Prediction of Turbulent Flow in Two-Dimensional Channel
with Turbulence Promoters

(1st Report, Numerical Prediction by Large Eddy Simulation)

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Fully developed incompressible turbulent flow in a two-dimensional channel has been simulated numerically. Turbulence promoters were arranged intermittently on one side of the channel walls. The two walls and promoters were heated individually. A large-scale flow field and a temperature field were calculated by integrating the filtered, three-dimensional time-dependent Navier-Stokes equations and time-dependent energy equation. Motions of turbulence field with small scales were simulated using eddy-viscosity models. The Reynolds and Péclet numbers were set at 1.1×10^4 and 7.7×10^3, respectively. Only 9,000 grid points were distributed and three kinds of averaging were used in order to complement the grid points. Mean-velocity profiles, turbulence statistics, time-dependent features of flow and mean-temperature profiles are in good agreement with the experimental data. Calculations were carried out using a HITAC M 200/280 H system computer of the University of Tokyo.

Turbulence, Numerical Prediction, Large Eddy Simulation, Channel Flow, Turbulence Promoter, Mean Velocity, Shear Stress, Turbulence Statistics, Large Scale Field, Eddy Viscosity Model

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1. Introduction

Complicated dynamics of fluid flows, especially of turbulent flows, is for the most part analyzed experimentally with the aid of accumulated data on related phenomena. Some of the major reasons for this could be the non-linear characteristics of equations of motion in flow fields and the difficulty of describing the microscopic field motions. The growing capacity of fluid machinery and diversification of working fluids make it difficult to carry out full-scale experiments, and theoretical or numerical analyses are thus all the more important. At present, theoretical approximations based on experimental data in model test are individually adopted in predicting the flow field. The recent great increase in data processing speeds and memory capacity of computers has rendered possible numerical simulations of the fluid motion and some studies on calculations of turbulent flows have been reported(1)-(3). Concerning the calculations of these flows, many problems arise, such as the lack of universal validity for turbulent models, requirements for simple boundary conditions and existence of numerical instability in calculation.

For methods to simulate the turbulent flow around a body or in a straight pipe/parallel channel, Boussinesq model, k-ε model, Reynolds stress closure models and large eddy simulation have been proposed. Large eddy simulation (abbreviated as LES), a relatively new approach to the calculation of turbulent flows, divides the flow field into two parts: One is a field with large scale motion, where the flow behavior is calculated directly by using time-dependent three-dimensional Navier-Stokes equations and the other is a field with small scale motion, where the flow behavior is modeled without direct calculations. This idea stems from the empirical facts that the structures of large scale vortices depend on individual flows and permit no universal modelling, and that those of small scale vortices have nearly isotropic characteristics and are much more amenable to universal modelling. Calculations of turbulent flows with solid boundaries by LES require an enormous computer memory capacity and considerable time. Therefore there have been few examples published of calculations by LES. Deardorff, for the first time, applied a two-dimensional channel flow with infinite Reynolds number, in which 24×20×14 grid points are used, and showed that the distributions of time mean velocities and turbulence intensities coincide relatively well with those of
experiments\(^{(4)}\). Schumann devised the subgrid scale (SGS) Reynolds stress in the small-scale flow field into isotropic and non-isotropic parts, and introduced a transport equation of SGS turbulence energy\(^{(5)}\). Horiiuchi calculated a two-dimensional channel flow with 10×21×10 grid points introducing a non-uniform grid system and simulated time-dependent structures of turbulence\(^{(6)}\). Moin et al also calculated a two-dimensional channel flow with 64×63×128 grid points taking the inner layer of the wall boundary layer into account and clarified the detailed structure of turbulence\(^{(7)}\). All these studies show that LES is one of the promising methods for turbulence simulation, but has its applications limited to one-dimensional equilibrium flow fields.

In this study, in order to establish the numerical calculation method in the non-equilibrium flow fields with separation, the flow behavior in a two-dimensional channel with turbulence promoters is calculated numerically using LES and \(k-e\) model. In this first report, the application of LES and its numerical results are described, and the comparison between calculation and experimental results in wall temperature profiles is also attempted.

2. Basic equations of large-scale flow field

In LES, physical quantity \(f\) is divided into the following two parts.

\[
f = \tilde{f} + f'\tag{1}
\]

where \(\tilde{f}\) is a component in large-scale flow field and \(f'\) is one of small-scale flow field. The large-scale component is defined as follows\(^{(8)}\):

\[
f(x, x_1, x_2) = \int_{x_1}^{x} G(x, x_1) f(x_1, x_2) dx_1 dx_2\tag{2}
\]

The Gaussian-type filter function \(G(x, x_1)\) in \(x_1\) direction, is described as follows.

\[
G(x, x_1) = (\frac{6}{\pi D_1^2})^{1/4} \exp \left[ -\frac{1}{6} (\frac{x-x_1}{D_1})^2 \right] \tag{3}
\]

where \(D_1 = h_1, h_1\) is computational grid size in \(x_1\) direction. Integration in Eq. (2) is performed over the whole flow field \(D\). Figure 1 shows the flow field composed of two parallel flat planes. Rectangular cylinders with square cross section are installed on the lower plane at equal intervals. Co-ordinate axis \(x_1\) is defined in streamwise direction, \(x_2\) in the direction normal to upper and lower planes and \(x_3\) in spanwise direction of rectangular cylinders.

Navier-Stokes, continuity and energy equations are filtered by Eq. (2) assuming an incompressible fluid and its constant properties regardless of temperature variation. Velocity \(u_1\), coordinate \(x_1\) and time \(t\) are non-dimensionalized by streamwise mean velocity \(U\), double length of the channel width \(L\) and \(L/U\), respectively. \(T\) means an excess of temperature over the mean temperature \(\overline{T}\) in the inlet cross section.

\[
\begin{align*}
\frac{\partial \tilde{u}_1}{\partial t} + \frac{\partial \tilde{u}_1}{\partial x_1} \tilde{u}_1 &= -\frac{\partial \tilde{f}}{\partial x_1} - \frac{\partial \tilde{u} u_1}{\partial x_1} - \frac{\partial \tilde{D}}{\partial x_1} + \frac{1}{Re} \frac{\partial^2 \tilde{u}_1}{\partial x_1^2} \tag{4} \\
\frac{\partial \tilde{u}_1}{\partial t} + \frac{\partial \tilde{u}_1}{\partial x_1} \tilde{u}_1 &= -\frac{\partial \tilde{f}}{\partial x_1} - \frac{\partial \tilde{u} u_1}{\partial x_1} - \frac{\partial \tilde{D}}{\partial x_1} + \frac{1}{Re} \frac{\partial^2 \tilde{u}_1}{\partial x_1^2} \tag{5} \\
\frac{\partial \tilde{T}}{\partial t} + \frac{\partial \tilde{T}}{\partial x_1} \tilde{u}_1 &= -\frac{\partial \tilde{f}}{\partial x_1} - \frac{\partial \tilde{u} u_1}{\partial x_1} - \frac{\partial \tilde{D}}{\partial x_1} + \frac{1}{Re} \frac{\partial^2 \tilde{T}}{\partial x_1^2} \tag{6}
\end{align*}
\]

where \(\tilde{f} = \tilde{p} + \tilde{u} \tilde{u}_1 / 3\) and pressure \(\tilde{p}\) is non-dimensionalized by \(\rho U^2 / \nu\) and \(\tilde{u}\) is Reynolds number and \(\rho = \rho U / \nu\) is Prandtl number, where \(\nu\) : kinematic viscosity, \(\gamma\) : thermal diffusivity, \(L_{eu}\) and \(L_{et}\), \(C_{nu}\) and \(C_{et}\), \(R_{eu}\) and \(R_{et}\) in Eqs. (4)-(6) are Leonard, Cross and Reynolds terms, respectively and are defined as follows.

\[
\begin{align*}
L_{eu} &= \tilde{u}_1 \tilde{u}_1 - \tilde{u}_1 \bar{u}_1 \tag{7} \\
L_{et} &= \tilde{T} \tilde{u}_1 - \bar{T} \tilde{u}_1 \tag{8} \\
C_{nu} &= \tilde{u}_1 \tilde{u}_1 + \tilde{u}_1 \bar{u}_1 \tag{9} \\
C_{et} &= \tilde{T} \tilde{u}_1 + \tilde{T} \bar{u}_1 \tag{10}
\end{align*}
\]

In this paper, the models derived from the Taylor series expansion of \(\tilde{u}_1 \bar{u}_1 \) and \(\tilde{T} \bar{u}_1 \),

\[
\begin{align*}
L_{eu} &= \frac{\partial \tilde{u}_1 \partial \tilde{u}_1}{24 \partial x \partial x} \tag{11} \\
L_{et} &= \frac{\partial \tilde{T} \partial \tilde{u}_1}{24 \partial x \partial x}
\end{align*}
\]

are applied to Leonard terms, and the well-known formulae

\[
\begin{align*}
C_{nu} &= \frac{\partial \tilde{u}_1 \partial \tilde{u}_1}{24 \partial x \partial x} - \frac{\partial \tilde{D}}{24 \partial x \partial x} \tag{12} \\
C_{et} &= \frac{\partial \tilde{T} \partial \tilde{u}_1}{24 \partial x \partial x} - \frac{\partial \tilde{D}}{24 \partial x \partial x}
\end{align*}
\]

are used for Cross terms.

3. SGS eddy diffusivity models

In LES, the principal transport of turbulence is performed in the large-scale flow field and the influences of modelling the small-scale flow field are relatively small. In this paper, the eddy diffusivity models
are used for Reynolds terms. $K$ and $K_t$ are eddy diffusivities in velocity and temperature, respectively, and are given by Smagorinsky models:

$$K_t = C_d K, \quad K_r = C_r K$$

where $C=0.1, C_r=1.2$ are SGS parameters and $\Delta t$, a representative length in SGS flow field, can be expressed as follows.

$$\Delta t = (\Delta t_1 + \Delta t_2 + \Delta t_3)/3$$

4. Numerical method and boundary conditions

Equations (4)-(6) and (10)-(14) are transformed into a series of difference equations and solved simultaneously using SMAC method with the central difference formula in space and the Adams-Bashforth type difference scheme in time. Figure 2 shows an outline of computational procedure. A staggered grid system is used, where velocities are defined at the center of grid surface, and pressure, temperature and eddy diffusivities at the center of grid volume. Grid intervals are constant and equal in $x$, $x$, $y$ directions, that is, $h_1 = h_2 = h_1(=h$ =const.).

Considering the case that turbulence promoters are installed periodically, the computational region can be confined by lines ABCDEFGA on $x-x$, plane in Fig.3, where $d_0 = d_2 = d$, $p/d = 5.0$ and $L/(2d) = 3.2$ in Fig.1. Computational grid points are $40 \times 25 \times 9$ as shown in Fig.3.

The boundary conditions are as follows: non-slip condition of velocity is adopted at wall boundaries and the periodicity of velocity is applied to inlet and outlet sections.

$$\bar{u} = 0 \quad \text{at walls}$$

$$\bar{u}_h = \bar{u}_m \quad \text{at inlet and outlet sections}$$

The quadratic distribution of $K$ near wall boundary is assumed,

$$K = \frac{x}{K_0 x_0} \left[ a + (1-a) \left( \frac{x}{x_0} \right)^2 \right]$$

where $x$ is a distance from the nearest wall, $x_0$ is the distance extended under the influence of emphatic non-isotropy, $K_0$ is a value of $K$ at the distance $x_0$ and $a$ is a constant. In this paper, as an example, $x_0 = 3h/2$ and $a = 7/4$ are selected. For the boundary condition in temperature, heat fluxes are given at wall boundaries and the periodicity of heat fluxes is assumed at inlet and outlet sections.

$$q_w = -c_p (Kr + \frac{L}{Pr} \frac{\partial T}{\partial n}) \quad \text{(at walls)}$$

$$\left( \frac{\partial T}{\partial x} \right)_w = \left( \frac{\partial T}{\partial x} \right)_{in} \quad \text{(at inlet and outlet sections)}$$

where $c_p$ is a specific heat at constant pressure, $n$ is a coordinate normal to wall with inward direction, and $q_w$ are the given heat fluxes.

$$q_w = \begin{cases} 0.2927 \text{ kW/m}^2 & \text{(planes ABC and DE)} \\ 0.1530 & \text{(plane CD)} \\ 0.1490 & \text{(plane GF)} \end{cases}$$

The boundary condition for $K_r$ is also given by Eq.(16).

Zero-initial conditions are applied for velocities, pressure and temperature. In this paper, non-dimensional computational grid width $h$ equals $1/50$ and non-dimensional time step $\Delta t$ is $1/1000$. The relation between $h$ and $\Delta t$ is so determined as to satisfy the numerical stability condition$^{10}$. Convergence condition in computation is:

$$\text{Max} \left[ \text{diff}(\frac{\partial u}{\partial x}) \right]_h < 0.001$$

where $\text{diff}(\partial u/\partial x)$ denotes a difference formula of $\partial u/\partial x$.

$N$ is number of iterations. Reynolds number and Pécellet number

Fig.2 Outline of computational procedure

Fig.3 Computational region and grid distribution
in computation of this paper are $1.1 \times 10^4$ and $7.7 \times 10^3$, respectively.

5. Computational results and discussion

Physical quantities such as velocity, pressure, temperature and so on are calculated at each grid point and each time step. In order to compare with the experimental results, three kinds of average operations are utilized. $\langle \rangle$ means the spatial average of $f$ in $x$ direction, $\langle \rangle$ denotes the spatial average in $x_1$-$x_2$ plane and $\langle \langle \phi \rangle \rangle$ is the time averaged value of $\phi$. 1000 time steps are chosen as the integration time, after the influences of initial conditions vanish.

5.1 Time-dependent characteristics of stream function

In a steady laminar flow, stream lines are often designated in order to diagnose the flow behavior macroscopically. Although there exists no steady stream line in a turbulent flow, instantaneous stream function $\langle \phi \rangle$ is calculated from the computed velocity in large scale flow field $\langle \dot{u} \rangle$.

$$
\langle \phi \rangle = \int \langle \dot{u} \rangle dx
$$

Figure 4 shows time-dependent characteristics of quasi-stream lines. From the figure, it is observed that large scale vortices appear behind the turbulence promoter and they change with time.

5.2 Mean velocity profiles

Figure 5 shows the computed profiles of mean velocities in sections II and III (see Fig.1). The profile in section III coincides well with that in section II near the upper wall ($x_1=0.5$), and this coincidence suggests that turbulence promoters on the lower wall have relatively small effects upon the mean velocity profile near the upper wall in this study. Figure 6 shows a detailed profile of mean velocity near the upper wall. Abscissa $y'$ denotes the distance from the upper wall and is non-dimensionalized by $L/2-d$. In the figure, the computed profile is compared with the experimental measurement in a

![Fig.4 Time dependent characteristics of quasi-streamlines](image1)

![Fig.5 Mean-velocity profiles](image2)

![Fig.6 Mean-velocity profiles near upper wall](image3)
two-dimensional channel without turbulence promoter by Laufer[11]. Although Reynolds numbers in both cases are somewhat different, the two flows have an almost identical tendency in the mean velocity profile near the straight wall.

5.3 Profiles of turbulence intensities

In LES, velocity fluctuations are composed of fluctuations in large-scale flow field and SGS turbulence. The former is introduced from

\[ \overline{\nu^2} = \overline{\nu^2} \]  

and the latter from Eqs (9), (12) and (13). Figures 7(a) and (b) show the profiles of turbulence intensities in sections II and III, where \( \overline{(u'^2 + v'^2)} \) in ordinate denotes the sum of the fluctuating component of and SGS turbulence. Both profiles have bigger values on the side of lower wall with turbulence promoters. Figure 8 shows the profiles of turbulence shear stress. From the figure, it can also be seen that the turbulence energy is more pronounced on the side of lower wall. Figure 9 shows the computed turbulence shear stress in section II. The experimental profile of turbulence shear stress by Laufer mentioned before is also shown in the figure. Both profiles are quite different and this fact suggests that the effects of turbulence promoters upon the turbulence intensities are quite pronounced even near the upper wall.

Hanjalic and Launder found by their detailed experiments that the maximum point of the mean velocity profile differs from the zero point of the turbulence shear stress in two-dimensional channel flows with asymmetric boundary conditions[12], coordinate of point M where the mean velocity profile has a maximum in Fig.6 is obviously different from that of Z where the turbulence shear stress is zero in Fig.9, and therefore this numerical simulation well describes this interesting empirical fact.

In Fig.10, the contours of instantaneous velocity fluctuations in the large scale flow field \( \overline{\nu} \) are shown, and solid and broken lines denote positive and negative values of \( \overline{\nu} \), respectively. From the
figure, it can be seen that the contours of $\omega^r$ extend in streamwise direction at the center of the flow field and that, on the contrary, those of $\omega^s$ indicate an isotropic nature.

5.4 Profiles of wall temperature

The computed profile of wall temperature is compared with the experimental one in Fig.11. Abscissa $S$ denotes the non-dimensional distance along the lower wall ABCDE and the upper wall GF (see Fig.1). It appears from the figure that the qualitative tendency of the wall temperature is similar, especially behind the turbulence promoter.

6. Conclusions

The numerical prediction by LES was applied to a non-equilibrium turbulent flow with separation. The features of this computation system are as follows: (1) the construction of LES is extended to a two-dimensional problem; (2) the SMAC method is improved and the Adams-Bashforth difference scheme is introduced to avoid the artificial viscosity; and (3) boundary conditions are imposed in consideration of non-isotropy in turbulence near solid boundaries.

The following were elucidated from the computation:

Fig.11 Wall temperature profiles

1) Mean velocity profiles near the wall without turbulence promoters have a comparatively good agreement with experimental results in a two-dimensional equilibrium channel flow.

2) Turbulence promoters have greater influence upon the profiles of turbulence intensity than upon the mean velocity profiles.

3) The empirical fact that the maximum point of the mean velocity profile differs from the zero point of the turbulence shear stress in an asymmetric flow field is described.

4) The qualitative tendency of the computed wall temperature is similar to that of experimental measurement, especially behind the turbulence promoter.

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References