A Study on Performance of Centrifugal Pumps

Driven in Two-Phase Flow

(1) Prediction of Hydraulic Torque*

By Hideo FUJIE**

Hydraulic torques of centrifugal pumps driven in two-phase flow are estimated using gas and liquid velocities at the impeller exit which are obtained from momentum conservation equations of both phases in the impeller, in the equations friction loss coefficients between the impeller wall and the phases as well as on the gas-liquid interface are chosen so as to fit a calculated value to a test datum in an appropriate condition. The estimated values show pretty good agreement with test data for pressures of 3.3 and 6.9 MPa and void fractions of 30 - 70% in several operation modes of normal directional flow and rotation. Although the agreement is not good in operation modes of reverse directional flow and rotation, the variation of the estimated torque with void fraction shows a tendency similar to that of the test data.

Key Words: Hydraulic Machine, Multiphase Flow, Centrifugal Pump, Hydraulic Torque, Momentum Equation.

1. Introduction

Behavior of centrifugal pumps driven in two-phase flow has not been made clear sufficiently, and several experimental and theoretical studies⁽¹⁾⁽²⁾⁽³⁾⁽⁴⁾⁽⁵⁾ on relationships among flow rate, void fraction, pump head and hydraulic torque of pumps have been performed in recent years.

One of these experiments⁽¹⁾ shows that the ratio of the hydraulic torque to the density of the two-phase flow decreased to about 70% of that obtained in single-phase flow under the experimental condition of 6.9 MPa pressure and 30 - 70% void fractions with the rated flow rate and the rated rotation speed. However, the theoretical explanation of such results has not been satisfactory.

Mikielwicz et al.⁽⁴⁾ estimated the performance of pumps driven in two-phase flow considering the relative velocity between the gas and the liquid, however, they ignored the void fraction dependency of the relative velocity in impellers, and could not explain the change in the hydraulic torque and the pump head with the change in the void fraction.

Zarechnak et al.⁽⁵⁾ tried to obtain the pump performance using a thermodynamic model, but they have not explained the void fraction dependency of the hydraulic torque.

In this report, it is tried to predict the hydraulic torque using the

velocities of the gas and the liquid at the impeller exit which are obtained from momentum conservation equations of both phases in the impeller.

2. Nomenclature

- A : cross-sectional flow area
 G : mass flow rate
 H : pump head
 i : specific enthalpy of fluid
 k : friction loss coefficient
 $NT = (T_{Tp} / \rho_{Tp}) / (T_R / \rho_1)$: normalized torque
 P : pressure
 Q : volumetric flow rate
 r : distance from center of impeller
 $S = V_g / V_l$: slip ratio
 s : distance along flow path
 T : hydraulic torque
 $u = r\omega$: circumferential velocity
 v : fluid velocity in static coordinate system

$$\bar{v} = \frac{\rho_g Q_g v_g + \rho_l Q_l v_l}{\rho_g Q_g + \rho_l Q_l}$$
 w : fluid velocity in coordinate system rotating with impeller

$$\bar{w} = \frac{\rho_g Q_g w_g + \rho_l Q_l w_l}{\rho_g Q_g + \rho_l Q_l}$$
 $x = \rho_g Q_g / (\rho_g Q_g + \rho_l Q_l)$: steam quality
 α : void fraction
 β : angle between u and w
 δ : angle between u and v
 ν : kinematic viscosity
 ρ : density of fluid
 σ : surface tension
 ω : angular velocity

Subscripts

- A : value for designated condition
 d : delivery side
 g : gas phase

* Received 5th April, 1982.

** Researcher, Energy Research Laboratory, Hitachi Ltd., 1168 Moriyama-cho, Hitachi-shi, Ibaraki-ken.

l : liquid phase
 lg: gas-liquid interface
 R : value for rated condition
 s : suction side
 SP: single-phase flow
 TP: two-phase flow
 TO: condition for null hydraulic torque

Superscripts

C : reverse flow and reverse rotation
 N : normal flow and normal rotation

3. Hydraulic torque in two-phase flow

Hydraulic torque of a centrifugal pump driven in two-phase flow is obtained as the difference in the angular momentum of the fluids between the exit and the inlet of the impeller. In the following sections, the equations expressing the hydraulic torque are given for two cases, that is, in the first case the fluids flow in normal direction (i.e. from suction to delivery) and the impeller rotates in the normal direction, and in the second case the fluids flow in reverse direction (i.e. from delivery to suction) and the impeller rotates in the reverse direction.

3.1. Case of normal flow and normal rotation

Hydraulic torque T_{TP} of a pump in this case is given by the following equation:

$$T_{TP}^N = r_d \{ \rho_{gd} Q_{gd} (u_d - w_{gd} \cos \beta_{gd}) + \rho_{ld} Q_{ld} (u_d - w_{ld} \cos \beta_{ld}) \} - r_s \{ \rho_{gs} Q_{gs} v_{gs} \cos \delta_{gs} + \rho_{ls} Q_{ls} v_{ls} \cos \delta_{ls} \} \quad (1)$$

And for stationary states, the following equation holds:

$$\rho_{gd} Q_{gd} + \rho_{ld} Q_{ld} = \rho_{gs} Q_{gs} + \rho_{ls} Q_{ls} \quad (\equiv \rho_{TP} Q_{TP}) \quad (2)$$

where $Q_{TP} = Q_g + Q_l$.

Further it is assumed that at the impeller inlet the gas and the liquid have the same flow direction in the static coordinate system, and at the impeller exit both fluids have the same flow direction in the coordinate system rotating with the impeller (see Fig.1), i.e. the following equations hold:

$$\delta_{gs} = \delta_{ls} \quad (\equiv \delta_s) \quad (3)$$

$$\beta_{gs} = \beta_{ls} \quad (\equiv \beta_s) \quad (4)$$

Therefore using the following relations:

$$\bar{v}_s = \frac{\rho_{gs} Q_{gs} v_{gs} + \rho_{ls} Q_{ls} v_{ls}}{\rho_{TP} Q_{TP}} \quad (5)$$

$$\bar{w}_d = \frac{\rho_{gd} Q_{gd} w_{gd} + \rho_{ld} Q_{ld} w_{ld}}{\rho_{TP} Q_{TP}} \quad (6)$$

equation (1) is expressed as follows:

$$T_{TP}^N = \rho_{TP} Q_{TP} \{ r_d (u_d - \bar{w}_d \cos \beta_d) - r_s \bar{v}_s \cos \delta_s \} \quad (7)$$

However, since $v_s \cos \delta_s$ is usually small, this term is neglected in the following.

3.2. Case of reverse flow and reverse rotation

Hydraulic torque T_{TP} of a pump in this case is given by the following equation:

$$T_{TP}^C = r_d (\rho_{gd} Q_{gd} v_{gd} \cos \delta_{gd} + \rho_{ld} Q_{ld} v_{ld} \cos \delta_{ld}) - r_s \{ \rho_{gs} Q_{gs} (u_s - w_{gs} \cos \beta_{gs}) + \rho_{ls} Q_{ls} (u_s - w_{ls} \cos \beta_{ls}) \} \quad (8)$$

Equation (2) holds as in the former case, and it is assumed that at the impeller inlet (i.e. at the delivery side of the impeller in this case) the gas and the liquid have the same flow direction in the static coordinate system, and at the impeller exit (i.e. at the suction side of the impeller) both fluids have the same flow direction in the coordinate system rotating with the impeller, that is, the following equations hold:

$$\delta_{gd} = \delta_{ld} \quad (\equiv \delta_d) \quad (9)$$

$$\beta_{gs} = \beta_{ls} \quad (\equiv \beta_s) \quad (10)$$

Therefore using the following relations:

$$\bar{v}_d = \frac{\rho_{gd} Q_{gd} v_{gd} + \rho_{ld} Q_{ld} v_{ld}}{\rho_{TP} Q_{TP}} \quad (11)$$

$$\bar{w}_s = \frac{\rho_{gs} Q_{gs} w_{gs} + \rho_{ls} Q_{ls} w_{ls}}{\rho_{TP} Q_{TP}} \quad (12)$$

equation (8) is expressed as follows:

$$T_{TP}^C = \rho_{TP} Q_{TP} \{ r_d \bar{v}_d \cos \delta_d - r_s (u_s - \bar{w}_s \cos \beta_s) \} \quad (13)$$

Thus, when the shape of a pump and the conditions of fluids flowing into the pump as well as the fluids velocities relative to the rotating impeller at the exit are given, the hydraulic torques of the pump driven in the two-phase flow are obtained from equation (7) in the case of normal flow and normal rotation and from equation (8) in the case of reverse flow and reverse rotation.

4. Velocities of gas and liquid at impeller exit

The velocities of the gas and the liquid at the impeller exit are obtained from momentum conservation equations of the gas and the liquid phases in the impeller for both cases of normal flow and

Fig.1 Velocities at Inlet and Exit of Impeller

normal rotation as well as reverse flow and reverse rotation as described in the following sections.

4.1. Case of normal flow and normal rotation

Assuming that the gas and the liquid flow along the vane in the impeller except the vicinity of the inlet, both fluids have the same flow direction in the coordinate system rotating with the impeller, and have the same pressure in a plane perpendicular to the flow direction, the momentum conservation equations of the gas and the liquid are written as follows;

$$A_g \rho_g w_g \frac{dw_g}{ds} = -A_g \frac{dP}{ds} + A_g \rho_g \omega^2 r \sin \beta$$

$$-k_g \rho_g w_g^2 + k_{lg} (w_l - w_g)^2 \quad (14)$$

$$A_l \rho_l w_l \frac{dw_l}{ds} = -A_l \frac{dP}{ds} + A_l \rho_l \omega^2 r \sin \beta$$

$$-k_l \rho_l w_l^2 - k_{lg} (w_l - w_g)^2 \quad (15)$$

Eliminating dP/ds from these two equations, the following equation is obtained:

$$(\rho_l - \rho_g) \omega^2 r \sin \beta + k_g \frac{\rho_g w_g^2}{A_g} - k_l \frac{\rho_l w_l^2}{A_l}$$

$$- k_{lg} (w_l - w_g)^2 \left(\frac{1}{A_g} + \frac{1}{A_l} \right)$$

$$= \rho_l w_l \frac{dw_l}{ds} - \rho_g w_g \frac{dw_g}{ds} \quad (16)$$

If the shape of the pump (A and β), the friction loss coefficients (k_g , k_l and k_{lg}) in the impeller, the velocities of the fluids (w_{gs} and w_{ls}), void fraction (α_s) and steam quality (x_s) at the inlet are given, the velocities of the gas w_{gd} and the liquid w_{ld} at the impeller exit are obtained from equation (16) and the following equations:

$$A_g = A \cdot \alpha \quad (17)$$

$$A_l = A \cdot (1 - \alpha) \quad (18)$$

$$\rho_g A_g w_g = G \cdot x \quad (19)$$

$$\rho_l A_l w_l = G \cdot (1 - x) \quad (20)$$

The liquid is accelerated by the centrifugal force in the impeller and reaches a high velocity while approaching the impeller exit, and the friction force on the channel wall may become so large as to balance with the centrifugal force in the vicinity of the exit. Therefore it is assumed here that the liquid is not accelerated any more in the vicinity of the exit, i.e. the following equation is assumed:

$$(dw_l/ds)_d = 0 \quad (21)$$

Further, when the change of the flow cross-sectional area along flow direction is small ($dA/ds = 0$) and the volumetric changes of the gas and the liquid in the impeller are small, the following equation is derived from equations (18) - (21):

$$(dw_g/ds)_d = 0 \quad (22)$$

Thus equation (16) is rewritten as follows;

$$(\rho_l - \rho_g) \omega^2 r \sin \beta + k_g \frac{\rho_g w_{gs}^2}{A_{gs}} - k_l \frac{\rho_l w_{ls}^2}{A_{ls}}$$

$$- k_{lg} (w_{ls} - w_{gs})^2 \left(\frac{1}{A_{gs}} + \frac{1}{A_{ls}} \right) = 0 \quad (23)$$

Estimating the friction loss coefficients k_g , k_l and k_{lg} in the equation by the method described in the next chapter, the exit velocities of the gas and the liquid are obtained from equation (23).

4.2. Case of reverse flow and reverse rotation

Adopting similar assumptions as in the former section, e.g. both fluids flow along the impeller vane etc., the momentum conservation equations in this case are written as follows:

$$A_g \rho_g w_g \frac{dw_g}{ds} = -A_g \frac{dP}{ds} - A_g \rho_g \omega^2 r \sin \beta$$

$$- k_g \rho_g w_g^2 - k_{lg} (w_l - w_g)^2 \quad (24)$$

$$A_l \rho_l w_l \frac{dw_l}{ds} = -A_l \frac{dP}{ds} - A_l \rho_l \omega^2 r \sin \beta$$

$$- k_l \rho_l w_l^2 + k_{lg} (w_l - w_g)^2 \quad (25)$$

Further, assuming similar equations as in the former section as follows:

$$(dw_l/ds)_d = 0 \quad (26)$$

$$(dw_g/ds)_d = 0 \quad (27)$$

the following equation is obtained from equations (24) - (27):

$$(\rho_l - \rho_g) \omega^2 r \sin \beta - k_g \frac{\rho_g w_{gs}^2}{A_{gs}} + k_l \frac{\rho_l w_{ls}^2}{A_{ls}}$$

$$- k_{lg} (w_{ls} - w_{gs})^2 \left(\frac{1}{A_{gs}} + \frac{1}{A_{ls}} \right) = 0 \quad (28)$$

The exit velocities of the gas and the liquid are obtained from this equation together with equations (17) - (20) using the friction loss coefficients k_g , k_l and k_{lg} estimated from the next chapter.

5. Estimation of friction loss coefficients

The friction loss coefficients k_g , k_l and k_{lg} in equations (14) and others are considered to be functions of the velocity distribution in the impeller and the contact areas of the gas and the liquid with the wall, that is, functions of the rotating speed of the impeller and the void fraction in the impeller. However, the functional forms are not clear, the coefficients are obtained assuming that they have no dependency on the rotating speed and void fraction in the following.

Although, in the following, only those cases where the volumetric flow rate at the impeller inlet is equal to the rated flow rate Q_R are dealt with, the results can be extended to the cases where the ratio of the flow rate to the rotating speed Q/ω has the same value if the changes of the fluid densities and the steam quality through the impeller and the friction loss coefficients have little dependency on the flow rate Q and the rotating speed ω .

5.1. Coefficients of friction loss between fluids and wall

When hydraulic torque of a pump becomes null under the operation condition of the normal flow and normal rotation, the pressure difference between the inlet and the exit of the impeller is usually very small. Therefore, under this condition, the following equation holds approximately:

$$dP/ds = 0 \quad (29)$$

Further, since the following equations:

$$(dw_r/ds)_d = 0 \quad (21)$$

$$(dw_e/ds)_d \approx 0 \quad (22)$$

are assumed to hold as stated in the preceding chapter, equations (14) and (15) are rewritten as follows:

$$A_{gd} \rho_{gd} \omega_{r0}^2 r_d \sin \beta_d = k_g \rho_{gd} w_{gd}^2 - k_{Lg} (w_{Ld} - w_{gd})^2 \quad (30)$$

$$A_{Ld} \rho_{Ld} \omega_{r0}^2 r_d \sin \beta_d = k_L \rho_{Ld} w_{Ld}^2 + k_{Lg} (w_{Ld} - w_{gd})^2 \quad (31)$$

On the other hand, if the impeller rotates with a speed at which the hydraulic torque for single-phase flow is null and the exit velocity of the gas is equal to that of the liquid (this means that the mean exit velocity w_d defined by equation (6) is equal to $w_d = Q_R/A_d$ which is the exit velocity of single-phase flow), it is deduced from equation (7) that the hydraulic torque for two-phase flow is also null. In other words, if the hydraulic torque for two-phase flow is null at the rotation speed where the hydraulic torque for single-phase flow is null, the exit velocity of the gas is equal to that of the liquid, i.e. $w_{gd} = w_{Ld}$. Therefore the last term of the right hand side of equations (30) and (31) becomes zero for the condition stated above, and the following equations are derived:

$$k_g = A_{gd} \omega_{r0}^2 r_d \sin \beta_d / w_{gd}^2 = \alpha_d A_d \omega_{r0}^2 r_d \sin \beta_d / w_d^2 \quad (32)$$

$$k_L = A_{Ld} \omega_{r0}^2 r_d \sin \beta_d / w_{Ld}^2 = (1 - \alpha_d) A_d \omega_{r0}^2 r_d \sin \beta_d / w_d^2 \quad (33)$$

The coefficients k_g and k_L are obtained from equations (32) and (33), in which the exit void fraction α_d is obtained from experiments where the hydraulic torques for two-phase flow as well as for single-phase flow are null.

5.2. Coefficients of friction loss on gas-liquid interface

When a hydraulic torque T_{TPA} is measured in an experiment with a rotation speed ω_A and a steam quality x_{dA} at the impeller exit, the mean exit velocity w_{dA} is calculated from the following equation which is obtained from equation (7):

$$w_{dA} = \left\{ r_d \omega_A - \frac{T_{TPA}}{r_d Q_R} \left(\frac{x_{dA}}{\rho_{gd}} + \frac{1-x_{dA}}{\rho_{Ld}} \right) \right\} / \cos \beta_d \quad (34)$$

On the other hand, the exit velocities of the gas w_{gdA} and the water w_{LdA}

under the above described condition are obtained from the following equations:

$$w_{dA} = x_{dA} w_{gdA} + (1 - x_{dA}) w_{LdA} \quad (35)$$

$$w_d = \frac{Q_{TPA}}{A_d} = \left(\frac{x_{dA}}{\rho_{gd}} + \frac{1-x_{dA}}{\rho_{Ld}} \right) / \left(\frac{x_{dA}}{\rho_{gd} w_{gdA}} + \frac{1-x_{dA}}{\rho_{Ld} w_{LdA}} \right) \quad (36)$$

Thus, the coefficient of friction loss on the gas-liquid interface k_1 is obtained from equations (19), (20)⁸ and (23) using rotation speed ω_A and steam quality x_{dA} of the experiment as well as the exit velocities w_{gdA} and w_{LdA} obtained from equations (35) and (36).

6. Comparison with test results

Hydraulic torques of a pump which is used in a series of tests⁽³⁾ made by Combustion Engineering Inc. (C.E.) are calculated using the method stated in the preceding chapters, and compared with the test results. The performances of the pump under the rated condition are shown in Table 1. The sizes of the impeller are inferred from figures in published papers concerning the test, and are shown in Table 2.

6.1. Estimation of friction loss coefficients in impeller

The relation between the void fraction α_s at the pump inlet and the hydraulic torque T_{TP} for the rated (volumetric) flow rate and 45% of the rated speed is obtained from the test made by C.E. and shown in Fig.2. (Explanation of the solid line in the figure is given in the following section.) It is seen from the figure that the hydraulic torque T_{TP} becomes null when the inlet void fraction α_s is 0 and when α_s is 0.65. In the latter case, i.e. when $\alpha_s = 0.65$, the relative velocity of the gas to the liquid is zero at the impeller exit as stated in section 5.1. Though the relative velocity at the impeller inlet is not clear in the test, usually in horizontal pipes the relative velocity is small and it is assumed zero in the test. Therefore the exit void fraction α_d is equal to the inlet one α_s in the test, i.e. $\alpha_d = \alpha_s = 0.65$. Thus

Table 1. Performance of Pump Used in C.E. Test

Rated Rotation Speed	4500 r.p.m.
Rated Flow Rate	0.22 m ³ /s
Rated Head	77 m
Rated Hydraulic Torque	418 N m

Table 2. Shape of Pump Impeller Used in C.E. Test
(Inferred from Figures in Published Papers)

Diameter of Mouth Ring	0.15 m
Diameter of Impeller Inlet	0.09 m
Diameter of Impeller Exit	0.20 m
Angle between Impeller Vane and Periphery	30 °

the coefficients k_g and k_l are calculated from the following equations which are obtained from equations (32) and (33):

$$k_g = 0.65 A_d (0.45 \omega_R)^2 r_d \sin \beta_d / w_d^2 \quad (37)$$

$$k_L = 0.35 A_d (0.45 \omega_R)^2 r_d \sin \beta_d / w_d^2 \quad (38)$$

For the rated flow rate and the rated speed, the relation between the inlet void fraction α_s and the hydraulic torque T_{TP} is shown in Fig.3. (Explanation of the solid line in the figure is given in the following section.) According to the figure, the normalized torque N_T is about 0.7 when the inlet void fraction α_s is 0.4 and the pump inlet pressure is 6.9 MPa. From this result, the friction loss coefficient k_{lg} on the gas-liquid interface is obtained as follows.

First, the exit steam quality x_d is calculated from the following equation:

$$x_{dA} = \left\{ \frac{\rho_g S_s \alpha_s}{\rho_g S_s \alpha_s + \rho_L (1 - \alpha_s)} \right\} \frac{(i_{gs} - i_{Ls})}{(i_{gd} - i_{Ld})} + \frac{(i_{Ls} - i_{Ld})}{(i_{gd} - i_{Ld})} \quad (39)$$

where the exit enthalpies i_{gd} and i_{Ld} are estimated using the pump exit pressure P_{dA} measured in the test, and it is assumed again that the relative velocity of the gas to the liquid is zero at the inlet.

Next, hydraulic torque T_{TPA} is calculated from normalized torque N_T using the following equation:

$$T_{TPA} = N_T Q_R r_d (r_d \omega_R - w_{dR} \cos \beta_d) \quad (40)$$

Using x_{dA} and T_{TPA} calculated from equations (39) and (40) as well as k_l and k_g obtained from (37) and (38), the coefficient k_{lg} is calculated as follows:

$$k_{Lg} = 4.8 \rho_g (7.0 \text{ MPa}) \frac{A_d \omega_R^2 r_d \sin \beta_d}{w_d^2} \quad (41)$$

where $\rho_g(7.0 \text{ MPa})$ means the density of saturated steam at 7.0 MPa which is the pump delivery pressure measured in the test at the rated flow rate, rated rotation speed and 40% inlet void fraction.

6.2. Case of normal flow and normal rotation

The hydraulic torque T_{TP} is calculated as a function of the inlet void fraction α_s from equations (6) and (7) in which the gas velocity w_{gd} and the liquid velocity w_{ld} at the impeller exit are obtained from equation (23) where the values of the coefficients k_g , k_l and k_{lg} are given by equations (37), (38) and (41).

The calculated results are shown in Fig.2 and Fig.3 with solid lines. Differences between the calculated values and the test data become large for void fractions less than 0.3 and higher than 0.7. These differences may be attributed to the following two assumptions, that is, (1) the pressure at the impeller exit is equal to that at the pump delivery, and (2) the friction loss coefficients are constant for any void fraction, though the coefficients might have different values for void fractions less than 0.3 and higher than 0.7 where the flow patterns may be

bubble flow or mist flow instead of stratified flow which is most probable for void fractions between 0.3 - 0.7.

6.3. Case of reverse flow and reverse rotation

The hydraulic torque for reverse flow and reverse rotation is obtained from equation (13), if the circumferential velocity component of the fluid flowing into the impeller from the vortex chamber, i.e. $v_d \cos \beta_d$, is known. This value is estimated assuming that a velocity distribution of two-phase flow in the vortex chamber is the same as that for single-phase flow whose volumetric flow rate is equal to the sum of the flow rates of the liquid and the gas.

According to the characteristic curve of the pump used in the test of C.E. (See Fig.4), $T/T_R = 0.55$ when $(Q/Q_R)/(\omega/\omega_R) = 1.0$. Using these values, the following relation is deduced from equation (13):

$$r_d v_d \cos \beta_d = 0.55 (T_R / \rho_L Q_R) + r_d^2 \omega_R - r_d w_{dR} \cos \beta_d \quad (42)$$

Substituting this relation into equation (13), the following equation is obtained:

$$\frac{T_{TP}}{\rho_{TP} Q_{TP}} = 0.55 \cdot \frac{T_R}{\rho_L Q_R} + r_d^2 \omega_R \left(1 - \frac{\omega}{\omega_R}\right) - r_d (w_{dR} - w_s) \cos \beta_d \quad (43)$$

The gas velocity w_{gs} and the liquid velocity w_{ls} used in w_s in equation (43)

Fig.2 Hydraulic Torque vs. Inlet Void Fraction
 $P = 6.9 \text{ MPa}$, $Q = Q_R$, $\omega = 0.45 \omega_R$

Fig.3 Hydraulic Torque vs. Inlet Void Fraction
 $P = 6.9 \text{ MPa}$, $Q = Q_R$, $\omega = \omega_R$

are obtained from equation (28). It is assumed here that the friction loss coefficients k_g , k_l and k_{lg} for reverse flow and reverse rotation have the same value as for normal flow and normal rotation, though the coefficients might have different values. Hydraulic torques for those cases in which the coefficient k_{lg} has 1/2 of the value for normal flow and normal rotation are also calculated to estimate the effect of the coefficient on the calculated hydraulic torque.

The calculated values are compared with test data in Fig.5 for the case of $Q_{TP} = -0.7Q_R$ and $\omega = -0.7\omega_R$, and in Fig.6 for the case of $Q_{TP} = -0.7Q_R$ and $\omega = -0.5\omega_R$. The broken lines in the figures show the results obtained using 1/2 value for the coefficient k_{lg} .

The calculated value is higher than the test datum for zero void fraction in Fig.5. A reason for this difference is considered as follows, that is, it is assumed in the calculation that the fluids are in saturated state even for zero void fraction at the impeller inlet and some void is contained at the impeller exit, though in the test for zero void fraction it is presumed that the liquid at the impeller inlet had enough subcooling and no void was left even at the impeller exit.

The calculated values in Fig.5 and Fig.6 do not show good agreement with the test data. This may be attributed to the assumptions that the circumferential component of the inlet velocity, $v_d \cos \delta_d$, for two-phase flow is equal to that for single-phase flow and the friction loss coefficients for reverse flow and reverse rotation are equal to those for normal flow and normal rotation.

Fig.4 Homologous Torque Curve of C.E. Pump for Single-Phase Flow

6.4. Extension to cases of different system pressures

Although the coefficients k_g and k_l are considered to have little dependency on the fluid pressure as inferred from the form of equations (14) and (15), the friction loss coefficient k_{lg} is supposed to vary with the fluid pressure and it is assumed here that the coefficient k_{lg} is proportional to the density of the gas, i.e.

$$k_{lg} \propto \rho_g \quad (44)$$

The calculated values using the assumption are shown in Fig.7 and Fig.8 with test data of C.E.. Since the test data for 3.3 MPa pressure show wide scattering, the agreement of the calculated values with the test data is not clear. However, the pressure dependency of the calculated hydraulic torque is similar to that of the test results.

In Fig.8 which shows the hydraulic torque for the rated volumetric flow rate and the rated rotation speed, the agreement of the calculated value with the test data is not good except the vicinity of void fraction 0.4 which is used to obtain the proportionality constant of relation (44). This may be attributed again to the assumption that the friction loss coefficient k_{lg} is constant for any void fraction.

Fig.5 Hydraulic Torque vs. Inlet Void Fraction

$P = 6.5 \text{ MPa}$, $Q = -0.7Q_R$, $\omega = -0.7\omega_R$

Fig.6 Hydraulic Torque vs. Inlet Void Fraction

$P = 6.9 \text{ MPa}$, $Q = -0.7Q_R$, $\omega = -0.5\omega_R$

7. Conclusions

Hydraulic torques of a centrifugal pump driven in two-phase flow are estimated using gas and liquid velocities at the impeller exit which are obtained from momentum conservation equations of both phases in the impeller. In the equations friction loss coefficients between the impeller wall and the both phases as well as on the gas-liquid interface are chosen so as to fit a calculated value to a test datum in an appropriate condition.

The calculated hydraulic torques are compared with test data obtained by Combustion Engineering Inc., and the following conclusions are obtained.

(1) The calculated hydraulic torques for normal flow and normal rotation show pretty good agreement with the test data of C.E. with the exception of void fractions less than 0.3 and higher than 0.7. The disagreement in the void fraction range stated above may be mainly attributed to the assumption that the friction loss coefficient, k_{lg} , on the gas-liquid interface is constant for any void fraction though the coefficient might vary with flow patterns of two-phase flow which are dependent on the void fractions.

(2) The calculated hydraulic torques for reverse flow and reverse rotation do not show good agreement with the test data. This may be mainly attributed to the assumption that the circumferential component of the inlet velocity, $V_d \cos \delta_d$, from the vortex chamber to the impeller for two-phase flow is equal to that for single-phase flow with the same volumetric flow rate though the stream lines of the two-phase flow in the vortex chamber might be different from that of the single-phase flow.

(3) The hydraulic torque for 3.3 MPa system pressure is predicted pretty well using the test data of 6.9 MPa system pressure, if the friction loss coefficient k_{lg} on the gas-liquid interface is assumed to be proportional to the density of the gas.

The author wishes to express his appreciation to The Tokyo Electric Power Co., Inc. and the Committee of Joint Study Program of Japan BWR Utilities for their valuable advice on this research and to Mr. Atsuo Yamanouchi for his helpful advice in the preparation of this paper.

Fig.7 Effect of System Pressure on Hydraulic Torque

$$Q = Q_R, \omega = 0.45 \omega_R$$

Fig.8 Effect of System Pressure on Hydraulic Torque

$$Q = Q_R, \omega = \omega_R$$

8. References

- (1) Winks, R.W., EPRI NP-135 (1976.11.)
- (2) Runstadler Jr., P.W. and Dolan, F.X., Proc. Polyphase Flow in Turbomachinery, ASME (1978.12.) 65
- (3) Kennedy, W.G. et al., EPRI NP-1556 (1980.9.)
- (4) Mikielewicz, J. et al., ASME 76-WA/FE-29 (1976.12.)
- (5) Zarechnak, A. et al., Proc. ANS/ASME/NRC International Topical Meeting on Nuclear Reactor Thermal Hydraulics (1980.10.)