Vortex Shedding from a Two-Dimensional Blunt Trailing Edge
with Unequal External Free-Stream Velocities*

By Hisataka TAMURA**, Masaru KIYA***
and Mikio ARIE****

The frequency of vortex shedding from a two-dimensional blunt trailing edge of a plate was experimentally investigated when external free-stream velocities on both sides are not equal. Reynolds number $Re$ (based on the thickness of the trailing edge and the mean velocity) ranged from 190 to 3,000 and the low-to-high velocity ratio $K$ was varied from 0 to 1.0. In the range $K \geq 0.78$, the relation between Strouhal number and Reynolds number was almost the same as that for the case where the ratio is unity. The level of Strouhal number was higher for $K < 0.78$ than for $K > 0.78$. The vortex-shedding patterns can be classified into four modes which are clarified in the $Re - K$ domain. Moreover, the critical Reynolds number beyond which the vortex shedding from the blunt trailing edge occurs were obtained as a function of $Re$ and $K$.

Key Words: Vortex Shedding, Trailing Edge, Strouhal Number, Wake, Unsteady Flow

1. Introduction

Most of vibrations and noises which occur in fluid machines are induced by velocity fluctuations around machine elements such as blades of turbomachines, an exhaust of jet engines, a combustion chamber, etc. The frequency of these vibrations and noises well coincides with the frequency of vortex shedding from a trailing edge, the noise level due to the vortex shedding being often higher than that due to other sources. The turbo-fan engines are a good example where the noise reduction is successfully achieved. The noise level of the turbo-fan jet engines is reduced by the control of the vortex shedding frequency near the exit of the nozzle, i.e. by changing the velocities of the main jet and the bypass air jet.

A flow in the mixing region of the main jet and the bypass jet is in nature the same as one behind a blunt trailing edge with unequal free-stream velocities. There are a few papers dedicated to studies in this category. Olsen et al. (1) and Olsen-Karchmer (2) experimentally studied the level of the lip noise near a conamnular nozzle and a circular nozzle with a splitter plate, respectively. Boldman et al. (3) numerically calculated the process of formation of vortices behind a blunt trailing edge by a discrete-vortex method, together with some comparison of the calculated results with experiments. These studies are performed in a limited range of Reynolds numbers, so that the dependence of the vortex-shedding properties on Reynolds number is not well understood.

The present paper describes results of an experimental study which was made to clarify the properties of vortex shedding from the blunt trailing edge with unequal free-stream velocities in a Reynolds number range $190 \leq 3,000$.

2. Nomenclature

$f$ : predominant frequency of velocity fluctuation in wake
$f_1$ : lower frequency of two spectrum-peak frequencies in low-velocity side
$f_2$ : higher frequency of two spectrum-peak frequencies
$H$ : thickness of plate
$K$ : velocity ratio $= U_2/U_1$
$Re$ : Reynolds number $= UH/\nu$
$St$ : Strouhal number $= fH/U$
$\bar{u}$ : local time-mean velocity
$U$ : longitudinal velocity fluctuation
$U'$ : average velocity $= (U_1^2 + U_2^2)^{1/2}$
$U_1$ : higher free-stream velocity
$U_2$ : lower free-stream velocity
$\sigma_x, \sigma_y, \sigma_z$ : Cartesian coordinates (see Fig.2)
$\delta, \delta^*, \theta$ : boundary layer thickness, displacement thickness and momentum thickness, respectively, at trailing edge
$\nu$ : kinematic viscosity
$\rho$ : density

subscripts
1 : high-velocity side
2 : low-velocity side

3. Experimental apparatus and procedure

Experiments were performed in a recirculating water channel whose test section
was of a rectangular shape with dimensions of 30cm in width, 15cm in depth and 1.4m in length. Most parts of the channel were constructed of transparent acrylic-resin plates. A blunt trailing-edge plate tested in this experiment was 3.0cm thick and 50cm long, as shown in Fig. 1. The center of the plate was set to coincide with the center of the channel. The leading edge of the plate was so shaped that an approaching flow would be divided smoothly on both sides of the front stagnation point. Moreover, in order to make the boundary-layer thickness at the trailing edge as small as possible, spanwise slits of 4mm in width were provided on both sides of the plate at the same distance upstream of the edge. Water sucked through the slits was returned to a downstream reservoir of the water channel by a small pump to keep the surface level constant. Unequal free-stream velocities on both sides of the plate were produced by installing rectangular block of polyvinylchloride honeycomb with hexagonal cells (dimension of the cells = 3mm) of length 5cm and 11cm and several polyvinylchloride gauges (15 mesh) in the test section as seen in Fig. 1.

The time-mean longitudinal velocity in the mid-depth plane (z = 0), where observations were made, was measured by a hydrogen-bubble technique or a propeller velocimeter of a small diameter 3mm. The two-dimensionality of the free-streams was fairly good within a region z = 25mm (0.67H). Fluctuating component of the longitudinal velocity was detected by platinum hot-film sensor coated with quartz (51µ in diameter) whose output was processed by a digital signal processor to obtain the r.m.s. value and power spectrum of the velocity fluctuation.

All experiments were performed by adjusting the suction-flow rate so as to have a minimum boundary layer thickness just before the trailing edge with the aid of the hydrogen-bubble technique. The time-mean higher free-stream velocity \( U_1 \) was varied in a range 0.45 \( \sim \) 11cm/s. The ratio of the lower free-stream velocity to the higher free-stream velocity \( K \) was within the range of 0 \( \sim \) 1.0.

The frequency of vortex shedding from the trailing edge of the plate was detected by a hot-film probe which was mounted in the vortex formation region of the wake. The vortex shedding frequency was determined on the basis of the power spectra of the velocity fluctuations in the wake. Positions of detection of the velocity fluctuation (which were somewhat dependent on Reynolds number and the velocity ratio) were determined by observing visualized flow patterns in the wake: they were in the range of \( z/H \) = 1.17 \( \sim \) 5.5 and \( y/H \) = 0.5 \( \sim \) 1.67. In order to avoid errors in the determination of the vortex-shedding frequency for small velocity ratios, the measurements were not made in regions where a pairing or a merging of vortices occurred.

The definition sketch of the flow and main symbols are shown in Fig. 2. The origin

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Fig. 1 Test plate and its installation in the water channel.

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Fig. 2 Co-ordinate system and definition of symbols.

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Fig. 3 Distributions of time-mean velocities and turbulence intensities just before the blunt trailing edge (\( z/H = -0.5 \)).
of the co-ordinate system \((x', y', s')\)

is located at the middle of the rear face of the plate of thickness \(H = 55m\). The \(x'\) axis is in the downstream direction, the \(y'\) axis is in the vertical direction and the \(z'\) axis is normal to the \(x'\) and \(y'\) axes so as to form a right-handed co-ordinate system. Reynolds number \(Re\) based on the thickness of the plate and the mean velocity \(V\) ranged from 190 to 3,000.

4. Results and discussion

Parameters which describe the properties of the vortex-shedding from the blunt trailing edge are Reynolds number \(Re\), the velocity ratio \(K\), the boundary-layer characteristics at the trailing edge and the free-stream turbulence intensity \((\varepsilon_{\infty}^{1/2})/U\). It was difficult to control all the four parameters systematically, so that it was intended in the present experiments to keep the boundary layer thickness as small as possible by the boundary-layer suction. Effects of the free-stream turbulence on the vortex-shedding properties were not examined although we expect that the effects exist.

Examples of the velocity and turbulence-intensity profiles just before the trailing edge \((z'/H = -0.5)\) are shown in Fig. 3. Throughout the present experiments, the non-dimensional boundary-layer thickness \(\delta_1'/H\) and \(\delta_2'/H\) were in the range 0.02 \(\sim\) 0.03 and the turbulent levels \((\varepsilon_{\infty}^{1/2})/U\) and \((\varepsilon_{z}^{1/2})/U\) in the range 0.6 \(\sim\) 0.92.

Figure 4(a) shows the Strouhal number \(St\) of vortex-shedding as a function of Reynolds number for the case of \(K = 1.0\). Critical Reynolds number \(Re_c\) at which the vortex-shedding was first observed was about 290. Fanning and Mueller's\(^{10}\) numerical study on the wake behind the blunt trailing edge shows that the vortex-shedding occurs when \(Re \geq 300\) but not at \(Re = 100\). At Reynolds numbers below critical, a pair of twin vortices were observed behind the trailing edge by the flow visualization. Strouhal number increases with increasing \(Re\) until it attains a maximum between 0.275 and 0.31 at \(Re \leq 1,000\) and then decreases with \(Re\). Moreover, Strouhal number obtained in this experiment seems to join smoothly to the value 0.2 \(\sim\) 0.24 at Reynolds number of the order of \(10^9\). Boundary-layer properties at the trailing edge in these studies \((2) \sim (7)\) are summarized in Table 1.

The \(St-Re\) curves for velocity ratio \(K = 0.9\) and 0.78 are almost the same as that for

![Graphs showing Strouhal number vs. Reynolds number for different values of K.](image)

**Fig. 4** Strouhal number \(St\) plotted against Reynolds number \(Re\) for various velocity ratios \(K\).

- \(\times\): present study (high-velocity side).
- \(\diamond\): present study (low-velocity side).
- \(\ominus\): uncertainty in Strouhal number on high- and low-velocity sides, respectively.
- \(\bigcirc\): present result for \(K = 1.0\), \(\square\): Shirahama-Toyoda \((5)\), \(\triangle\): Boldman et al. \((3)\), \(\bigtriangledown\): Fanning-Mueller \((4)\), \(\triangledown\): Olsen-Karchmer \((2)\), \(\bullet\): Bearman \((6)\), \(\bigstar\): Bearman \((7)\)
$K = 1.0$ (see Fig. 4(b) and (c)). Moreover, the vortex-shedding frequencies are the same on both sides of the trailing edge. This fact shows that even if there exists a difference of less than 20% in the free-stream velocities, Strouhal number can be obtained from results for $K = 1.0$ by using the mean velocity $U$ as the representative velocity. The data of Boldman et al. (3) for $K = 0.75$ and $Re = 1.75 \times 10^6$ are found to coincide with those for $K = 1.0$ when the same procedure was applied.

Critical Reynolds number was about 390 for $K = 0.64$, as seen in Fig. 4(d). Strouhal numbers in the range $390 \leq Re \leq 480$ were almost the same on both sides and higher by about 20% than those for $K = 0.78$. In this Reynolds-number range, flow patterns in the near wake (see Fig. 10) seem to suggest that vortices on the low-velocity side are induced by rolled-up vortices on the high-velocity side. Accordingly, the vortices on the low-velocity side are probably much weaker than those on the high-velocity side; the former vortices are engulfed into the latter shortly after their formation.

Figure 5 shows the power spectra of the fluctuating velocity measured at appropriate positions on the high- and low-velocity sides. In the range $480 \leq Re \leq 800$, the power spectra have one dominant peak on the high-velocity side and two dominant peaks on the low-velocity side. The higher-peak frequency in the latter coincides with the predominant frequency on the high-velocity side. The height of lower-frequency peak increases with an increase of $Re$, while that of the higher-frequency peak decreases, being overcome by the former until there remains only one peak in the spectra (see the spectrum for $Re = 813$).

The results for $K = 0.44$ are shown in Fig. 4(e). Flows behind the trailing edge in this velocity ratio were qualitatively the same as ones for $K = 0.64$ except that the positions of hot-film probe:

high-velocity side ($x/H = 4.0$, $y/H = 1.17$)
low-velocity side ($x/H = 1.167$, $y/H = -0.6$)

![Fig. 4](image)

Fig. 4 For caption see the preceding page.

![Fig. 5](image)

Fig. 5 Power spectra of velocity fluctuation in the near wake ($K = 0.64$).

| Table 1 |
|---|---|---|
| References | $Re$ | $St$ | Characteristic of boundary layer | Remarks |
| Olsen-Karchmer (2) | $1.9 \times 10^6$ | 0.28 | Acoustic pressure fluctuation | Circular nose with splitter plate |
| | $1.8 \times 10^6$ | 0.16 | Velocity fluctuation | |
| Boldman et al. (3) | $2.0 \times 10^6$ | 0.197 | | Turbulent boundary layer |
| Fanning-Muller (6) | 16,295 | 0.202 | 0.321 | 0.121 | Numerical study |
| Shittama-Troend (1) | $9.5 \times 10^5$ | 0.196 | 0.186 | 0.046 | Turbulent boundary layer |
| | $1.5 \times 10^5$ | 0.2 | 0.096 | 0.030 | |
| | $3.3 \times 10^5$ | 0.205 | 0.025 | 0.017 | |
| Bearman (4) | $2.42 \times 10^5$ | 0.241 | 0.25 | ditto |
| | $4.08 \times 10^5$ | 0.241 | 0.25 | ditto |
| Bearman (7) | $2.3 \times 10^5$ | 0.242 | 0.2 | 0.025 | 0.017 | ditto |
vortex shedding occurs only from the high-velocity side in a Reynolds-number range 290 ≤ Re ≤ 500.

As shown in Fig. 4(f), the critical Reynolds number is approximately 260 for K ≠ 0.3. In this case, vortices are formed by the Kelvin-Helmholtz instability of the shear layer only on the high-velocity side in the range of 260 ≤ Re ≤ 1,300 where Strouhal number is between 0.35 and 0.45. Although the vortex shedding was also observed on the low-velocity side when Reynolds number was larger than 1,300, the vortex-shedding frequency was so irregular that most of the power spectra showed no clear peaks.

In the case of K = 0.0 i.e. when U₂ = 0, flow patterns (in Fig. 8) are brought about only by the Kelvin-Helmholtz instability of a separated shear layer. A vortex merging or coalescence was observed to occur in the region x/H ≥ 2.3 ∼ 3.0. The critical Reynolds number Rec was about 300. Strouhal number S_{T} which was measured at the position ( x/H = 1.33, y/H = 0.5 ∼ 0.83 ) showed a wide scatter in the range 0.4 to 0.7 (see Fig. 4 (g)).

Figure 6 shows the vortex-shedding frequency f₁ plotted against the higher freemstream velocity U₁ for K = 0.0. Sato(8) demonstrated that behavior of a separated shear layer is strongly dependent on the momentum thickness of a laminar boundary layer just before the trailing edge and that the predominant frequency f of the velocity fluctuation (due to the formation of vortices) in the shear layer is approximately proportional to U₁^{3/2}, U₁ being the freemstream velocity. The frequency f₁ in the present experiment is not inconsistent with the relation f₁ = U₁^{3/2}. The authors feel that the scatter of data in Fig. 6 should be attributed to an unsystematic change in the boundary layer thickness with the freemstream velocity U₁ due to the boundary layer control by suction.

From the above-mentioned results, it is possible to classify four modes of the vortex shedding from the blunt trailing edge in terms of Reynolds number Re and the velocity ratio K.

(i) First mode: The rolled-up vortices are found only in the shear layer on the high-velocity side. The shear layer on the low-velocity side is engulfed into the vortices without experiencing the rolling up, so that there is no significant velocity fluctuation on the low-velocity side. The flow patterns in the wake in this mode are shown in Fig. 8 (K = 0.0, Re = 930) and 9 (K = 0.45, Re = 940).

(ii) Second mode: The rolled-up vortices on the high-velocity side induce an oscillatory flow behind the trailing edge. The shear layer on the low-velocity side is deformed by this oscillation to form a 'pseudo' vortex which is so weak that it is immediately engulfed into the vortices on the high-velocity side. Predominant frequencies of the velocity fluctuation are the same on both sides. The flow patterns in this mode are shown in Fig. 10.

(iii) Third mode: This mode appears at almost the same velocity ratio as in the second mode but at higher Reynolds numbers where a particular unstable wave makes an appearance in the shear layer on the low-velocity side. The vortices formed by this instability are much weaker than those on the high-velocity side, so that the former vortices are immediately engulfed into the latter vortices. Therefore, on the low-velocity side, there exist two dominant peaks in the velocity spectra: the higher-frequency peak reflects the vortex shedding from the high-velocity side. Figure 11 shows the flow patterns in this mode.

(iv) Fourth mode: The vortices found on the low-velocity side are strong enough to produce a periodic vortex-shedding by interaction with the vortices on the high-velocity side. We have only predominant frequency at a given combination of Reynolds number and the velocity ratio. For low velocity ratios in this mode the convection velocities of vortices are slightly different on both sides, so that the vortex shedding becomes unstable i.e. the power spectra of the fluctuation show a broad peak.

Figure 7 illustrates the four modes of the vortex formation and shedding in the Re-K domain. The critical Reynolds number which is approximately 290 at K = 1.0 increases gradually with decreasing K to

![Fig. 7 Four modes of vortex shedding in the Re-K domain](image-url)
Fig. 8 An example of flow patterns in the first mode. Flow direction is from left to right. The high- and low-velocity sides correspond to upper and lower sides, respectively, of the photographs. (This is the same in the following figures.) Vortex merging is observed in the region \( x/H \approx 2.3 \sim 3.0 \). \( x = 0.0, \quad Re = 930 \). Time interval of exposure \( \Delta t = 0.5 \) sec.

Fig. 9 Another example of flow patterns in the first mode. Vortices are formed due to the Kelvin-Helmholtz instability of the shear layer on the high-velocity side. Rolling up of the shear layer on the low-velocity side does not occur. \( x = 0.45, \quad Re = 540, \quad \Delta t = 1.5 \) sec.

Fig. 10 An example of flow patterns in the second mode. Rolled-up vortices on the high-velocity side induce an oscillation in the near wake to give rise to a 'pseudo' vortex on the low-velocity side. \( x = 0.63, \quad Re = 460, \quad \Delta t = 1.5 \) sec.

Fig. 11 An example of flow patterns in the third mode. A particular instability wave appears in the shear layer on the low-velocity side and interacts with vortices on the high-velocity side. \( x = 0.63, \quad Re = 380, \quad \Delta t = 1.5 \) sec.
become 380 at $K = 0.57$. When the velocity ratio is less than 0.57, there emerges a region where the vortex shedding occurs only from the high-velocity side. If we confine our attention to cases where the vortex shedding occurs on both sides, $Re_c$ increases rapidly with decreasing $K$ until $K = 0.57$.

5. Conclusions

Properties of vortex-shedding from a blunt trailing edge with unequal free-stream velocities were experimentally investigated in a Reynolds-number range 190–3,000. Main results of the present study may be summarized as follows:

1. If Reynolds number $Re$ and Strouhal number $St$ are defined in terms of the average velocity $U = (U_1 + U_2)/2$, $U_1$, $U_2$ being the higher and lower free-stream velocities, respectively, the relation $Re \sim St$ for velocity ratios $K = U_2/U_1 \geq 0.78$ can be approximated by that for $K = 1.0$.

2. The vortex-shedding patterns can be classified into four modes which are clarified in the $Re-K$ domain.

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