Study on Welded Structure Gears*
(5th Report, Analyses of Root Stresses and Tooth Deflections
by 3-Dimensional Finite Element Method)

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The root stresses and tooth deflections of welded structure gears (thin rim) are computed by the 3-dimensional finite element method (FEM).

The effects of shapes of tooth, the partitions and constraint conditions of gear models on the root stresses and tooth deflections of a thin-rimmed gear with ribs and a web are discussed. A suitable gear model and constraint condition for the analyses of the root stresses and tooth deflections of welded gears by FEM are indicated. The root stresses and tooth deflections computed by FEM for the rack-shaped tooth model adopted in this report agree well with those measured. The root stresses and tooth deflections of a symmetric type of single web gear are clarified to a considerable extent.

Key Words : Gear, Welded Structure, Root Stresses, Tooth Deflections, Web, Rim, Rib, 3-Dimensional FEM

1. Introduction

In the previous papers^{(1)~(4)}, the weldability and the bending fatigue strength of rim materials and the effects of rim and web thicknesses on the root stress distributions and the bending fatigue strength of gear teeth were clarified to a considerable extent. The effects of web arrangements on the static root stresses were investigated by carrying out a static loading test. Furthermore the effects of web arrangements on the dynamic behavior of the welded structure gears were investigated by measuring the dynamic load and the vibration accelerations of gears under different running conditions.

It may be considered unreasonable to apply these results to gears with various dimensions and web arrangements because these results are those obtained for the gears with certain limited dimensions and web arrangements. It takes much cost and time to carry out the experiments on these

gears with various dimensions and web arrangements. It might be, therefore, reasonable to apply the 3-dimensional finite element method (FEM) to the analyses of the root stresses and tooth deflections including deformation of rim and web parts of thin-rimmed gears with various dimensions and web arrangements. It is desirable that the gear model for FEM analysis is effective and as simple as possible, since FEM analysis generally requires much labor and an enormous memory of the computer, when the form of the model is complex and the number of elements is large.

In the present paper, the possibility of simplifying the gear model was examined by computing the root stresses and tooth deflections including deformation of rim and web parts by FEM for a cylindrical shell model without tooth and gear models in which the teeth are approximated by a rectangle and a rack. The validity of the computing conditions in this FEM analysis was investigated by comparing the computed root stresses and tooth deflections with the measured ones.

Furthermore the root stresses and tooth deflections of a symmetric type of single web gear were clarified considerably.

2. Analysis by 3-dimensional finite element method

2.1 Computer program system for 3-dimensional analysis

The computer program used in this computation is SANAS-F, which was developed by Mitsubishi Heavy Industries Ltd. and

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is applicable to the static and dynamic analyses of structures. The special merits of SANAS-F are: (1) functions for analysis are plentiful, (2) analysis of structure with internal constraints and of compound members is possible. This program is of medium scale in itself, in which numbers of elements and nodes should be less than 1600 and 800 respectively. SANAS-F can be extended, however, to a multipurpose program with various functions by connecting modules, which have their own functions.

2.2 Gear models

A 3-dimensional finite element analysis was made on a symmetric type of single web gear with ribs. The main dimensions are module $m = 7$, pressure angle $\alpha_0 = 20^\circ$, number of teeth $z = 69$, face width $b = 100$ mm, rim thickness $l_w = 2m$ (m : module), web thickness $b_w = 12$ mm. Figure 1 shows the web arrangement and dimensions of a single web gear with ribs. The analyses were made on a gear model with three ribs and

Fig.1 Structure and dimensions of single web gear with ribs

Fig.3 Dimensions of rectangle tooth (G.Y)

Fig.4 Dimensions and mesh pattern of rack tooth (G.U)

Table 1 Numbers of elements and nodes

Gear model	Sign	Tooth part		Rim part		Web part		Rib part		Total	
		No. of nodes	No. of elements	No. of nodes	No. of elements	No. of nodes	No. of elements	No. of nodes	No. of elements	No. of nodes	No. of elements
Cylindrical shell model	G.X	—	—	264	240	120	192	165	120	458	552
Rectangle tooth model	G.Y	132	100							590	652
Rack tooth model	G.U										

two teeth (above rib and between ribs).

In order to determine a model suitable for the 3-dimensional finite element analysis, the root stresses and tooth deflections including deformation of rim and web parts were computed for a cylindrical shell model G.X ($l_w = 2m$) without tooth, a rectangle tooth model G.Y in which a gear tooth is approximated by a rectangle whose thickness is equal to the root thickness of a rack of $m = 7$, $\alpha_0 = 20^\circ$ and a rack tooth model G.U in which the gear tooth is approximated by a rack. The gear models are divided into 24 (15°) meshes on the circumference and 10 meshes in the face width direction. The numbers of elements and nodes for each gear model are shown in Table 1. Figure 2 shows a mesh pattern of the gear model. Figures 3 and 4 show the shapes and dimensions of rectangle and rack teeth respectively. A rack tooth is approximated by four rectangles inscribed inside it as shown in Fig.4 and it consists of 50 elements and 66 nodes.

Fig.2 Mesh pattern of gear model

2.3 Partition of gear model and constraint conditions

The gear models (G.X, G.Y, G.U) are mainly divided into 24 (15°) meshes on the circumference and 10 meshes in the face width direction and only the nodes of inside circumference of web are fixed. In order to examine the effects of partition of gear model and constraint condition on

Fig.5 Constraint condition for G.U- 15° -10(20)

Fig.6 Positions of nodes for G.U and G.U- 5° -10

(a) G.X

(b) G.Y, G.U

Fig.7 Directions of load and deflection

the accuracy of the computed results, the root stresses and tooth deflections are computed for the gear models G.U- 5° -10 and G.U- 15° -10(20) (Fig.5) and the computed results are compared with the results for G.U (Fig.2). G.U- 5° -10 is divided into 72 (5°) meshes on the circumference and 10 meshes in the face width direction and only the nodes of inside circumference of web are fixed. G.U- 15° -10(20) is divided into 72 (5°) meshes on the circumference and 10(20) meshes in the face width direction and the nodes of both ends of rim are fixed as shown in Fig.5. Figure 6 shows the positions of nodes for G.U and G.U- 5° -10.

2.4 Computation of root stresses and tooth deflections due to concentrated load

The root stresses and tooth deflections including deformation of rim and web parts were computed for a concentrated normal tooth load $P = 10$ kN which was applied to eleven points along the tip of a tooth one after another as shown in Fig.4. Figure 7 shows the loading positions and the directions of load and deflections for each model. The root stresses σ_c on compressive side were computed for the elements at E-zone 4 in Fig.4 and the deflection δ_a in the direction of the line of action at the loading point in Fig.7 is taken as the tooth deflection.

2.5 Computation of root stress distribution in longitudinal direction due to distributed load

The load distribution on a line of contact of gear tooth can be computed by the following Eq.(1)⁽⁵⁾ (see Fig.8).

$$\int_0^b K(x, \xi) p(\xi) d\xi = w(x) \quad (1)$$

where $p(\xi)$: distributed load, $K(x, \xi)$: deflection at x when a unit load is applied to ξ on the line of contact, $w(x)$: deflection of gear tooth, b : face width.

Consequently the distributed load p_j ($j = 1, 2, \dots, 11$) is obtained by solving Eq.(2).

Fig.8 Relation between load and deflection

stresses and tooth deflections.

4. Computed and experimental results and discussions

4.1 Partition of gear model, constraint conditions and accuracy of computed results

Figure 11 shows the root stresses on compressive side and the deflections in the direction of line of action of a gear tooth above rib computed for G.U and G.U-5°-10 for a concentrated normal tip load $P = 10$ kN. Arrows in this figure indicate the positions of load application and the computed root stresses are ones for the element at E-zone 4 in Fig.4. It is seen from this figure that the root stresses and the deflections computed for G.U agree well with those for G.U-5°-10, and the differences between the cases of G.U and G.U-5°-10 are almost zero in the root

(a) Root stress

(b) Deflection

1 MPa = 0.102 kgf/mm²
1 kN = 102 kgf

Fig.12 Effects of element mesh in face width direction and constraint condition on accuracy of computation

stresses and about 10% in the deflections. The load and the root stress distributions in tooth trace direction obtained from the results of Fig.11 for G.U also agree well with those obtained for G.U-5°-10, and the differences between the cases of G.U and G.U-5°-10 are within 5% in both the load and the root stress distributions. Hence it might be considered sufficient for the analyses of root stresses and tooth deflections of a thin-rimmed gear by 3-dimensional finite element method to use the gear model with 24 (15°) meshes on the circumference.

Figure 12 shows the root stresses on compressive side and the deflections in the direction of line of action of a gear tooth above rib computed for G.U and G.U-15°-10(20). Figure 13 shows the load and the root stress distributions obtained from the results of Fig.12. The computed results for G.U-15°-10 agree well with those for G.U-15°-20 as seen in Figs.12, 13. It might be considered sufficient for the thin-rimmed gear used in this investigation to use the gear model with 10 meshes in the face width direction. The deflections computed for G.U-15°-10(20) differ appreciably from those for G.U, which agree well with the measured ones as shown in Fig.19. Hence the nodes on the inner circumference of the web disk of the gear model are fixed in the present analyses with 3-dimensional finite element method.

The partitions of elements (24 (15°) meshes on the circumference, 10 meshes in face width direction) and the constraint condition (nodes of inner circumference of

Fig.13 Load and root stress distributions of G.U and G.U-15°-10(20)

Fig.14 Computed deflection for G.X

web disk:fixed) adopted in this investigation might be considered valid.

4.2 Root stresses and tooth deflections due to concentrated load

Figure 14 shows the deflections of the gear tooth between ribs computed for G.X. Figures 15 and 16 show the root stress on compressive side and the tooth deflections computed for G.Y and G.U for a concentrated normal tip load $P = 10$ kN. Figure 17 shows the deflections of gear tooth above rib computed for the gear models G.X, G.Y and G.U. Since the computed tooth deflections of G.X are considerably smaller than those of G.U and show tendencies different from the case of G.U, which agree well with the measured ones as shown in Fig.19, the cylindrical shell model G.X is considered unsuitable for the 3-dimensional finite element analysis. The root stresses and tooth deflections near the loading point of G.Y are considerably smaller than those of G.U and the differences between these corresponding values become smaller as the distance from the loading point increases. This might be because the thickness variation in tooth height direction differs between G.Y and G.U. Since the root stresses and deflections of the teeth above rib and between ribs are almost equal, the root stress and

(a) Root stress

(b) Deflection

1 MPa = 0.102 kgf/mm²
1 kN = 102 kgf

Fig.15 Computed root stress and deflection for G.Y

tooth deflection analyses for either of the teeth above rib and between ribs are considered to be satisfactory. Figures 18 and 19 show the computed root stresses on compressive side and deflections of the tooth between ribs of G.U as compared with the measured ones of actual gear for a concentrated normal tip load $P = 10$ kN.

(a) Root stress

(b) Deflection

1 MPa = 0.102 kgf/mm²
1 kN = 102 kgf

Fig.16 Computed root stress and deflection for G.U

1 kN = 102 kgf

Fig.17 Computed deflections for each model

The measured root stresses and deflections are corrected considering the differences in the load value and loading point between the computation and the measurement (see Fig.10). It is seen from these figures that the computed results agree well with the measured ones and the computing conditions used in this investigation are considered valid for the analyses of root stresses and tooth deflections of a thin-rimmed gear by 3-dimensional finite element method. Since the computed results

for the rack tooth model agree fairly well with the measured ones, this gear model is considered sufficient for the analyses of welded gears.

4.3 Root stress distribution in longitudinal direction due to distributed load

Figure 20(a) shows the load distributions along the line of contact at the tip of the tooth computed by Eq.(2) for G.X, G.Y and G.U. In Eq.(2) δ_i ($i=1, 2, \dots, 11$) is taken as unity for the case where the deflection is considered equal along the tooth trace under normal distribution of loads. Figure 20(b) shows the root stress distributions on compressive side computed by Eq.(3) for the load distributions shown in Fig.20(a). The stress values for G.X are the circumferential ones on the rim surface. Since the load distribution for G.X differs from the cases of G.Y and G.U as shown in Fig.20(a) and the root stress distribution of G.X is considerably smaller than those of G.Y and G.U as shown in Fig.20(b), the gear model G.X is found unsuitable for the analyses of root stresses and tooth deflections of a thin-rimmed gear by 3-dimensional finite element method. The load distribution of G.Y is almost the same as that of G.U and the root stress of G.Y is 30 per cent smaller than that of G.U. This might be because the thickness variation in tooth

(a) Above rib

(b) Between ribs

1 MPa = 0.102 kgf/mm²
1 kN = 102 kgf

Fig.18 Comparison of computed root stress (G.U) with measured one

1 kN = 102 kgf

Fig.19 Comparison of computed deflection (G.U) with measured one

(a) Load distribution

(b) Root stress distribution

1 MPa = 0.102 kgf/mm²
1 kN = 102 kgf

Fig.20 Load and root stress distributions for each model

height direction differs between G.Y and G.U.

5. Conclusions

The main results obtained from this investigation are summarized as follows.

(1) The gear model with 24 (15°) meshes on the circumference and 10 meshes in the face width direction is satisfactory for the 3-dimensional FEM analyses of the root stresses and tooth deflections of such a thin-rimmed gear as used in this experiment. As the constraint condition the inner circumference of the web disk of the gear model should be fixed.

(2) The computed root stresses and deflections for the rack tooth model agree well with the measured ones for the actual gear. The computed results for the rectangle tooth model are different from those for the rack tooth model.

(3) The computed root stresses and deflections for the cylindrical shell model are lower than those for the rack tooth model. Therefore the cylindrical shell model is unsuitable for 3-dimensional FEM

analysis of a thin-rimmed gear.

(4) The load and root stress distributions of a symmetric type of single web gear show the highest value at the middle of face width (just above the web).

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