Pressure Distributions around an Airfoil Placed in
a Periodically Fluctuating Air Flow*

(1st Report, A Flat Plate Approximation)

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This paper describes theoretically and experimentally obtained pressure distributions around an airfoil placed in a periodically fluctuating air flow. The following facts are found out: (1) When the amplitude of fluctuating velocity is small, the time averaged pressure distributions are nearly equal to those in steady flows. (2) The amplitude of fluctuating pressures increases approximately in proportion to that of air velocity. Furthermore, the values are fairly well predicted by the flat plate airfoil theory. (3) The angle of lead of the pressure fluctuation to the velocity becomes smaller in the decelerating regions and larger in the accelerating regions. The above is not well predictable by the theory. (4) It is clear that it is necessary to consider the distribution of the airfoil thicknesses in order to calculate the time averaged values and phase angles with high accuracy.

Key Words: Unsteady Flow, Pressure Distribution, Flat Plate, Theoretical Analysis

1. Introduction

Many theoretical flutter analyses of an airfoil and cascade in steady flow have been carried out according to the oscillating wing theory. Recently gust flows have been studied about interferences between stator and rotor cascades. For automotive exhaust turbines, it is, however, necessary to consider fluctuations of both incidence angle and velocity into the cascades, because they are driven by pulsating exhaust gases from engine cylinders. Usually airfoil characteristics have been studied with regard to the effect of fluctuating incidence angle of the inlet flow with constant velocity⁽¹⁾, because it is considered that the fluctuation of incidence angle has greater effect on

cascade performances than the velocity fluctuation. Prof. Horlock⁽²⁾, however, pointed out that it might be necessary to take account of the latter effect. For instance, the fluctuating lift in this case was worked out in references (3) and (4) and the fluctuating pressure distributions on airfoils in references (5) - (7). There are, however, few experimental studies and especially fewer reports on the effects of uniformly fluctuating velocity field.

This paper describes theoretical and experimental pressure distributions around an airfoil placed in a periodically fluctuating air flow. The result of the flat plate approximation does not adequately explain the experimental results and it is clear that it is necessary to consider the distribution of airfoil thicknesses in order to calculate the time averaged values and phase angles with high accuracy.

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Nomenclature

- b : half chord length = $c/2$
- c : chord length
- c_p : pressure coefficient
- k : reduced frequency = $\omega b/U_\infty$
- p : pressure
- t : time
- u, v : velocities in directions of x and y , respectively
- x, y : orthogonal coordinates
- U : velocity of main flow
= $U_\infty(1 + \delta \cos \omega t)$
- α : attack angle
- δ : velocity amplitude ratio of fluctuating air flow

- θ : phase lag of pressure fluctuation to velocity
 ρ : density
 ω : angular frequency
 Π : pressure difference (defined by Eq.(12))
 Kc_0 : pressure summation (defined by Eq.(13))
 Subscripts
 + : upper surface of airfoil ($y=+0$)
 - : lower surface of airfoil ($y=-0$)

2. Theoretical analysis assuming a flat plate airfoil

Two dimensional irrotational flow of inviscid and incompressible fluid is assumed in order to calculate the flow field around an airfoil placed in a spatially uniform flow at sinusoidally varying velocity. As shown in Fig.1, the mid-chord is taken as the origin of coordinates and the chord of the blade as x -axis. In the figure, the point C_0 is placed on the wind tunnel wall at distance l from the origin ($x=l \sin \alpha$, $y=-l \cos \alpha$, where l is considered sufficiently large), the point C_1 denotes the infinite upstream point and C_2 the infinite upstream point corresponding to the point C_0 . It is assumed that the vortices in the wake are carried along the x -axis at main flow velocity.

Basic equations are as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (3)$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \quad (4)$$

Approximating the airfoil as a flat plate, the induced velocities on both upper and lower surfaces of the blade are represented by the following relations using the boundary conditions on the blade surfaces;

$$u'_z = \pm \frac{1}{2} \gamma_a(x, t), (-1 \leq x \leq 1) \quad (5)$$

$$v'_z = -U \sin \alpha, (-1 \leq x \leq 1) \quad (6)$$

where,

$$\left. \begin{aligned} u'_z &= u'(x, y = \pm 0, t) \\ v'_z &= v'(x, y = \pm 0, t) \\ U &= U_\infty(1 + \delta \cos \omega t) \end{aligned} \right\} \quad (7)$$

and the vorticity γ_a on the blade surface is given below as the sum of the quasi-steady vorticity γ_0 and the vorticity γ_{ow} induced by the wake vortices,

$$\gamma_a(x, t) = \gamma_0(x, t) + \gamma_{ow}(x, t), \quad (8)$$

Using the flat plate airfoil assumption,

Fig.1 Coordinate system

the induced velocity in the upstream of the blade is expressed as

$$u'(x, y=0, t) = 0, (x \leq -1), \quad (9)$$

Pressure distributions on both upper and lower surfaces are derived from Eqs.(1) - (9) taking into account the terms up to the second order of the induced velocity.

$$\begin{aligned} p_z - p_{c_0} &= -\rho \frac{\partial}{\partial t} \int_0^x U \cos \alpha dx \\ &- \rho \frac{\partial}{\partial t} \int_{-1}^x u'_z dx - \rho U \cos \alpha u'_z \\ &- \rho U \sin \alpha v'_z - \frac{1}{2} \rho (u'^2_z + v'^2_z), \quad (10) \end{aligned}$$

In the above, p_{c_0} denotes the pressure at C_0 and p_z is represented by

$$p_z = p(x, y = \pm 0, t), (-1 \leq x \leq 1), \quad (11)$$

Both the difference and sum of pressures on the upper and lower surfaces are given as

$$\begin{aligned} \Pi = p_- - p_+ &= \rho \frac{\partial}{\partial t} \int_{-1}^x \gamma_a(x, t) dx \\ &+ \rho U \cos \alpha \gamma_a(x, t), \quad (12) \end{aligned}$$

$$\begin{aligned} Kc_0 = p_- - p_{c_0} + p_+ - p_{c_0} \\ = -2\rho \frac{\partial}{\partial t} \int_0^x U \cos \alpha dx \\ + \rho U^2 \sin^2 \alpha - \frac{1}{4} \rho \gamma_a(x, t)^2, \quad (13) \end{aligned}$$

In the case of the flat plate airfoil, the quasi-steady vorticity γ_0 on the airfoil is expressed by

$$\begin{aligned} \gamma_0(x, t) &= 2U \sin \alpha \sqrt{\frac{1-x}{1+x}}, \\ &(-1 \leq x \leq 1), \quad (14) \end{aligned}$$

The vorticity induced by the vortices in the wake is formulated by Kármán-Sears⁽¹⁾ as

$$\gamma_{ow}(x, t) = \frac{1}{\pi} \sqrt{\frac{1-x}{1+x}} \int_{-1}^{\infty} \frac{\gamma_w(\xi, t)}{\xi - x} \sqrt{\frac{\xi+1}{\xi-1}} d\xi \quad (15)$$

where γ_w denotes the vorticity in the wake and is represented, taking account of the terms up to $\delta^2(8)$, as

$$\begin{aligned} \gamma_w(x, t) &= \frac{1}{2} \delta^2 \Gamma_s \operatorname{Re} \left\{ \frac{e^{-ikx}}{K_0(ik) + K_1(ik)} \right\} - \delta \Gamma_s \operatorname{Re} \left\{ \frac{e^{-ikx} e^{i\omega t}}{K_0(ik) + K_1(ik)} \right\} \\ &- \frac{1}{2} \delta^2 \Gamma_s \operatorname{Re} \left\{ \left[\frac{e^{-ikx}}{K_0(ik) + K_1(ik)} - \frac{e^{-i2kx}}{K_0(i2k) + K_1(i2k)} \right] e^{i2\omega t} \right\}, \quad (16) \end{aligned}$$

In Eq.(16), $\Gamma_s = 2\pi U_\infty \sin \alpha$, $k = \omega b / U_\infty$, K_0 and K_1 are modified Bessel functions of the second kind of the 0-th and 1st orders, respectively, and $\text{Re}\{\}$ indicates the real part of complex number. The relations

$\cos \alpha \approx 1$, $\sin \alpha \approx \alpha$ are hereafter assumed because of a small attack angle α . Substituting Eqs.(14) - (16) into Eqs.(12) and (13), the following relations are obtained by neglecting the terms higher than δ^2 .

$$\Pi = \frac{\rho U_\infty \Gamma_s}{\pi} \sqrt{1-x^2} \delta \text{Re}\{ike^{i\omega t}\} + \frac{\rho U_\infty \Gamma_s}{\pi} \sqrt{\frac{1-x}{1+x}} \left[\delta \text{Re}\{[C(k)-1]e^{i\omega t}\} + \frac{\delta^2}{2} \times \text{Re}\{[2C(k)-C(2k)-1]e^{i2\omega t}\} \right] + \frac{\rho U_\infty \Gamma_s}{\pi} \sqrt{\frac{1-x}{1+x}} \left[1 + \frac{\delta^2}{2} + 2\delta \text{Re}\{e^{i\omega t}\} + \frac{\delta^2}{2} \text{Re}\{e^{i2\omega t}\} \right] \dots (17)$$

$$K_{C_0} = -\frac{\rho U_\infty \Gamma_s}{\pi} \frac{1-x}{\alpha} \delta \text{Re}\{ike^{i\omega t}\} + \frac{\rho U_\infty \Gamma_s}{\pi} \alpha \frac{x}{1+x} \left[1 + \frac{\delta^2}{2} + 2\delta \text{Re}\{e^{i\omega t}\} + \frac{\delta^2}{2} \text{Re}\{e^{i2\omega t}\} \right] + \frac{\rho U_\infty \Gamma_s}{\pi} \alpha \frac{1-x}{1+x} \left[\delta \text{Re}\{B\} + \delta^2 \text{Re}\{C\} - \frac{\delta^2}{2} \text{Re}\{D\} - \frac{\delta^2}{2} \text{Re}\{B^2\} \right] \dots (18)$$

where,

$$C(k) = K_1(ik)/K_0(ik)$$

$$\text{Re}\{B\} = \text{Re}\{Ie^{i\omega t}/K_0(ik)\}$$

$$\text{Re}\{C\} = \text{Re}\{Ie^{i2\omega t}/K_0(ik)\}$$

$$\text{Re}\{D\} = \text{Re}\{Ie^{i2\omega t}/K_0(2k)\}$$

$$I = \int_1^\infty \frac{e^{-i\alpha\xi}}{\xi-x} \sqrt{\frac{\xi+1}{\xi-1}} d\xi$$

$$K(iz) = K_0(iz) + K_1(iz)$$

and $C(k)$ is called Theodorsen function⁽⁹⁾.

The pressure is considered a difference from the atmospheric pressure p_a , hence the relation is

$$K_a = p_- - p_a + p_+ - p_a = K_{C_0} + 2\rho l' \frac{\partial U}{\partial t} \dots (19)$$

where l' is a distance from C_0 to the tunnel exit.

The pressure on the blade surface is calculated from Eqs.(17) - (19), that is,

$$p_+ - p_a = \frac{1}{2}(K_a \mp \Pi) \dots (20)$$

Then the pressure coefficient c_p is given by the relation,

$$c_{p\pm} = (p_\pm - p_a) / (\frac{1}{2}\rho U_\infty^2) \dots (21)$$

3. Experimental equipment and test procedure

The experiment was conducted using a blowdown wind tunnel with six butterfly

valves at its inlet which produced a sinusoidal velocity fluctuation in air flow. A general view of test apparatus is shown in Fig.2(a) and of the test section in Fig.2(b). Referring to the steady flow characteristics of the test wind tunnel, the maximum air velocity is about 30 m/s, the intensity of turbulence is about 0.3 % and the nonuniformity is about 0.1 % except the boundary layer of the tunnel wall. The air velocity is also considered uniform in the case of a periodically fluctuating flow. The test airfoil was NACA 65A010 and its attack angle was set at the quarter chord point. The pressure distributions on the blade surfaces were measured by semi-conductor pressure transducers put on the inner surfaces of the blade. Fig.3 shows the test airfoil which is made of bronze with a chord length c of 150 mm and a blade width of 297 mm. Air velocity was measured by a hot wire anemometer at the point A and by a pitot tube at the point B, respectively.

Experiments were conducted at three attack angles ($\alpha = 0^\circ, 5^\circ, 7^\circ$) with several velocity levels and fluctuating frequencies. Measured data were recorded by a cassette recorder, converted to digital data by an FFT analyzer and processed by an electronic computer. Table 1 shows round numbers of experimental reduced frequency k and velocity amplitude

Fig.2 Test apparatus

ratio δ of fluctuating velocity. Symbols in the table are used in Figs.5 - 7. Reynolds number based on U_∞ and c varies between 5.7×10^4 and 2.2×10^5 .

4. Comparison of experimental and calculated results

The airfoil is set at $-1 \leq r \leq 1$ in calculation but at $0 \leq x/c \leq 1$ for comparison of experiment and prediction. Considering the end effect l' is taken equal to 258 mm in Eq.(19).

The higher harmonics more than 20 Hz were cut off from velocity and pressure waves by Fourier transform. The procedure was found adequate because the maximum frequency was about 8.4 Hz in the experiment. The experimental data may be approximated as

$$\left. \begin{aligned} U &= U_\infty(1 + \delta \cos \omega t) \\ p &= \bar{p} + |\Delta p| \cos(\omega t - \theta) \end{aligned} \right\} \dots\dots\dots (22)$$

Then, the pressure coefficient c_p is represented as

$$\left. \begin{aligned} c_p &= (p - p_a) / \left(\frac{1}{2} \rho U_\infty^2 \right) \\ &= \bar{c}_p + |\Delta c_p| \cos(\omega t - \theta) \\ \bar{c}_p &= (\bar{p} - p_a) / \left(\frac{1}{2} \rho U_\infty^2 \right) \end{aligned} \right\} \dots\dots\dots (23)$$

Fig.3 Test blade

4.1 Pressure waves

An example of pressure waves on the suction side is shown in Fig.4 at the conditions of $\alpha = 7^\circ$, $k = 0.53$ and $\delta = 0.14$. The abscissa represents a nondimensional time.

The origin of the time is set at the instant when the velocity takes the maximum value. In the figure, symbols represent the experimental pressure coefficients on the blade surface and solid lines indicate the calculated results. The figure shows the fact that the waves are approximated by the time averaged values and the fundamental waves. In this example the calculated results agree well with the experimental.

4.2 Time averaged pressure distribution

Fig.5 shows the time averaged pressure coefficient $\bar{c}_p$. The figure (a) is for $\alpha = 0^\circ$, (b) for $\alpha = 5^\circ$ and (c) for $\alpha = 7^\circ$. The symbols represent the experimental results corresponding to the cases in Table 1. The solid lines in the figure indicate the calculations on a flat plate airfoil at the conditions of $\delta = 0.1$ and $k = 0.1 - 0.6$, and the predicted results have the same values for various reduced frequencies. In the case (a) the attack angle is 0.1° in calculation, then two solid lines appear in the figure. Furthermore the calculation is carried out only in the case of $\delta = 0.1$, because there

Table 1 Test conditions

Symbol	⊖	+	△	×	◇	⧢
k	0.5	0.4	0.3	0.2	0.2	0.2
δ	0.1	0.1	0.2	0.2	0.1	0.1

Fig.4 Pressure waves

is about 2 % difference in $\bar{c}_p$ within the experimental range of $\delta=0.1 - 0.2$. The figure shows that it is necessary to consider the distribution of the blade thicknesses in order to predict with high accuracy the pressure distributions at $\alpha=0^\circ$, and on the pressure side in case of $\alpha \neq 0^\circ$. The experimental pressure distributions in the figure are equal to those in the steady flow. From the above, it is clear that the time averaged pressure distributions are almost the same in the ranges of $\alpha=0^\circ - 7^\circ$, $k=0.17 - 0.53$ and $\delta=0.10 - 0.23$ as those in the steady flow.

4.3 Amplitude distribution of pressure fluctuation

Fig.6 shows the amplitude distributions of fundamental pressure coefficients $|\Delta c_{pi}|$ divided by δ . The figure (a) is for $\alpha=0^\circ$, (b) for the pressure side at $\alpha=7^\circ$ and (c) for the suction side at $\alpha=7^\circ$. In calculation δ is 0.1 and k is 0.1 - 0.6. The calculated results in Fig.(a) are for the

suction side at $\alpha=0.1^\circ$. The amplitude ratio δ has no effect on $|\Delta c_{pi}|/\delta$ according to the calculation. The attack angle α also affects little except in the region near the leading edge on the suction side. As k becomes larger the amplitude increases over the whole region of the blade, because the first terms in Eqs.(17) and (18), that is, in Eqs.(12) and (13) become larger than the other terms in those equations. Fig.6 shows a comparatively good agreement between the calculated and experimental results except in the region near the leading edge on the suction side. Further, the figure indicates that the amplitudes near the leading edge are greater than those in the other regions and that those on the suction side larger than those on the pressure side. It is clear that the amplitudes near the leading edge on the suction side increase but those on the pressure side do not change as α becomes large.

Fig.5 Time averaged pressure distribution

Fig.6 Distribution of pressure amplitudes

Fig.7 Distribution of phase differences

4.4 Phase difference of pressure wave and velocity fluctuation

Fig.7 shows the phase difference ($-180^\circ \leq \theta \leq 180^\circ$) defined by Eq.(22). The test and calculating conditions are same as those for Fig.6. Fig.(a) is for $\alpha = 0^\circ$, (b) for the pressure side at $\alpha = 7^\circ$ and (c) for the suction side at $\alpha = 7^\circ$. All the values of θ are negative which implies that a pressure fluctuation lead to a velocity wave. According to the results of experiment, the lead of phase becomes small in the decelerating region and great in the accelerating region along the chord. The figure shows that the flat plate approximation cannot predict, with high accuracy, the test results on the phase difference at $\alpha = 0^\circ$ and on the pressure side in case of $\alpha \neq 0^\circ$, because the distribution of the blade thicknesses is not taken into account in the method. In Fig.(a) the calculation represents the result on the suction side at $\alpha = 0.1^\circ$ and is little influenced by k . However the phase difference is affected strongly by k as α becomes larger. In calculation δ has little effect on θ , then it is reasonable to compare the calculation for $\delta = 0.1$ with the test results. Finally it is clear that the phase differences on both pressure and suction sides show a tendency to produce uniform distribution of $\theta = -90^\circ$ with an increasing k .

5. Conclusion

The pressure distributions around an airfoil placed in a uniform air flow with a sinusoidally fluctuating velocity were

calculated by the flat plate approximation method and measured for the blade of NACA 65A010. According to the results, it is clear that the distribution of pressure amplitudes is well predicted by the method but that the time averaged pressure and the phase difference between pressure and velocity waves are not estimated with high accuracy by the method because the distribution of the blade thicknesses is not taken into account.

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