\sqrt{LC}$. The principle of suspension is well known⁽⁵⁾, so its explanation can be omitted here.

The analysis of static and dynamic characteristics of the system is based on few assumptions as follows:

1. Magnetic circuits in each pair-pole are mutually independent.
2. Rotor moves in translational mode.
3. Displacements of rotor are in vertical direction (x-direction).
4. Tuning capacitance C and resistance R of each tuned-circuit are constant and not changed by altering the frequency and amplitude of tuning current. Furthermore, each inductance L_i is represented by a same function of gap g_i .

2.1 Basic equations

Assuming that the rotor is not displaced in the horizontal direction ($Y=0$), the mean gaps at each pair-pole, referring to Fig. 1, are represented by

$$\left. \begin{aligned} g_1 &= (g_{11} + g_{12})/2 = g_0 + X \cos(\pi/8) \\ g_2 &= (g_{21} + g_{22})/2 = g_0 \\ g_3 &= (g_{31} + g_{32})/2 = g_0 - X \cos(\pi/8) \\ g_4 &= (g_{41} + g_{42})/2 = g_0 \end{aligned} \right\} \text{----- (1)}$$

Although each inductance L_i at each pair-pole is considered a function of mean gap g_i from the above-mentioned assumption, it can also be considered a function of the rotor's displacement X by Eqs. (1). Then, the inductance can be expressed in general in the form of

$$L = G(X) \text{----- (2)}$$

In practice, it would not be easy to obtain the function except in the case of a relatively simple magnetic circuit. The inductance of the electro magnet with an eight-pole stator shown in Fig. 1 has been calculated in reference 6 as follows

Fig. 1 Passive magnetic suspension with an eight-pole stator

Fig. 2 Equivalent tuned-circuit

$$L_i = \frac{\mu_0 N^2}{2\pi^2} \left\{ \frac{A\pi^2}{g_i} + \frac{16(\pi m^2 - A)}{g_i + h} + \frac{32}{3}\pi(g_i + h) + 8\pi^2 m \right\}, \quad (i=1,2,3,4) \text{--- (3)}$$

The small difference in inductance between the experimental and analytical values considerably affects the static and dynamic characteristics. If the inductance cannot be calculated exactly, it may be better to use the value of the empirical equation as follows rather than that of the analytical equation.

$$L = G(X) = \frac{\beta}{X + \alpha} + \gamma \text{----- (4)}$$

where α, β, γ are constant values determined by the experimental results. Then the inductances at each pair-pole are

$$\left. \begin{aligned} L_1 &= G(X) \\ L_2 &= L_4 = G(0) \\ L_3 &= G(-X) \end{aligned} \right\} \text{----- (5)}$$

We introduce a new function subsequently by the following equation

$$\left. \begin{aligned} G'(X) &= -\frac{dG(X)}{dX} = \frac{\beta}{(X + \alpha)^2} \\ G''(X) &= \frac{1}{2} \frac{d^2 G(X)}{dX^2} = \frac{\beta}{(X + \alpha)^3} \end{aligned} \right\} \text{----- (6)}$$

The basic equation described in the tuned-circuit shown in Fig. 2 is

$$\frac{d(L_i I_i)}{dt} + R I_i + \frac{1}{C} \int I_i dt = E_s, \quad (i=1,2,3,4) \text{--- (7)}$$

The magnetic attractive forces at each pair-pole can be expressed by

$$F_i = -\frac{1}{2} I_i^2 \frac{dL_i}{dg_i} \text{----- (8)}$$

By summing the X-directional components of the forces expressed by Eq. (8), the restoring force in X direction is obtained⁽⁶⁾

$$F_x = (F_1 - F_3) \cos \frac{\pi}{8} - \frac{2F_0}{g_0} X \sin^2 \frac{\pi}{8} \quad (9)$$

where

$$F_0 = F_2 = F_4 \quad (10)$$

The equations of motion are given by assuming the motion of the translational mode.

$$M \frac{d^2 X}{dt^2} + D \frac{dX}{dt} = W - 2F_x - F_0 \quad (11)$$

2.2 Static characteristics

Let us first suppose that the rotor is in an equilibrium state. Then the inductances as well as the gap distance have constant values.

$$\left. \begin{aligned} X &= X_0 \\ L_1 &= G(X_0) \\ L_2 &= L_4 = G(0) \\ L_3 &= G(-X_0) \end{aligned} \right\} \quad (12)$$

Substituting Eqs. (12) into Eq. (7), we have a linear equation describing the tuned-circuit. When we define the source voltage as

$$E_s = V_s \sin \omega t \quad (13)$$

the steady-state solutions of the linear equation are easily obtained as

$$J_{i0} = I_{i0} \sin(\omega t - \phi_{i0}) \quad (14)$$

where

$$\left. \begin{aligned} I_{i0} &= \frac{V_s}{\sqrt{R^2 + (L_{i0}\omega - 1/C\omega)^2}} \\ \phi_{i0} &= \tan^{-1} \frac{L_{i0}\omega - 1/C\omega}{R} \end{aligned} \right\} \quad (15)$$

Substitute Eqs. (12), (14) into Eqs. (8) and neglect the double-frequency component and the following equation are derived

$$\left. \begin{aligned} F_{i0} &= \frac{1}{4} I_{i0}^2 \frac{G'(X_0)}{\cos(\pi/8)} \\ F_{20} &= \frac{1}{4} I_{20}^2 \frac{G'(0)}{\cos(\pi/8)} = F_{40} = F_0 \\ F_{30} &= \frac{1}{4} I_{30}^2 \frac{G'(-X_0)}{\cos(\pi/8)} \end{aligned} \right\} \quad (16)$$

The restoring force is derived from Eqs. (9), (16) as

$$F_x = (F_{i0} - F_{30}) \cos \frac{\pi}{8} - \frac{2F_0}{g_0} X_0 \sin^2 \frac{\pi}{8} \quad (17)$$

In relation to the equation of motion (11), the balance of all static forces on the rotor is

$$2F_{20} = W_0 - F_0 \quad (18)$$

2.3 Dynamic characteristics

Now we suppose that a disturbing force w acts on the rotor and the displacement of it is varied with time. In this case, the inductance is a function of time and Eq. (7) is a nonlinear equation. If the disturbance is very small and variation of the rotor position is very slow compared with the variation of the tuned-coil current, the solution of Eq. (7) will be obtained in the form similar to Eq. (14).

$$J_i = I_i \sin(\omega t - \phi_i) \quad (19)$$

These parameters can be divided into a time-dependent part and a static (or steady) equilibrium part, assuming that the deviation from the equilibrium is very small: they are given by

$$\left. \begin{aligned} W &= W_0 + w, \quad X = X_0 + x \\ L_1 &= L_{10} + l_1, \quad L_3 = L_{30} + l_3 \\ I_1 &= I_{10} + i_1, \quad I_3 = I_{30} + i_3 \\ \phi_1 &= \phi_{10} + \phi_1, \quad \phi_3 = \phi_{30} + \phi_3 \\ F_1 &= F_{10} + f_1, \quad F_3 = F_{30} + f_3 \end{aligned} \right\} \quad (20)$$

$$F_x = F_{x0} + f_x$$

Here, we define the dimensionless parameters and variables as follows

$$\left. \begin{aligned} w' &= \frac{w}{2W_0}, \quad x_n = \frac{x}{g_0} \\ l_{ni} &= \frac{wl_i}{R}, \quad i_{ni} = \frac{i_i}{I_{i0}} \\ f_{ni} &= \frac{f_i \cos(\pi/8)}{2W_0}, \quad f_{xn} = \frac{f_x}{2W_0} \\ F_{ni} &= \frac{F_{i0} \cos(\pi/8)}{W_0}, \quad F_{n0} = \frac{F_0 \sin^2(\pi/8)}{W_0} \\ x_{ni} &= \frac{g_0 G'(X_0)}{G'(X_0)}, \quad x_{n3} = \frac{g_0 G'(-X_0)}{G'(-X_0)} \\ \lambda_{ni} &= \frac{g_0 \omega G'(X_0)}{R}, \quad \lambda_{n3} = \frac{g_0 \omega G'(-X_0)}{R} \\ M_n &= \frac{\omega^2 g_0}{4W_0} M, \quad D_n = \frac{\omega^2 g_0}{4W_0} D \\ Q_{Li} &= \frac{L_{i0}\omega}{R}, \quad Q_c = \frac{1}{RC\omega} \end{aligned} \right\} \quad (21)$$

Substitute Eqs. (20) into Eq. (19), (7) and neglect the second and higher order terms which tend to zero and subtract the static part presented in Eq. (12) to (18). Also, making the Laplace transforms and normalizing the perturbed dynamical equations using Eq. (21), we obtain a transfer function relating the changes in amplitude of tuned-coil current to the changes in inductance.⁽⁴⁾

$$i_{ni}(S) = \frac{H_{ni}(S)}{H_{oi}(S)} l_{ni}(S) \quad (i=1, 3) \quad (22)$$

where

$$\left. \begin{aligned} H_{oi}(S) &= Q_{Li}^2 S^4 + 2Q_{Li} S^3 \\ &\quad + (1 + 2Q_{Li} Q_c + 2Q_{Li}^2) S^2 \\ &\quad + 2(Q_{Li} + Q_c) S + 1 + (Q_{Li} - Q_c)^2 \\ H_{ii}(S) &= Q_{Li} S^4 + S^3 \\ &\quad + (2Q_{Li} + Q_c) S^2 + S + Q_{Li} - Q_c \end{aligned} \right\} \quad (23)$$

In the same manner as used in Eqs. (22), (23), the transfer function relating the changes in inductance to the changes in rotor position is obtained by substituting Eqs. (20) into (1) and by using Eqs. (5), (21)

$$l_{ni}(S) = \lambda_{ni} x_n(S) \quad (i=1, 3) \quad (24)$$

We substitute Eq. (19) into (8) and neglect the double-frequency component. Then the attractive magnetic forces at each pair-pole are given as derived in Eqs. (16)

$$F_i = \frac{1}{4} I_i^2 \frac{G'(X)}{\cos(\pi/8)}$$

From the above equation and Eqs. (9), (20), the following two normalized transfer functions are derived by introducing a dimensionless parameter into Eq. (21)

$$f_{ni}(S) = F_{ni} \{ i_{ni}(S) - x_{ni} x_n(S) \} \quad (25)$$

$$f_{xn}(S) = f_{n1}(S) + f_{n3}(S) - F_{n0} x_n(S) \quad (26)$$

The equation of motion (11) leads to the transfer function similarly.

$$\begin{aligned} (M_n S^2 + D_n S) x_n(S) \\ = -f_{xn}(S) + w_n(S) \end{aligned} \quad (27)$$

From these equation mentioned above such as Eqs. (22), (24), (25), (26), (27), a block diagram of the passive magnetic suspension system with an eight-pole stator can be gained as in Fig. 3. It is evident in Fig. 3 that the positive loop of horizontal attractive force at pair-poles 2, 3

(F_{n0}) is one of the causes of dynamic instability. But this positive feedback loop is not the main cause of the instability, because dynamic stability cannot be attained without an oil damper even if the coil currents at the horizontal pair-pole are made zero ($F_{n0}=0$).

Fig. 3 Block diagram for passive magnetic suspension system

From the above equations or the block diagram, a characteristic equation for this magnetic suspension system can be derived as

$$(M_n S^2 + D_n S - F_{n0} - \kappa_{n1} F_{n1} - \kappa_{n3} F_{n3}) \times H_{01}(S) H_{03}(S) + \lambda_{n1} F_{n1} H_{03}(S) \times H_{11}(S) + \lambda_{n3} F_{n3} H_{01}(S) H_{13}(S) = 0 \quad (28)$$

Rearranging the above equation (28) into a power series of S , we have a ten-order algebraic equation.

$$\sum_{n=0}^{10} A_n S^n = 0 \quad (29)$$

The coefficients and roots of Eq. (29) can be numerically calculated by a digital computer. Then the dynamic stability can be obtained directly from the numerical roots of Eq. (29) rather than from the Routh-Hurwitz criterion. The conjugate root with maximum real part of the complex roots is expressed by

$$S = \sigma_n + j\Omega_n \quad (30)$$

where,

$$\left. \begin{aligned} \text{logarithmic decrement} &= -2\pi\sigma_n / \Omega_n \\ \text{natural frequency} &= \omega_n \end{aligned} \right\} \quad (31)$$

Therefore, when $\sigma_n = 0$, the system becomes unstable.

3. Experimental Apparatus

The experimental apparatus for a passive magnetic suspension with an eight-pole stator is schematically shown in Fig. 4. In the figure, the radial load of the rotor (1) is supported by two passive magnetic suspensions (2), while the thrust load is supported by the repulsion of a permanent magnet (5). The housing (4) is filled with a floating oil (8) in order to assure damping. Since the axial stiffness is made very weak, the rotor is fairly free to move axially. If the axial rotor position is sensed by the position sensor (6), this apparatus can be utilized as a high precision tiltmeter which can measure

Fig. 4 Schematic of experimental device
① Rotor ② Stator ③ Stator Holder ④ Housing ⑤ Permanent Magnet ⑥ Position Sensor ⑦ Oil Pipe ⑧ Floating Oil

a very small inclination of the base supporting the housing (4). The radial position can be approximately known by measuring the changes in amplitude of the voltage across the tuning coil, which is approximately proportional to the rotor's displacement.

(a) Rotor

(b) Stator

Fig. 5 Rotor and stator of passive magnetic suspension

The rotor and stator are schematically shown in Fig. 5 (a), and (b). The stator consisted of laminated silicon steel plates with 0.5 mm thickness, and the rotor consisted of laminated 122 plates with 0.4 mm thickness. The formal wire of 0.29 mm diameter was used for tuned-coil of 365 turn/pair-pole. The other principal dimensions are shown in Table 1.

4. Experimental Results

As shown in Fig. 6, the method of measuring the damping coefficient of floating oil is such that a cantilever with a rotor installed at one end is made to vibrate freely, and then the observation of the damped vibration mode through the oscilloscope signaled from a strainmeter will give the damping coefficient. Note that an additional mass is needed due to the small magnitude of rotor inertia with

respect to the large damping force. The equivalent total mass M_e of the rotor including the additional mass and the cantilever mass is given by $M_e = K/\omega_{air}^2$, where K is the stiffness of the cantilever and ω_{air} is the frequency of natural vibration in the air. As the logarithmic decrement δ and the frequency ω_{oil} of natural vibration in oil can be easily obtained, the damping coefficient is approximately given by

$$D = M_e \omega_{oil} \zeta / \pi$$

The experimental values are listed in Table 1. It should be noted that the value of D in Table 1 is obtained when the rotor is placed near the center of the stator.

The experimental results of the inductance L of tuned-coil for the displacement X are shown in Fig. 7. See the reference (4) about the measuring circuit for the inductance. The measuring configuration is similar to that shown in Fig. 6, but the oil is drained off. The displacement of the rotor which is fixed on a gap adjuster was read through a comparator. The single-dotted chain lines shown in Fig. 7 are calculated by Eq. (3). The theoretical value agrees well with the experimental one at small X , but the larger the X , the larger is the difference between the theoretical and experimental results, i.e., a leakage magnetic flux becomes large with an increased displacement X . In this system, a rather large gap is favorable because of the large equilibrium position. Then, it would be adequate to use an empirical equation which agrees comparatively well with the experimental results at a large gap. The empirical equation leads to Eq. (4) from Fig. 7. The constants are listed in Table 1. Henceforth the empirical equation (4) is used to calculate the inductance.

The experimental and analytical results of the terminal voltage of coil 1 for the rotor's displacement X are shown in Fig. 8. This figure shows the resonant

Table 1. Fundamental Values

W_0	0.492	N	N	365
M	50.2	g	h	0.5 mm
F_b	81.3	mN	m	3.2 mm
R	5.0	Ω	A	20 mm ²
C	4.668	μF	α	0.495 mm
g_0	0.400	mm	β	1.940 mH·mm
D	11.0	Kg/s	γ	4.657 mH

Fig. 6 Schematic of test apparatus measuring damping coefficient of floating oil

curves of LCR series tuned-circuit and Q -value which is about 8. Since this magnetic suspension system utilizes the resonant curve, it is desirable to provide a large Q -value for the stiffness and the load capacity of the magnetic suspension. From Fig. 8, the resonant peak is found the largest at the source frequency $f=750$ Hz; but this is not always favorable because the restoring force is given as the sum of the vector of the attractive force at each pole

The restoring force of the magnetic suspension can be obtained from the measurement of strain in the cantilever shown in Fig. 6. In this case, the floating oil and additional mass were removed and a thin thickness plate instead of the cantilever was used. The analytical and experimental results of restoring force of the suspension are shown in Fig. 9. In this figure, if dF/dX is positive, the suspension system is statically stable. But if the restoring curve comes in the second or fourth quadrant, the rotor cannot spring back to the original position under a small

Fig. 7 Inductance of coil vs. displacement of rotor

Fig. 8 Terminal voltage of coil vs. displacement X

Fig. 9 Restoring force F_x versus displacement X

load, which is termed the non restorative region. From Fig. 9, it is clear that there are some differences between the calculated and experimented values, which is probably due to measuring the error arising from the low source voltage. The measurement of the restoring force at the high source voltage was too difficult because the self-excited vibration force was strong.

An example of calculation of the static stable region and equilibrium position is shown in Fig. 10. The left region from the broken line is the statically unstable region. The double-dotted chain line ($X \approx g_0$) indicates the contactless limit where the rotor contacts stably with the stator. The single-dotted chain lines indicate the dynamically stable limit calculated by the characteristics equation (29). As is obvious from the figure, when the source voltage is made higher or the source frequency lower, the rotor approaches the center of the stator and becomes more statically stable. However, at the same time, the rotor becomes dynamically unstable as it approaches the limitation of dynamic instability as shown in Fig. 10 by a single-dotted chain line. Consequently, a stable equilibrium is reached in the region surrounded by these three curves. As a matter of course, the larger the damping coefficient D , the wider the stable region.

When the equilibrium position is changed by changing the source frequency, the variations of the voltage across the coils 1, 2, 3 become as shown in Fig. 11. The calculated values shown by a full line in this figure agree well with the experimental value. Then it is clear that the analysis on the static characteristics is valid. From Fig. 11, it is found that the terminal voltage becomes lower, bringing the circuit 3 farther from resonance as the source frequency becomes higher.

The statically and dynamically stable region given by analysis and experiment are shown in Fig. 12 when source frequency and source voltage are changed. In this figure, the solid line shows the calculated

Fig. 10 Equilibrium position versus stable region

Fig. 11 Terminal voltage of coil versus supply frequency at equilibrium position

value of the dynamically unstable threshold where a self-excited oscillation occurs, and the broken line does the statically unstable threshold where the rotor sticks to the stator. Consequently, stable equilibrium is reached in the region surrounded by these two curves. Marks $\circ$, $\bullet$ in Fig. 12 indicated the experimental values when $D=11$ Kg/s. The experimental results of the stable region agree well with the analytical results, so it seems that the analysis of the dynamic characteristics is valid. From this figure, it is noted that the higher the source voltage, the more dynamically unstable the magnetic system becomes under constant source frequency; or the higher the source frequency, the more dynamically stable the system becomes under constant source voltage. It is clear from the calculation of the dynamic stability that the dynamically stable region is expanded as the damping coefficient D is larger.

Fig. 12 Analytical and experimental stable regions
O, ●; experimental values
($D=11$ Kg/s)

(a) Self-exciting oscillation
($f=920$ HZ, $V_s=4.5$ Vrms)

(b) Damping oscillation
($f=920$ HZ, $V_s=4.0$ Vrms)

Fig. 13 Amplitude of terminal voltage of coil 1 in self-exciting and damping oscillation

If the rotor is displaced by a small quantity, the change in amplitude of the terminal voltage across the coil 1 is approximately proportional to the change in the displacement. Therefore, the change in the displacement can be estimated by measuring the terminal voltage of each coil. An example of a self-excited and damped vibration is shown in Fig. 13.

Figure 14 shows the experimental and analytical results of dimensionless damped natural frequency under various source voltages and constant source frequency. The full line in this figure indicates the

Fig. 14 Damped natural frequency versus supply voltage

analytical curve of Ω_n calculated by Eq. (29), where the dimensionless complex root having a maximum real part is represented as $S = \sigma_n + j\Omega_n$. It is found from this figure that two different modes occur in the experimental data; one is a conical mode caused by striking the end of the rotor, and the other is a translational mode which gradually changes from the conical mode.

5. Conclusions

The static and dynamic characteristics of a passive magnetic suspension with an LCR tuned-circuit suspending a radial load in a floating oil are analyzed, and the analytical results are compared with the experimental ones. From this investigation, the primary results may be summarized as follows:

1. The validity of the analysis on the static and dynamic characteristics is confirmed by experimental results.
2. The statically and dynamically stable region is surrounded by three curves such as curves of statically and dynamically stable limit and physical contactless limit ($X_0 \approx g_0$).
3. The higher the source voltage, the more dynamically unstable the magnetic system becomes under constant source frequency; or the higher the source frequency, the more dynamically stable the system becomes under constant source voltage.

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