Submarine Cable Kink Analysis

By Tetsuro YABUTA**

Cable kink is well recognized as a significant problem in use of submarine cables. Cable kink occurs when a cable loop forms due to torsion and tension action, and then tightens under subsequent tension increase. However, few investigations have been reported on this cable kink phenomenon. The relation between cable mechanical properties and cable kink has not been clarified.

This paper reports the results of studies on cable kink phenomenon using potential energy method. It was made clear that cable kink occurs when cable deformation changes from torsional deformation to bending deformation. Results also revealed that cable kink phenomenon is controlled by only four parameters; cable slack, cable twist, cable diameter and ratio of bending rigidity to torsional rigidity.

Key Words: Elasticity, Structural Analysis, Cable Kink Analysis

1. Introduction

Cable kinking is well recognized as a significant problem in the use of cables for oceanic applications[1]. In oceanic applications, a cable loop occurs because of torsional stress due to the helical strength member, when tension on the cable is temporarily reduced. When the cable is retensioned, the loop will decrease in diameter, will kink, and may cause the cable to be damaged.

Few investigations have been report- ed on this cable kinking phenomenon, and the relation between cable mechanical properties and cable kink has not been clarified. Previous analyses[2]~[3] using force equilibrium were done from a viewpoint that the cable loop would kink or reopen. These results only showed the tendency that cable kinking seldom occurs when the cable is easy to twist and hard to bend. However, these analyses could not clarify well the unstable phenomenon of cable kinking. Additionally, although there is also an example of study which uses the potential energy method to investigate whether or not cable loop forms under torsional torque[6], this phenomenon has not been clarified completely.

The author clarified theoretically that cable kink is easy to occur when residual cable twist is large, using the potential energy method[7]. This paper studies cable kink mechanism from the viewpoint of the cable slack by continuing the previous analysis.

Fig.1 Cable kink phenomenon

2. Cable Kink Phenomenon

Figure 2 shows a kinking phenomenon. When the tension is reduced, the cable begins to deform into a helical shape, as shown in Fig. 2(b), from twisted cable under tension, as shown in Fig. 2(a). As cable slack becomes large, helical deformation becomes large and cable loop shown in Fig. 2(c) forms due to rotation of helical shape. When cable loop is retensioned, cable loop decreases in diameter and kinks, as shown in Fig. 2(d). However, in the process of decreasing cable loop diameter, shown in Fig. 2(d), cable loop may reopen under some specific conditions.

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The coordinates used for the analysis are shown in Fig. 3. Cable is assumed to be helical as shown below:

\[ r = R, \quad Z = b \theta. \]  

Curvature \( \kappa \) and torsion \( \phi \) are obtained as follows:

\[ \kappa = R/(R^2 + b^2), \quad \phi = b/(R^2 + b^2). \]  

If \( k \) is one pitch length of the helix, the following relation is obtained, when \( R/k \ll 1 \):

\[ b = \frac{1}{2\pi} \left( 1 - \frac{1}{2} \frac{2}\pi \frac{R}{k} \right)^2. \]  

Using Eqs. (2) and (3), curvature \( \kappa \) and torsion \( \phi \) can be rewritten as

\[ \kappa = \frac{(2\pi)^2 R}{k^2}, \quad \phi = \frac{2\pi}{k} \left( 1 - \frac{1}{2} \frac{2\pi R}{k} \right)^2. \]  

Cable twist \( \tau \) is defined by the following formula, using Eq. (4):

\[ \tau = \Delta \tau_0 + \phi = \Delta \tau_0 + \frac{2\pi}{k} \int_0^\kappa \frac{2\pi R}{k} \frac{2\pi R}{k} \]  

where \( \Delta \tau_0 \) is initial twist, which torsion \( \phi \) in Eq. (4) cannot represent. From Eq. (5), initial twist \( \tau_0 \) can be expressed by

\[ \tau_0 = \lim_{k \to 0} \Delta \tau_0 + \frac{2\pi}{k}. \]  

In order to determine the stability of cable loop, potential energy will be obtained. Torsional strain energy \( V_T \) and bending strain energy \( V_B \) for one pitch of the cable helical deformation are defined by

\[ V_T = \frac{1}{2} GJ(\tau_0 - \frac{2\pi R}{k})^2 k, \]
\[ V_B = \frac{1}{2} EI(\frac{2\pi R}{k})^2 k, \]

where \( EI \) is bending rigidity and \( GJ \) is torsional rigidity. Using Eqs. (4), (5) and (6), Eq. (7) is rewritten by

\[ V_T = \frac{1}{2} GJ(\tau_0 - \frac{2\pi R}{k})^2 k, \]
\[ V_B = \frac{1}{2} EI(\frac{2\pi R}{k})^2 k. \]  

External work \( W \) will be obtained. Displacement \( \Delta k \) of the cable end from the condition in Fig. 2(a) to condition shown in Fig. 2(b) is defined as

\[ \Delta k = k - 2b = \frac{1}{2} \frac{2\pi R}{k}. \]  

Using Eq. (9), external work \( W \) is obtained by
\[ W = -P\Delta L = -P \left( \frac{2\pi R}{L} \right)^2 \]  
(10)

From Eqs. (8) and (10), potential energy \( U \) is given as follows:
\[
U = V_T + V_B - W = \frac{1}{2} GJ\left( \tau_0 - \frac{2\pi R}{L} \right)^2 + \frac{1}{2} E \left( \frac{2\pi R}{L} \right)^4 + \frac{P}{2} \left( \frac{2\pi R}{L} \right)^2 .
\]  
(11)

Cable deformation equilibrium is obtained by the following formula:
\[
\frac{3U}{3R} = \frac{1}{2} GJ\left( -2\frac{2\pi}{L} \tau_0 + \left( \frac{2\pi R}{L} \right)^2 \tau_0 + 2\alpha \left( \frac{2\pi R}{L} \right)^4 \right) + 2P \left( \frac{2\pi R}{L} \right)^2 = 0,
\]  
(12)

where \( \alpha = EI/GJ, P = P/GJ \).

Equilibrium stability is given by the following second differential calculus for potential energy:
\[
\frac{5U}{5R^2} = \frac{1}{2} GJ\left( -2\frac{2\pi}{L} \tau_0 + 3\left( \frac{2\pi R}{L} \right)^2 \tau_0 + 2\alpha \left( \frac{2\pi R}{L} \right)^4 \right) + 2P \left( \frac{2\pi R}{L} \right)^2 ,
\]  
(13)

where

\[
F = -2\left( \frac{2\pi}{L} \right) \tau_0 + 2\alpha \left( \frac{2\pi R}{L} \right)^4 + 2P \left( \frac{2\pi R}{L} \right)^2 .
\]  

(1) If \( F > 0 \), then \( R = 0 \) is an equilibrium whose stability is given as follows:
\[
\frac{3U}{3R} = \frac{1}{2} GJF > 0.
\]  
(14)

Equation (14) indicates that \( R = 0 \) is a stable equilibrium.
(11) If \( F < 0 \), then the equilibrium is given as follows from Eq. (12):

\[ R = 0 \text{ or } R = R_0 = \frac{\tau_0}{2\pi} \left( \frac{2\pi R}{L} \right)^{-1/2} . \]  
(15)

Equilibrium stability \( (R=0) \) is an unstable equilibrium because \( (\frac{3U}{3R})R=0 < 0 \) from Eq. (14). Equilibrium stability \( (R=R_0) \) is given by
\[
\frac{5U}{5R^2} = \frac{1}{2} GJF(-2P) > 0.
\]  
(16)

Equation (16) indicates that \( R = R_0 \) is a stable equilibrium. These results show that the condition, under which cable begins to take a helical deformation, is

\[ F = 0 \text{. Then, tension } P \text{ is given by the following relation:}
\[
P \leq GJ\tau_0 \left( \frac{2\pi}{L} \right)^2 - EI \left( \frac{2\pi R}{L} \right)^2 = P_0 .
\]  
(17)

Since cable begins to take a helical shape when tension \( P \) decreases, the condition wherein cable begins to deform by the maximum tension, agrees with an actual case. The condition which gives \( (P_0)_{\text{MAX}} \) is obtained by
\[
\varepsilon = 4\pi a/\tau_0 .
\]  
(18)

From Eq. (18), \( (P_0)_{\text{MAX}} \) is given by
\[
(P_0)_{\text{MAX}} = GJ\tau_0 2/4a .
\]  
(19)

When \( P \leq (P_0)_{\text{MAX}} \), cable begins to take a helical deformation, whose pitch length \( \lambda \) is given by Eq. (18).

The following formula is defined as an initial cable twist:
\[
\tau_0 = 2\pi L .
\]  
(20)

Equation (20) indicates that cable is twisted in one turn at cable length \( L \). Substitution of Eq. (18) into Eq. (20) gives the following relation.
\[
\lambda = 2aL .
\]  
(21)

When cable is homogeneous and elastic, \( \varepsilon = 2L \) because \( a = 1+\varepsilon \). This result means that the initial cable deformation begins in the pitch of cable length corresponding to two turn cable twists. When \( R \) is nearly equal to zero, the relation between \( \Delta \) and \( R \) is given from Eq. (9):
\[
R = \left( \frac{2\pi \Delta}{\lambda L} \right)^{1/2} = \left( \frac{al_0 \Delta}{\varepsilon^2} \right)^{1/2} .
\]  
(22)

where \( \Delta \) is a displacement corresponding to one pitch \( \lambda \).

3.2 Cable loop formation and reopening

This section deals with both cable loop formation shown in Fig. 2(a), and cable loop reopening, shown in Fig. 2(e). The model used for the analysis is shown in Fig. 4.

![Fig. 4 Analysis model for both formation and reopen of cable loop](image)
The model indicates that the cross point of the cable loop shifts a little in the Z direction, since it is considered that cable loop begins to form and begins to reopen.

Cable deformation is assumed to be the same helical deformation[7], as in Sec. 2.1. In this analysis, nondimensional parameter \( \lambda(\delta \equiv \pi) \) is introduced:

\[
r = R \begin{cases} 
Z = RA \phi, & 0 \leq \phi \leq \pi, \\
Z = 0, & -\pi/2 \leq \phi \leq 0 
\end{cases}
\]

Then, bending strain energy \( V_B \), and torsional strain energy \( V_T \) stored in the cable loop and external work \( W \) are represented as follows[7], using Eqs. (2) and (7):

\[
V_B = \frac{1}{2} \frac{\pi \lambda^2}{R} (1 + \lambda^2)^{-3/2} + \frac{1}{2} \frac{\pi \lambda^2}{R} 
\]

\[
V_T = \frac{1}{2} \frac{\pi G J R}{R} \left( \frac{1}{1 + \lambda^2} \right)^2 (1 + \lambda^2)^{1/2} 
\]

\[
W = -2\pi n P + \pi \lambda^2 R \left( 1 + \lambda^2 \right)^{1/2} 
\]

Using Eq. (24), potential energy \( U \) is given by

\[
U = V_B + V_T - W 
\]

Equilibrium state is obtained by

\[
\frac{\partial U}{\partial R} = 0, \quad \frac{\partial U}{\partial \lambda} = 0 
\]

Equation (26) is expressed as follows considering small terms up to the second order when \( \lambda \ll 1 \):

\[
\frac{\partial U}{\partial \lambda} = -\frac{3\pi \lambda^2}{2R} + \frac{\pi G J}{R} 
\]

\[
+ \frac{\pi G J R}{2} \left( 1 - \frac{3\lambda^2}{4} \right) - 2P R a = 0 
\]

\[
\frac{\partial U}{\partial R} = \frac{\pi \lambda^2}{R} \left( 1 - \frac{3\lambda^2}{4} \right) - \frac{\pi G J \lambda^2}{2R} 
\]

\[
+ \pi G J R \left( 1 + \frac{1}{4} \lambda^2 \right) + 2\pi P = 0 
\]

The following relation is obtained from Eq. (28) in consideration of \( \lambda \ll 1 \):

\[
R = R_0 \left( \frac{2P + G J R^2}{2 \pi G J R^2} \right)^{1/2} 
\]

Substituting Eq. (29) into Eq. (27), the following expression is obtained:

\[
\lambda = \lambda_0 = \frac{-\frac{\pi G J}{3 \pi E I} - \frac{2 P R a}{2 \pi G J R^2}}{2 \pi G J R^2} 
\]

When cable is homogeneous and elastic, \( \lambda_0 \) is positive because the denominator is negative[7].

Stability of the equilibrium \( (\lambda = \lambda_0) \) is obtained by the following formula:

\[
\frac{\partial U}{\partial \lambda} = \frac{3\pi \lambda^2}{2R} + \frac{\pi G J}{R} 
\]

\[
\frac{\partial U}{\partial R} = \frac{\pi \lambda^2}{R} \left( 1 - \frac{3\lambda^2}{4} \right) - \frac{\pi G J \lambda^2}{2R} 
\]

\[
- \frac{3}{2} \pi G J R \lambda_0 - 2 P R a < 0 
\]

Since Eq. (31) indicates \( \lambda_0 > 0 \) is an unstable equilibrium. Although this analysis indicates that there is no stable equilibrium in the neighborhood of \( \lambda_0 \), there is a stable equilibrium wherein cable loop rotates at angle \( \phi_0 \) from the state (\( \delta = 0 \)) shown in Fig. 4.[7] The potential energy is shown schematically in Fig. 6, which indicates that there exists a stable equilibrium for a cable loop rotating at angle \( \phi_0 \). When cable is re-tensioned, cable rotating angle gradually decreases to zero and cable loop diameter also decreases. This phenomenon is in accord with Fig. 2(d). These theoretical results indicate that cable loop does not reopen and will kink if cable loop forms.

![Fig. 5 Rotating phenomenon of cable loop](image_url)

However, some experimental results, as described in Sec. 4, show that cable loop reopens when the cable loop diameter decreases due to external force. This is why there exists an initial displacement \( \lambda_I \) because of cable diameter \( d \) in the region where cable loop does not open:

\[
\lambda_I = d/R_0 
\]

As shown in Fig. 6, cable loop reopens if initial displacement \( \lambda_I \) is larger than an
unstable equilibrium \((\lambda_0)\) whose relation is given by
\[
\lambda_1 \geq \lambda_0.
\] (33)
Equation (33) is rewritten by
\[
R_0 \leq \left( \frac{3}{2} - 1 - \alpha \right) \frac{d}{tr} \left( \frac{d}{\alpha} \right)^{1/2}
\] (34)
where \(\alpha = EI/GJ\).
This result means that cable loop reopens when cable loop radius is less than the value obtained by Eq. (34).

Theoretical study clarifies a part of cable kink phenomenon. Moreover, this paper reports the results of studies on the whole part of cable kink phenomenon by additional experimental investigation. Theoretical results were also examined by comparison of experimental results.

Fig. 6 Change in potential energy

4. Experimental Investigation

Jacketed optical fibers shown in Fig. 7 were used for experimental investigation. The reason for test samples selection was as follows; it is difficult to carry out an experiment on actual submarine cable because of its large diameter, which needs an equally large testing apparatus. However, it is easy to use optical fiber in carrying out an experiment, because of its small diameter and easy handling. Additionally, experimental results obtained for optical fibers are easy to compare with theoretical values based on the elastic theory, because optical fiber deformation is elastic in large deformations.

4.1 Experimental method

Test samples shown in Fig. 7 were tested by the experimental method shown in Fig. 8. In the experiment, cable end displacement \(dL\) which is in accord with cable slack, cable deformation, applied tension and torsional torque, was measured after initial twist was given to the test sample. Measurements were conducted in detail, especially when cable loop formed and reopened.

4.2 Experimental results

Figure 9 shows the relation between cable end displacement \(dL\), torque \(T\) and applied tension \(P\). Cable end displacement \(dL\) corresponds to cable slack during laying and recovery shown in Fig. 1. (b), (c), (d), (e) in Fig. 9 are in accord with cable deformations (b), (c), (d), (e) in Fig. 2. That is, (b) is the growth of initial helical deformation, (c) is a cable loop formation, (d) is a cable loop diameter decreasing region under retention and (e) is cable loop reopening. As shown in Fig. 9, there happens a rapid change in torque when cable loop forms (b) region and when cable loop reopens (a) region. These results indicate that cable deformation suddenly transfers to another stable equilibrium as shown in Sec. 3. Torque gradually decreases in the growth process of initial deformation (b) and torque does not change when cable diameter decreases.

Fig. 7 Test sample

Fig. 8 Schematic figure of experimental method

Fig. 9 Torque and tension change due to cable slack
Figure 9 also shows that applied tension is almost zero in the growth process of initial deformation (b). However, applied tension increases when cable diameter decreases.

Figure 10 shows the relation between cable slack and the growth of helical deformation. The helical assumption model is valid for the analysis of initial helical deformation, since experimental results are in accord with calculated values obtained by Eq. (22):

\[ \tau_T = \alpha - 2s/L_0. \]  

(36)

Comparison between measured values and calculated values indicates validity of calculation. As shown in Eq. (36), residual twist \( \tau_T \) is almost the same as initial twist \( \tau_0 \) when \( L_0 \) is very long. Then, Eq. (35) is rewritten using Eq. (29):

\[ \Delta L = a^{1/2} L = L. \]  

(37)

Equation (37) shows that the necessary slack \( \Delta L \) for cable loop formation is almost the same as cable length \( L \) corresponding to one turn twist.

Figure 11 shows the relation between cable slack \( \Delta L \) and cable initial twist \( \tau_0 \) in the process of cable loop formation. As this figure shows, the larger the initial twist is, the smaller the cable slack which makes the cable loop. Calculated values are obtained by the following formula using Eq. (29). In this calculation, tension \( P \) is zero because the experimental result shown in Fig. 9 is zero:

\[ \Delta L = 2\pi R = 2\pi a^{1/2}/\tau_T, \]  

(35)

where \( \tau_T \) is residual twist in the cable loop.

Calculation uses the following relation, assuming that a decrease in cable twist during cable loop formation spreads out over the whole cable length \( L_0 \):

\[ \Delta L = a^{1/2} L = L. \]  

(37)

Although calculated values are almost the same as measured values, calculated values are larger than measured values. This error is assumed to be caused by a difference between the actual cable loop, which is not exactly a circle shown in Fig. 2, and the cable loop assumption for analysis that the loop is a circle. Figure 12 also shows that cable loop reopens in a large cable loop diameter if \( d/\tau_T \) is large, which indicates that cable diameter \( d \) is large and residual twist \( \tau_T \) is small. Reopen diameter of cable loop decreases with a decrease of \( d/\tau_T \).

Fig. 12 Relation between cable slack and ratio of cable diameter to residual twist when cable loop reopens.
In actual cable kink phenomenon, residual twist \( \tau_0 \) is almost the same as initial twist \( \tau_0 \), because cable twist relaxation is very small during cable loop formation, since cable length \( L_0 \) is very large.

5. Discussion

This section reports results of an investigation on cable kink phenomenon using both theoretical results in Sec. 3 and experimental results in Sec. 4.

Figure 13 shows a diagram of the cable kink phenomenon. The horizontal axis of Fig. 13 which arranges cable kink deformations indicates cable slack \( \Delta L \). Cable deformation can be measured by cable slack, whose turning points of deformation are given by

\[
\Delta L_1 = a^{1/2}L, \\
\Delta L_2 = \beta(dL)^{1/2},
\]

(39)

where \( \beta = \frac{2(\alpha + \frac{1}{3})a - 1)}{2} \), and \( L \) is the cable length corresponding to one turn twist.

Equation (39) shows that cable kink phenomenon is controlled by only three parameters, which are cable initial twist \( \tau_0 \) (or cable length \( L \) corresponding to one turn of initial twist), ratio \( a \) of bending rigidity to torsional rigidity, and cable diameter.

Classification of cable deformation by cable slack is done as follows:

i) When \( \Delta L < a^{1/2}L \), cable deformation is a helical deformation and becomes straight when cable slack becomes small, due to an increase in tension.

ii) When \( \Delta L = a^{1/2}L \), cable loop forms and the equilibrium transfers to a new stable equilibrium, as shown in Fig. 5.

iii) When \( \Delta L > a^{1/2}L \), cable loop rotates, as shown in Fig. 6, after cable loop formation. Cable loop does not reopen at \( \Delta L = a^{1/2}L \), when cable slack \( \Delta L \) decreases again. In this case, cable loop deformation is classified by the following slack.

iv) When \( \Delta L > \beta(dL)^{1/2} \), cable loop decreases in diameter and does not reopen during decrease in cable slack \( \Delta L \).

v) When \( \Delta L < \beta(dL)^{1/2} \), cable loop reopens and cable deformation becomes straight or takes a helical shape.

As this figure shows, cable loop reopens if cable is a homogeneous elastic body. However, there are some actual cases where cable loop does not reopen. The reason is as follows: in an actual case, cable deformation gives rise to plastic deformation and bending rigidity decreases very much in a small diameter of cable loop. As parameter \( a \) becomes small, because of bending rigidity decrease, opening radius for the cable loop becomes small, as shown in Eq. (24), and cable kink occurs due to decreasing cable loop diameter.

These results indicate that cable loop formation has to be eliminated in order to control cable kink. Therefore, the relation \( \Delta L < a^{1/2}L \) has to be kept between parameters of cable slack \( \Delta L \), cable mechanical parameter \( a \), and cable length \( L \) representing cable twist. Actually, the following factors apply work:

(a) Parameter \( a \) becomes large; that is, bending rigidity becomes large and torsional rigidity becomes small. (b) Cable initial twist becomes small (or cable length corresponding to one turn twist becomes large). (c) Cable slack becomes small during laying.

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![Fig. 13 Schematic figure of cable kink phenomenon](image-url)
6. Conclusion

Cable kink phenomenon was clarified from the viewpoint of cable slack by both theoretical analysis using energy method and experimental results. These results present a quantitative guide to preventing cable kink, although there were only qualitative results present in the previous work.

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(References)