Basic Research on Vibration and Noise of Internal Combustion Engine*

(2nd Report, Noise Analysis of Cylinder Block)

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Distributions and frequency characteristics of sound pressures, which are emitted from the surface of a cylinder block of an internal combustion engine, in the space around the cylinder block are analyzed. At first, vibratory transfer functions of the cylinder block are analyzed by the component mode synthesis method. Using modal dampings obtained in experiment of the cylinder block by the harmonic exciting test, a vibration analysis of the cylinder block is tried in consideration of damping. Next, distributions and frequency characteristics of sound pressures are analyzed using the vibratory transfer functions calculated by the component mode synthesis method. Besides, noise analysis is tried using vibratory transfer functions obtained in experiment, too. The calculated results agree nearly with the experimental ones.

Key Words: Vibration, Noise, Component Mode Synthesis Method, Finite Element Method, Internal Combustion Engine, Cylinder Block

1. Introduction

Distributions and frequency response functions (FRF in brief) of sound pressures emitted from the surface of a cylinder block of an internal combustion engine in the space around the cylinder block are analyzed. At first, distributions and FRF of sound pressures emitted from a flat steel plate are analyzed in order to examine availability of the technique used in the present paper. Next, a noise analysis of a cylinder block is tried.

First, the sound emission surface is divided into rectangular elements. The transfer functions of the nodal points obtained both by the experiment and by the component mode synthesis method^{(1),(2)} (CMS in brief) are used as the input data for the sound analysis.

2. Component Mode Synthesis Method

By CMS, it is possible to analyze the vibration of such a complex structure that it can hardly be analyzed by usual methods, for instance, the finite element

method (FEM in brief) and the transfer matrix method. A structure is divided into some components. Here, we consider a structure composed of two components (sub. in brief) as shown in Fig.1. The interior regions of sub.1 and sub.2 are called the regions a and b respectively, and the interface region between sub.1 and sub.2 is called the region c. The following equation of motion of sub.1 is formulated by FEM.

$$\begin{bmatrix} M_a^I & M_{ac}^I \\ M_{ca}^I & M_c^I \end{bmatrix} \begin{Bmatrix} \ddot{\delta}_a \\ \ddot{\delta}_c \end{Bmatrix} + \begin{bmatrix} K_a^I & K_{ac}^I \\ K_{ca}^I & K_c^I \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_c \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_c \end{Bmatrix} \quad \dots\dots\dots (1)$$

where [M] is the mass matrix, [K] is stiffness matrix, {δ} is the displacement vector, {F} is the force vector and the attached subscripts identify the regions.

Fig.1 Structure with two components
a, b: Internal regions
c: Interface region

The reduced mass matrix $[M_I]$ and the reduced stiffness matrix $[K_I]$ which are related to only the degrees of freedom of the region c are obtained by Guyan's method of reduction as written in the following equation.

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$$\left. \begin{aligned} [M_1] &= [T_1]^T [M_a^1] [T_1] + [T_1]^T [M_{ac}^1] \\ &\quad + [M_{ca}^1] [T_1] + [M_c^1] \\ [K_1] &= [T_1]^T [K_a^1] [T_1] + [T_1]^T [K_{ac}^1] \\ &\quad + [K_{ca}^1] [T_1] + [K_c^1] \end{aligned} \right\} \dots (2)$$

where $[T_1] = -[K_a^1]^{-1} [K_{ac}^1]$

The reduced mass matrix $[M_2]$, the reduced stiffness matrix $[K_2]$ and the transformation matrix $[T_2]$ of sub.2 are obtained in the same way. The displacement $\{\delta_c\}$ of the region c is expressed as

$$\{\delta_c\} = [\phi_c] \{\xi_c\} \dots (3)$$

where $[\phi_c]$ is the modal matrix of the following generalized eigenvalue problem.

$$([K_1 + K_2] - \omega^2 [M_1 + M_2]) \{x\} = \{0\} \dots (4)$$

The displacement $\{\delta_a\}$ of the region a is expressed as

$$\begin{bmatrix} M_a^1 & 0 & M_{ac}^1 \\ 0 & M_b^2 & M_{bc}^2 \\ M_{ca}^1 & M_{cb}^2 & M_c^2 + M_c^2 \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_b \\ \delta_c \end{Bmatrix} + \begin{bmatrix} C_a^1 & 0 & C_{ac}^1 \\ 0 & C_b^2 & C_{bc}^2 \\ C_{ca}^1 & C_{cb}^2 & C_c^2 + C_c^2 \end{bmatrix} \begin{Bmatrix} \dot{\delta}_a \\ \dot{\delta}_b \\ \dot{\delta}_c \end{Bmatrix} + \begin{bmatrix} K_a^1 & 0 & K_{ac}^1 \\ 0 & K_b^2 & K_{bc}^2 \\ K_{ca}^1 & K_{cb}^2 & K_c^1 + K_c^2 \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_b \\ \delta_c \end{Bmatrix} = [M_A] [A] \{\ddot{\xi}\} + [C_A] [A] \{\dot{\xi}\} + [K_A] [A] \{\xi\} = \{F\} \dots (9)$$

where $[C]$ is the damping matrix. Multiplying Eq.(9) by $[A]^T$, the degrees of freedom of the equation of motion are largely reduced, adopting the natural modes as the normalized coordinates, and the resultant equation becomes

$$[A]^T [M_A] [A] \{\ddot{\xi}\} + [A]^T [C_A] [A] \{\dot{\xi}\} + [A]^T [K_A] [A] \{\xi\} = [\hat{M}] \{\ddot{\xi}\} + [\hat{C}] \{\dot{\xi}\} + [\hat{K}] \{\xi\} = [A]^T \{F\} \dots (10)$$

The response $\{\xi\}$ of the structure when the external force $\{F\}e^{j\omega t}$ is given becomes

$$\{\xi\} = [\xi_0] \{d\} e^{j\omega t} \dots (11)$$

where $[\xi_0]$ is the modal matrix of the following generalized eigenvalue problem.

$$([\hat{K}] - \mu^2 [\hat{M}]) \{x\} = \{0\} \dots (12)$$

The following equation is obtained by substituting Eq.(11) into Eq.(10) and multiplying Eq.(10) by $[\xi_0]^T$.

$$(-\omega^2 [\xi_0]^T [\hat{M}] [\xi_0] + j\omega [\xi_0]^T [\hat{C}] [\xi_0] + [\xi_0]^T [\hat{K}] [\xi_0]) \{d\} = \begin{pmatrix} -\omega^2 \begin{bmatrix} m_1 & & 0 \\ & m_2 & \\ 0 & & m_l \end{bmatrix} + j\omega \begin{bmatrix} c_1 & & 0 \\ & c_2 & \\ 0 & & c_l \end{bmatrix} + \begin{bmatrix} k_1 & & 0 \\ & k_2 & \\ 0 & & k_l \end{bmatrix} \end{pmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ \vdots \\ d_l \end{Bmatrix} = [\xi_0]^T [A] \{F\} = \begin{Bmatrix} f_1 \\ f_2 \\ \vdots \\ f_l \end{Bmatrix} \dots (13)$$

where l is the degree of freedom of $\{\xi\}$. From Eq.(13), d_i becomes

$$d_i = \frac{f_i}{-\omega^2 m_i + j\omega c_i + k_i} \quad (i=1 \sim l) \dots (14)$$

$$\begin{aligned} \{\delta_a\} &= [T_1] \{\delta_c\} + [\phi_a] \{\xi_a\} \\ &= [T_1] [\phi_c] \{\xi_c\} + [\phi_a] \{\xi_a\} \dots (5) \end{aligned}$$

where $[\phi_a]$ is the modal matrix of the following generalized eigenvalue problem.

$$([K_a^1] - \omega^2 [M_a^1]) \{x\} = \{0\} \dots (6)$$

The displacement $\{\delta_b\}$ of the region b is similarly expressed as

$$\begin{aligned} \{\delta_b\} &= [T_2] \{\delta_c\} + [\phi_b] \{\xi_b\} \\ &= [T_2] [\phi_c] \{\xi_c\} + [\phi_b] \{\xi_b\} \dots (7) \end{aligned}$$

Combining Eqs.(3), (5) and (7), the displacement of the total structure becomes

$$\begin{Bmatrix} \delta_a \\ \delta_b \\ \delta_c \end{Bmatrix} = \begin{bmatrix} \phi_a & 0 & T_1 \phi_c \\ 0 & \phi_b & T_2 \phi_c \\ 0 & 0 & \phi_c \end{bmatrix} \begin{Bmatrix} \xi_a \\ \xi_b \\ \xi_c \end{Bmatrix} = [A] \{\xi\} \dots (8)$$

Using Eq.(8), the equation of motion of the total structure is expressed as

Substituting $\{d\}$ obtained from Eq.(14) into Eq.(12), $\{\xi\}$ is obtained. Substituting $\{\xi\}$ into Eq.(8), the displacement of the total structure is obtained.

3. Sound Pressure Analysis

The complex sound pressure $\dot{p}$ at a point which is distant r from the small sound source is given by

$$\dot{p} = j\rho_0 \frac{\omega^2 \dot{w}}{2\pi r} e^{j(\omega t - kr)} ds \quad (15)$$

where ρ_0 is the density of the air, ω is the angular frequency, $\dot{w}$ is the complex displacement, k is the acoustic wave number and ds is the area of the small sound source. Since the sound emission surface can be approximately regarded as an assembly of small sound sources, the sound pressure expressed by the complex number which is emitted from the whole sound emission surface is given by the integral

$$\dot{p} = j\rho_0 \frac{\omega^2}{2\pi} e^{j\omega t} \int_S \frac{\dot{w}}{r} e^{-jkr} ds \quad (16)$$

The sound emission surface is divided into rectangular elements as shown in Fig.2,

$$\dot{p}_e = j\rho_0 \frac{\omega^2}{2\pi R} e^{j(\omega t - kR)} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \{\dot{\delta}_e\}^T [B_e]^T [1, x, y, xy]^T \{1 + (\alpha x + \beta y)/R\} e^{jk(\alpha x + \beta y)} dx dy \quad (21)$$

The complex sound pressure emitted from the whole of the sound emission surface is obtained by adding up the results calculated by Eq.(21).

Fig.2 Rectangular element

Fig.3 Definition of coordinates
A: Measured point

and the displacement $\dot{w}_e$ in an element is assumed as

$$\dot{w}_e = \dot{\alpha}_1 + \dot{\alpha}_2 x + \dot{\alpha}_3 y + \dot{\alpha}_4 xy \quad (17)$$

where $\dot{\alpha}_1 \sim \dot{\alpha}_4$ are the complex coefficients defined by the displacements $\dot{w}_1 \sim \dot{w}_4$ at four nodal points,

$$\begin{Bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\alpha}_3 \\ \dot{\alpha}_4 \end{Bmatrix} = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/2a & -1/2a & 1/2a & -1/2a \\ 1/2b & 1/2b & -1/2b & -1/2b \\ 1/ab & -1/ab & -1/ab & 1/ab \end{bmatrix} \begin{Bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \end{Bmatrix} \\ = [B_e] \{\dot{\delta}_e\} \quad (18)$$

Referring to Eqs.(17) and (18), the displacement $\dot{w}_e$ in the element is given by

$$\dot{w}_e = \{\dot{\delta}_e\}^T [B_e]^T [1, x, y, xy]^T \quad (19)$$

Meanwhile, the distance r from ds to A in Fig.3 is approximated by

$$\frac{1}{r} \approx \frac{1}{R - (\alpha x + \beta y)} \approx \frac{1}{R} (1 + \frac{\alpha x + \beta y}{R}) \quad (20)$$

where $\alpha = \sin\theta \cos\phi$, $\beta = \sin\theta \sin\phi$. Referring to Eqs.(16), (19) and (20), the sound pressure emitted from the element is given by

4. Experiment

Figure 4 shows an outline of the experiment setup. In the present report, the sound pressure emitted from one side surface of a cylinder block excited by a shaker is measured. A baffle is set up around the cylinder block in order to make a half infinite space. The distribution of the sound pressures is measured on x - z plane and y - z plane, and FRFs of the sound pressure are measured at the points A and B in Fig.4.

Fig.4 Outline of experiment setup
A, B: Measured point

5. Calculation

A sound analysis is tried with the input data on the transfer functions obtained both by CMS and by experiment. The sound emission surface of the cylinder block shown in Fig.5 is divided into rectangular elements shown in Fig.6. The displacements of the nodal points are calculated both by CMS and by the harmonic exciting test. Substituting them into Eq.(21), the distributions and FRF of the sound pressures are obtained.

Figure 7 shows the division of the total cylinder block into 10 components and the division of each component into triangular finite elements for the vibration analysis. The mass and the stiffness matrices of each component are made by FEM. Using them and the modal dampings ζ_i obtained by the experiment, the displacements of the nodal points on the sound emission surface are obtained by CMS. Here, c_i in Eq.(14) is expressed with ζ_i as

$$c_i = 2\zeta_i \sqrt{m_i k_i} \quad \dots\dots\dots (22)$$

Fig.5 Cylinder block

Fig.6 Finite element division for sound analysis

Fig.7 Finite element division for vibration analysis

6. Sound Analysis of Flat Plate

The sound pressure emitted from a flat steel plate shown in Fig.8 is analyzed in order to check the availability of the sound analysis method adopted in the present report. Sound pressure is calculated in the same way as with the cylinder block. Figure 9 shows the distribution of sound pressures in case that the point F of the plate is excited at the natural frequency 1074Hz. The dotted line is the result measured in experiment, and the solid line is the one calculated by using the transfer functions obtained in experiment. Figure 10 shows FRF of the sound pressure at the point (-30cm, 30, 10). In this case the transfer functions calculated by FEM are used.

Fig.8 Finite element division of flat plate
F: Excited point
Thickness=4mm

Fig.9 Distribution of sound pressures from flat plate
Solid line: Calculation
Dotted line: Experiment

Fig.10 FRF of sound pressure from flat plate
Solid line: Calculation
Dotted line: Experiment

7. Sound Analysis of Cylinder Block

At first, the results calculated by using the transfer functions measured in experiment are shown. Figure 11 shows the distribution of sound pressures at the natural frequency 1005Hz on x-z plane. Figure 12 shows FRF of the sound pressure measured at the point A in Fig.4. The calculated results agree well with the experimental ones. They show that the sound analysis technique adopted in the present report is available.

Next, the results calculated by using the transfer functions obtained by CMS are shown. Figure 13 shows the distribution of sound pressures at the natural frequency 765Hz on y-z plane. Figure 14 shows FRF of the sound pressure at the point B. In Fig.14, the calculated result (solid line) does not agree well with the experimental one in the frequency range of 600~800Hz. This error may be made smaller by increasing the finite element division in the vibration analysis by CMS.

8. Conclusion

A sound analysis of a cylinder block is performed. At first, the transfer functions are calculated by the component mode synthesis method. Next, using the results of this calculation as the input data, the distribution and the frequency response function of sound pressures emitted from one side of the cylinder block excited by shaker are calculated. The calculated results are in good agreement with the experimental ones.

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Fig.11 Distribution of sound pressures from cylinder block
Solid line: Calculation
Dotted line: Experiment

Fig.12 FRF of sound pressure from cylinder block
Solid line: Calculation
Dotted line: Experiment
Measured point: A

Fig.13 Distribution of sound pressures from cylinder block
Solid line: Calculation
Dotted line: Experiment

Fig.14 FRF of sound pressure from cylinder block
Solid line: Calculation
Dotted line: Experiment
Measured point: B