Fundamental on Scoring Failure

(2nd report: A Model for Scoring and Critical Temperature)

By Aizoh KUBO*, Tetsuya NONAKA**,
Ritsuya IWAI***, Takeshi FUJITSU**,
and Hiroshi YABE****

From the observation and analysis of scoring phenomena on gear tooth flank, it is proposed that the scoring phenomena be dealt with as a thermal stability problem, whether the surface temperature of sliding-contacting body takes a steady value or diverges. A mathematical model for such scoring phenomena is introduced, which considers the heat transfer from lubricating oil film to the sliding-contacting body. The shapes of time to surface-temperature curves of this mathematical model correspond fairly well to those of measured temperature curves of gear tooth with and/or without scoring. This mathematical model is therefore considered well to express the basic characteristics for initiation and progress of scoring. Most of research on scoring today is worked out on the subject how to get a critical temperature against scoring for each kind of lubricants and contacting body materials. Here the general meaning of such research is clarified: to get critical temperature values is to get experimentally the limit to stability of this mathematical model.

Key words : Gear, Scoring, Scuffing, Critical temperature, Thermal stability.

1. Introduction

Many investigations have been carried out over a long time about scoring- or scuffing failure on lubricated surfaces, but the problem is still not well solved, because of the complex nature of scoring phenomenon. In this report, the scoring owing to thermal causes, that is, scoring failure under high sliding velocity and heavy load is dealt with, and a logical explanation for scoring initiation is introduced which is assumed from the nature of scoring shown in the previous report.^[1] A mathematical model for thermal scoring is constructed to explain the measured temperature - driving time relation. This mathematical model clarifies the physical meaning of the critical temperature for scoring initiation and also the empirically known effect of the temperature of lubricating oil. Some suggestions about the nature of thermal scoring which are given by this model are important for experimental investigations

about scoring and as actual design information to prevent scoring.

2. State of Scoring development on Gears and Scoring Model

Scoring tests were carried out using spur gears of module $m=4$ mm, tooth number $z_1/z_2=25/25$, and quality JIS 0. The gears were driven at a speed of 7000 rpm under the forced lubrication with #180 turbine oil. The oil supplying temperature was 37 °C and the flow rate was 4.5 l/min. The temperatures close to the tooth surface were measured during scoring test.^[1] Thermocouples for this measurement were made of 0.1 mm ϕ Alumel and Chromel wires, and stuck on the side of four teeth at the ending points of meshing (Fig.1).

The experiments and the theoretical analysis about dynamic load in spur gears and dynamic thermal- and heat generating conditions on the tooth flanks of spur gears have revealed the following nature about the occurrence and development of thermal scoring on gears with a gear ratio of unity^[1]: Once a scoring occurs at a tooth, the scoring develops on this tooth

Fig.1
Thermocouple for temperature measurement near tooth flank

* Associate professor, Faculty of Engineering, Kyoto University, Yoshida-Hommachi, Sakyo-ku, Kyoto.

** Graduate student, Division of Precision Mechanics, Graduate School of Engineering, Kyoto University, Yoshida-Hommachi, Sakyo-ku, Kyoto.

*** Engineer, Sumitomo Steel Co. Ltd., Nishinosho 1095, Wakayama City.

**** Professor, Faculty of Engr., Kyoto University, Yoshida-Hommachi, Sakyo-ku, Kyoto.

flank as long as the gears continue to run. This is because the temperature of the scored tooth is already high enough to maintain the condition of scoring occurrence, although the load sharing of the scored tooth at the meshing by tooth tip and tooth root decreases from the value before the initiation of scoring, where the sliding velocity between contacting driving- and driven tooth flanks is high. The heat generated on the initially scored tooth flank is conducted through the gear body and raises the temperatures of the successive teeth to fulfill the condition of scoring initiation. The scoring propagates in such a manner successively to the tooth flank of the neighboring tooth. But if the driving is stopped as soon as the scoring has occurred at a tooth and if the driving is started again under the same condition after sufficient cooling of gears, the scoring occurs then at a different tooth. Figure 2 shows some examples of the measured results about the change in teeth temperatures of seven gear pairs as a function of driving time. When a scoring does not occur (broken curves), temperature saturates and approaches a constant value. On the contrary when a scoring occurs (solid curves), temperature begins to rise suddenly after some driving time, showing a point of inflection. When the driving was stopped before temperature curve showed a point of inflection, the scoring never occurred. And when the driving was stopped after temperature curve showed a point of inflection, a slight or severe scoring was surely found on the tooth flank.

Now, the changes in the tooth forms with progress of scoring are represented in Fig.3 for spur gears and in Fig.4 for helical gears, which have been empirically known through many studies on the field of scoring of gears. Scoring on helical gear tooth initiates at a point close to the beginning point of meshing where the flash temperature concentrates and becomes very high, and it develops on the tooth flank

of approaching contact in the direction of the tooth trace. When it spreads over the face width, it occurs then at the ending point of meshing and develops on the tooth flank of recessing contact in the contra-direction of the tooth trace. Finally it covers the whole tooth flank except the neighborhood of pitch point.^[2] In order to get a general idea of how heat generation on a tooth flank changes when a scoring initiates and develops, suppose a scoring has occurred and developed as shown in Fig.3 or Fig.4, when gears of various dimensions shown in Table 1 transmit a load corresponding to a nominal Hertzian stress of 1000 MPa at the working pitch point. Figures 5 and 6 show the results, indicating how the maximum flash temperature max. T_f and frictional heat flux Q_{fr} [which is generated on a tooth flank during one meshing period] change with initiation and progress of scoring. T_f and Q_{fr} were calculated by

$$T_f = \frac{1.11 \mu p(x) \sqrt{V_1(x) - V_2(x)}}{\sqrt{\lambda_s \rho_s c_s \sqrt{2a(x)}}} \quad \dots\dots(1)$$

$$Q_{fr} = 9.8 \times 10^{-3} \mu \int_0^{t_{fr}} \left[\int_{u_{fr}} p(x) \sqrt{V_1(x) - V_2(x)} dx \right] dt \quad \dots\dots(2)$$

where

μ : coefficient of friction
 $p(x)$: distributed load at a point x on tooth flank (normal direction)

Table 1 Specifications of test gears

gear pair	A	B	C	D
tooth form	normal			
module (mm)	4			
press.angle (deg.)	20			
tooth number	20/92			
helix angle β_s (deg.)	0	10	20	30
tooth width (mm)	40.0	34.0	51.79	94.47
prof.modf.factor	0/0			
top clearl.factor	0.25			
contact ratio	1.682	1.640	1.519	1.326
overlap ratio	0	0.500	1.500	4.000

Fig.2 Some experimental examples of a change in teeth temperatures as a function of driving time

Fig.3 Change of the tooth form of spur gear with initiation and progress of scoring

Fig.4 Change of the tooth form of helical gear with initiation and progress of scoring (on plane of action)

- $V_1(x), V_2(x)$: tangential velocities at a point x on tooth flank of driving and driven gears respectively
 λ_s, ρ_s, c_s : thermal conductivity, density, specific heat of the material of gear tooth respectively
 $a(x)$: a half of Hertzian contact band at a point x
 $L(t)$: length of the line of contact at time t
 ϵT_z : time duration of a tooth contact

It is assumed that there is no dynamic load owing to gear vibration and that the load distribution $p(x)$ on $L(t)$ is equal to the static load distribution for helical gears with manufacturing- and alignment errors.^[2] The coefficient of friction between contacting tooth flanks is considered constant, even if a scoring occurs and develops. As seen in Figs.5 and 6, both the maximum flash temperature on tooth flank and the frictional heat generated decrease considerably on account of the tooth profile change owing to the occurrence and development of the scoring. It is almost impossible to think that this decreasing tendency could change much, even if the influence of dynamically increasing load is taken into consideration. Contrary to this result, it is always observed that, once an actual scoring occurs on a tooth flank, it always develops on this tooth flank explosively, so long as the driving condition is not relaxed. In addition to this, the contacting time duration of a tooth is very short, and if a scoring occurs on a specific contacting tooth flank in a moment, the teeth separate from each other at the next moment and are lubricated before next meshing comes. Summing up these results, it is seen that the critical temperature for the occurrence of scoring is as strongly influenced by steady bulk- or surface-temperature of the tooth flank as by the flash temperature. In order to understand the condition of occurrence of scoring, it is therefore indispensable to consider the state of

Fig.5 Changes in the maximum flash temperature with occurrence and development of scoring

Fig.6 Changes in frictional heat generation with occurrence and development of scoring

bulk temperature close to the contacting surface under boundary lubrication : The heat generated in boundary lubricating oil film is partly carried away by the flow of oil film and partly given to the contacting surfaces. The latter part of heat is

conducted into body and raises the surface temperature. To explain correctly the way of occurrence of scoring, it is therefore necessary to deal with the problem of the stability of the temperature of boundary lubricated surface: that is, making a model for the state of heat supply and dissipation to and from the boundary lubricated surfaces, and to consider such state of surface temperature related with occurrence of scoring as it will be in steady state and settle to a constant value or diverge.

3. Description of Scoring Model

3-1 Order of heat amount supplied to a lubricated surface

Let the x-axis be taken in the direction of oil film thickness and y-axis in the direction of flow of the oil film. Suppose that the oil film is in steady state and the oil film thickness h is constant throughout the length of the oil film. Consider the oil film at a time section $t=t$ as a sum of rectangular blocks of oil. Each assumed rectangular oil block stands perpendicular to surfaces (Fig. 7(a)), and its width is infinitesimally small. The oil in a block $C_1D_1C_2D_2$ with a width dy at a position y moves to the position of $C_1'D_1'D_2'C_2'$ (Fig. 7(b)) after small time dt .

Because dt is small enough, the magnitudes of C_1C_1' , D_1D_1' , C_2C_2' , and D_2D_2' are negligibly small in comparison with dy . Then the amounts of heat dQ_1 and dQ_2 given onto the surfaces 1 and 2 respectively during dt are expressed by

$$dQ_1 = \left[\int_0^l \phi_1 dy \right] dt, \quad dQ_2 = \left[\int_0^l \phi_2 dy \right] dt \quad (3)$$

where ϕ_1 and ϕ_2 are the mean heat flux through C_1D_1 and C_2D_2 respectively during dt . Equation (3) does not include heat which flows through A_1A_1' , B_1B_1' , A_2A_2' , and B_2B_2' , but this amount of heat is so small that it does not change the discussion, because the length is infinitesimally small, if dt is taken short enough. Now let the temperature of the oil film within one infinitesimal block be constant in the direction of y-axis during dt and the distribution of temperatures in the direction of the oil film thickness be expressed by

$$\theta(x, t) = \frac{\theta_2 - \theta_1}{h} x + \theta_1 + \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{h} \quad (4)$$

where θ_1 and θ_2 indicate the temperatures of surfaces C_1D_1 and C_2D_2 respectively at a time section $t=t$ (Fig. 8). The oil in the block $C_1D_1C_2D_2$ moves during dt to $C_1'D_1'D_2'C_2'$. Taking the coordinate system which moves with this movement, i.e., if the discussion is limited to within the block of oil $C_1D_1C_2D_2$ and its deformed $C_1'D_1'D_2'C_2'$, no fluid goes in and out of this block. The influence of inclination of x-axis is considered negligible during a sufficiently short time duration of dt . It is assumed that the temperature distribution in the oil film is decided in x-direction by thermal conduction, because there is no oil flow in this direction, and in y-

direction it is decided by mass temperature of a moving oil in which heat is generated, because the oil flow in this direction transfers much more heat than that of thermal conduction. In other words, thermal conduction between oil molecules is considered in x-direction, but neglected in y-direction. Neglecting the thermal conduction through the boundaries of oil blocks C_1C_2 and D_1D_2 , the state of thermal conduction in one block of oil film in the direction of x-axis is expressed by

$$c\rho \frac{\partial \theta}{\partial t} = \lambda \frac{\partial^2 \theta}{\partial x^2} + \Psi \quad (5)$$

where c , ρ , λ are specific heat, density, and thermal conductivity of the oil respectively, and Ψ is the heat generation rate in unit volume of the oil. When the rates of temperature rise of the surface at time section $t=t$ are given by

$$\left(\frac{\partial \theta}{\partial t} \right)_{x=0} = \delta_1, \quad \left(\frac{\partial \theta}{\partial t} \right)_{x=h} = \delta_2 \quad (6)$$

(a) State of oil film at a time section

(b) State of oil film after passage of infinitesimal time

Fig. 7 Model for the state of oil film

Fig. 8 Temperature distribution in a block in oil film

and also when ψ is not a function of x , the distribution of temperatures in the oil film block after the passage of time dt is given from Eq.(5) by

$$\theta(x, t+dt) = \frac{\theta_2 - \theta_1}{h}x + \theta_1 + \sum_{m=1}^{\infty} a_m \exp\left(-\frac{m^2 \pi^2}{h^2} \frac{\lambda}{c\rho} dt\right) \sin \frac{m\pi x}{h} + \left[\frac{\delta_2 - \delta_1}{h}x + \delta_1\right] dt + \frac{c\rho}{6\lambda h} \left\{ g(x) - \sum_{n=1}^{\infty} b_n \exp\left(-\frac{n^2 \pi^2}{h^2} \frac{\lambda}{c\rho} dt\right) \sin \frac{n\pi x}{h} \right\} \quad (7)$$

where

$$g(x) = (\delta_2 - \delta_1)x + 3\left(\delta_1 - \frac{\psi}{c\rho}\right)hx^2 - \left(2\delta_1 + \delta_2 - \frac{3\psi}{c\rho}\right)hx^3 \quad (8)$$

$$b_n = \frac{2}{h} \int_0^h g(\xi) \sin \frac{n\pi \xi}{h} d\xi \quad (9)$$

Putting the orders of power series which can well describe the temperature distribution of Eq.(7) as m^* and n^* , and considering that dt is short enough to fulfill the conditions

$$\left(\frac{m^* \pi}{h}\right)^2 \frac{\lambda}{c\rho} dt \ll 1, \quad \left(\frac{n^* \pi}{h}\right)^2 \frac{\lambda}{c\rho} dt \ll 1 \quad (10)$$

then from Eqs.(4) and (7) the temperature distribution is expressed by

$$\theta(x, t+dt) = \theta(x, t) - \sum_{m=1}^{m^*} a_m \left(\frac{m^2 \pi^2}{h^2} \frac{\lambda}{c\rho} dt\right) \sin \frac{m\pi x}{h} + \left[\frac{\delta_2 - \delta_1}{h}x + \delta_1\right] dt + \frac{c\rho}{6\lambda h} \sum_{n=1}^{n^*} b_n \left(\frac{n^2 \pi^2}{h^2} \frac{\lambda}{c\rho} dt\right) \sin \frac{n\pi x}{h} \quad (11)$$

The second term in the right side of Eq.(11) indicates a portion of temperature change brought about by such situation that the heat contained in the block of oil film at the time section $t=t$ is conducted in the x -direction during the passage of time dt . The third term indicates the influence of the change in the surface temperature of the body on the temperature distribution in the block of oil film. And the fourth term indicates a portion of temperature change brought about by such situation that the amount of heat generated in the oil film is conducted in the x -direction and given partly to the surfaces of the bodies. The amount of heat $\phi_1 dydt + \phi_2 dydt$, which is given during dt to the surfaces of the bodies through the block of oil film, is a difference between heat generated in this block of oil and heat spent to raise oil temperature, hence

$$\phi_1 dydt + \phi_2 dydt = \psi h dt - \int_0^h c\rho \{\theta(x, t+dt) - \theta(x, t)\} dx = \left(\psi - c\rho \frac{\delta_2 + \delta_1}{2}\right) h dt - \sum_{m=1}^{m^*} a_m \frac{m\pi}{h} \lambda dt (\cos m\pi - 1) + \sum_{n=1}^{n^*} b_n c\rho \frac{n\pi}{6h^2} (\cos n\pi - 1) \quad (12)$$

Considering the orders about ψ , δ_1 , δ_2 and h in each term on the right side of Eq.(12), the order of the first term is $[\psi h]$ because $\delta_1 + \delta_2$ is usually regarded as proportional to ψ . The order of the second term is $[1/h]$ because a_m is irrelevant to ψ , δ_1 , δ_2 and h as shown in Eq.(4). And the order of the third term is $[\psi h]$ because the order of b_n is $[\psi h^3]$ as seen from Eqs.(8) and (9). Although the amounts of heat $\phi_1 dydt$ and $\phi_2 dydt$ given to the surfaces 1 and 2 are not known, their orders are considered the same as the order of Eq.(12). Thus from Eqs.(3) and (12), the order of the amount of heat given to the lubricated surfaces of contacting bodies during time dt is

$$\text{order}[dQ_i] = \text{order}[\psi h] + \text{order}[1/h] \quad (13)$$

where $i=1, 2$

When the lubrication between mating surfaces is fully hydrodynamic and the shear stress in the oil film is expressed by $\eta \cdot du/dx$, supposing the distribution of velocities in the oil film to be approximately linear, the rate of heat generation in the oil film is given by

$$\psi = \eta \left(\frac{u_s}{h}\right)^2 \quad (14)$$

where

$u_s = u_1 - u_2$: sliding velocity
 u_1, u_2 : velocities of the surfaces 1, 2 respectively
 η : viscosity

When the frictional force F between mating surfaces is given by $F = \mu p$ (μ : coefficient of friction, p : applied pressure), the rate of heat generation is

$$\psi = \frac{\mu p u_s}{h} \quad (15)$$

Thus the order of ψh is considered to come between the order $[1]$ and the order $[\eta/h]$. On the other hand, the EHL theory shows that the oil film thickness h is proportional to about $\eta^{2/3}$, and then

$$\left. \begin{aligned} \text{order}[\eta/h] &= \text{order}[\eta^{1/3}] \\ \text{order}[1/h] &= \text{order}[\eta^{-2/3}] \end{aligned} \right\} \quad (16)$$

therefore

$$\text{order}[dQ_i] = \text{order}[\eta^\alpha] + \text{order}[\eta^{-2/3}] \quad (17)$$

where α is a constant between 0 and 3/11

After the passage of a short time dt , the temperature distribution at this new time is known. By assuming a new set of blocks of small width perpendicular to the surfaces in the oil film, the thermal state for the next time duration can be seen. As Eq.(17) is also valid for the order of heat amount given to the surfaces which is obtained by the successive procedure for each small duration of time, the order of heat flux $q = dQ_i/dt$ on the surface of the bodies is also equal to Eq.(17). Now, let θ be the representative temperature of the oil film, η_0 be the

viscosity of oil under atmospheric pressure and k_0 be a constant representing the property of oil. Then the viscosity is given approximately by

$$\eta = \eta_0 \exp(-k_0 \theta), \quad k_0 > 0 \quad (18)$$

Putting q for the heat flux from the oil film to the surfaces, it is expressed from Eqs.(17) and (18) by

$$q = q_a \exp\left(\frac{8}{11} k_0 \theta\right) + q_b \exp(-ak_0 \theta) \quad (19)$$

where

q_a, q_b : constant

3-2 Scoring model as thermal balance

Scoring failure is a phenomenon fixed on the contacting surface of body and not on the oil film. Using a function with a period T_n

$$I(t) = \begin{cases} 1, & (0 \leq t \leq T_c) \\ 0, & (T_c < t < T_n) \end{cases} \quad (20)$$

the supply heat flux q^* onto specific position of the contacting surface is given by

$$q^* = q I(t) \quad (21)$$

(Fig.9). Heat flux q_i dissipating from the definite position of the surface into atmosphere is well approximated by the heat flux radiated from the surface during the time interval $T_n - T_c$, in which the definite position of the surface is out of contact, because the non-contacting time duration $T_n - T_c$ is usually much longer than the contacting time duration T_c . Putting θ for the temperature of the surface and θ_0 for the temperature at an infinitely distant place (or the temperature of the oil in tank), dissipating heat flux is given by

$$q_i = \bar{K}(\theta - \theta_0) \quad (22)$$

where

$\bar{K}$: a constant which considers both the thermal radiation from the surface to the oil and the thermal conduction from the surface to the inside of the body

Now assuming the contacting bodies as a series of thermally equivalent masses as shown in Fig.10, and considering thermal balance about each equivalent mass, the following relation holds:

$$c_b \frac{d\theta}{dt} = q^* - q_i \quad (23)$$

Fig.9 Supply heat flux at a definite position of the surface as a function of driving time

where c_b is the heat capacity of a thermally equivalent mass, when the surface temperature θ is employed as a representative value. Taking the oil temperature at the entrance of the oil film for the representative value of the oil temperature which determines the state of the oil film, Eq.(23) becomes

$$c_b \frac{d\theta}{dt} = \left\{ q_a \exp\left(\frac{8}{11} k_0 \theta\right) + q_b \exp(-ak_0 \theta) \right\} \times I(t) - \bar{K}(\theta - \theta_0) \quad (24)$$

because the temperature at the oil in-let is very close to the surface temperature during no contacting situation ($T_c < t < T_n$).

By the way, it is impossible to measure the temperature of actually contacting surface : the temperature is measured at a point a little apart from the contacting surface. To relate the result of the theoretical analysis with the result of the measured tooth temperature in this case, the heat source of pulse form may be replaced by its time average value. Namely, employing $\bar{\theta}$ as the mean temperature of the contacting surface, the following equation holds :

$$c_b \frac{d\bar{\theta}}{dt} = \frac{T_c}{T_n} \left\{ q_a \exp\left(\frac{8}{11} k_0 \bar{\theta}\right) + q_b \exp(-ak_0 \bar{\theta}) \right\} - \bar{K}(\bar{\theta} - \theta_0) \quad (25)$$

It is questionable whether scoring failure could be produced on the surface of the contacting body by an instantaneous flash temperature which is generated by a moving heat source, or by thermal pulse for extremely short duration on contacting surface. If the mean temperature of the contacting surface plays a predominant role in occurrence of a scoring failure, Eq.(25) could explain the behavior of occurrence of scoring fairly well.

Fig.10 Model for the state of heat supply and heat dissipation in the contacting bodies

4. Behavior of Scoring Model and its Qualitative Description

Let Y_1 and Y_2 be two terms in Eq.(25) as

$$Y_1 = \frac{T_c}{T_n} \left\{ q_a \exp\left(\frac{8}{11} k_0 \bar{\theta}\right) + q_b \exp(-a k_0 \bar{\theta}) \right\} \quad (26)$$

$$Y_2 = \bar{K}(\bar{\theta} - \theta_0) \quad (27)$$

Y_1 indicates the quantity about heat supply and Y_2 indicates the quantity about heat dissipation. Whether the temperature of this system tends to diverge or saturate with progress of time is determined by the balance between these two terms. The relation between them is classified into two cases as shown schematically in Fig.11. In the case (A), as heat supply and heat dissipation become balanced when $\bar{\theta} = \bar{\theta}_{sat}$, the temperature will reach a steady state. On the contrary in the case (B), as heat supply is always larger than heat dissipation, the temperature will keep rising and will diverge, which leads to fulfill the condition for occurrence of a scoring failure.

Selecting $\bar{q}_a$ and k' properly, the state of change of Y_1 as a function of $\bar{\theta}$, Eq.(26), is well approximated by

$$Y_1 = \bar{q}_a \exp(k' \bar{\theta})$$

$$(k' > 0) \quad (28)$$

The behavior of surface temperature of contacting body up to the occurrence of scoring is expressed from Eq.(25) by

$$c_b \frac{d\bar{\theta}}{dt} = \bar{q}_a \exp(k' \bar{\theta}) - \bar{K}(\bar{\theta} - \theta_0) \quad (29)$$

and when the solution of Eq.(29) saturates and converges to a steady value, a scoring failure will not occur. When the solution diverges, a scoring will occur. It should be noted, however, that the scoring will occur even if the solution saturates, when the lubricating oil cannot stand its high temperature concerning its chemical property. Eq.(29) indicates that the factors which determine the occurrence of scoring are : ① $\bar{q}_a$ and k' which depend on the driving condition (load, sliding -, rolling velocity, coefficient of friction between contacting surfaces, and on-off ratio of the contacting surface) and the properties of the lubricating oil, ② c_b which depends on the material, geometrical shape of contacting body, const-

ruktion of the machine including the objective contacting body, and ③ $\bar{K}$ which is a function of the above mentioned arguments and also of the way of lubricating oil supply. Figure 12 shows an example of the driving time - temperature curves which are calculated by Eq.(29). It is recognized that there are two specific cases on the behavior of curves according to the relation between heat supply and heat dissipation; in one case, the temperature saturates to a steady value with passage of driving time, and in the other case, the curve has a point of inflection and the temperature rises suddenly. Comparing the state of the run of the driving time - temperature curve in this figure with that in Fig.2 which shows the run of actual tooth temperature as a function of driving time, they are recognized to be similar qualitatively. Equation (29), therefore, can explain the behavior of the temperature of sliding - rolling surfaces and the initiation and development of a scoring failure fairly well. It is obvious from this fact that whether or not a scoring occurs depends on whether the solution of Eq.(29) diverges or not.

Fig.11 Relation between heat supply (Y_1) and heat dissipation (Y_2)

Fig.12 Calculated example of driving time - temperature curves

5. Meaning of Critical Temperature for Scoring Resistance

As stated in the previous section, that the basic index for the behavior, indicating whether a scoring occurs or not is whether the driving time - temperature curve of Eq.(29) has a point of inflection or not. Putting θ_{crit} for the critical temperature which corresponds to the point of inflection of Eq.(29), θ_{crit} is obtained from $d^2\theta_{crit}/dt^2 = 0$ as

$$\theta_{crit} = \frac{1}{k} \ln \frac{\bar{K}}{k \bar{q}_a} \quad (30)$$

and it is a constant which is determined by the properties of the lubricating oil, method of lubrication, driving condition, material and configuration of the contacting bodies and the machine construction. The chained curve in Fig.12 is obtained by linking the points of inflection of driving time - temperature curves, and it shows the position of critical temperature for scoring resistance. Many experimental investigations about scoring failure until now have been made to find the critical temperature of lubricating oil against scoring. Chained curve in Fig.12, which shows the value of the critical temperature after some driving time has passed takes constant value, and the Eq.(30) indicate that the physical meaning of most investigations to find the critical temperature against scoring is in searching experimentally the limitation of thermal stability of sliding - rolling contacting surfaces, that is, in finding the point of inflection of driving time - temperature curve or finding the critical temperature of Eq.(29), i.e. the value of Eq.(30). Fig.12 shows, however, that the critical temperature θ_{crit} is low when the scoring occurs immediately after the driving is started. This means that the critical temperature against scoring which is determined by scoring test under such driving condition is somewhat lower than the one for the case when a scoring occurs after passage of a sufficiently long driving time.

In order to avoid a scoring failure even after a sufficiently long driving time, the temperature calculated from Eq.(29) at $t \rightarrow \infty$, that is the saturated temperature θ_{sat} , must be lower than θ_{crit} ,

$$\theta_{sat} < \theta_{crit} \quad (31)$$

Defining $G(\bar{\theta})$ from Eqs.(27) and (28) or the right side of Eq.(29) as

$$G(\bar{\theta}) = \bar{q}_a \exp(k\bar{\theta}) - \bar{K}(\bar{\theta} - \theta_0) \quad (32)$$

it is obvious from the explanation about Fig.11 that θ_{sat} is obtained as the solution of

$$G(\theta_{sat}) = 0 \quad (33)$$

On the other hand, θ_{crit} satisfies the relation

$$\left(\frac{dG}{d\bar{\theta}}\right)_{\bar{\theta}=\theta_{crit}} = 0 \quad (34)$$

and is equal to $\bar{\theta}$ when Eq.(32) takes the minimal value. Equation (32) decreases therefore monotonously in such region of $\bar{\theta} < \theta_{crit}$ (refer to Fig.11 (a)) and the

condition for no occurrence of scoring, Eq.(31), becomes

$$G(\theta_{crit}) < 0 \quad (35)$$

Thus in order to avoid scoring even after a sufficiently long time, the atmospheric oil temperature (for example the oil temperature in tank) θ_0 must be

$$\theta_0 < \frac{1}{k} \left(\ln \frac{\bar{K}}{k \bar{q}_a} - 1 \right) \quad (36)$$

namely θ_0 must be lower than a specific value determined by the properties of lubricating oil, the method of lubrication, driving condition, material and configuration of contacting bodies and the machine construction.

6. Conclusions

By summing up the results of tests about the behaviors of the change in gear tooth temperature measured in many cases when a scoring occurred and when did not occur, the occurrence of scoring is recognized as a problem of thermal stability on the boundary lubricated surfaces under sliding - rolling contact, and a model for this phenomenon is constructed. Strictly speaking, this model ignores some important factors on boundary lubrication, and assumes a concentrated mass system instead of an actual distributed mass system in calculating thermal conduction etc. In spite of such defects, however, the model can explain qualitatively well the state of temperature change close to the sliding - rolling contacting surfaces as a function of the driving time. The condition for occurrence of a scoring derived from this model agrees also with our empirical knowledge on the nature of scoring resistance. It is revealed theoretically that the physical meaning of many experimental investigations done until now to find the critical temperature of lubricating oil against scoring lies in finding the point of inflection of the driving time - temperature curve, that is, finding the limit of thermal stability problem expressed by the model mentioned in this paper.

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