A Numerical Study of Gas-Particle Supersonic Flow Past Blunt Bodies (The Case of Axisymmetric Flow) *

by Hiromu SUGIYAMA **

A numerical method (inverse method) was developed for a gas-particle supersonic flow past axisymmetric blunt bodies. This method is based on a stream function coordinate system, which makes it convenient to determine the shock layer flow fields and body shapes.

Using the present method, the gas-solid particle flows in the shock layers around blunt bodies (nearly spheres) were solved for freestream Mach number $M_\infty = 2$. The effects of freestream loading ratio and particle diameter on the shock layer thickness, the body surface pressure and the flow quantities along the stagnation streamline are shown, and the flow patterns (gas streamlines and particles streamlines) in the shock layers are also shown.

Key Words : Compressible Flow, Multiphase Flow, Gas-Particle Flow, Blunt Body Flow, Shock Layer Flow, Numerical Analysis

1. Introduction

High-speed gas flows including small solid or liquid particles have been studied by a number of authors in the last two decades in view of their important engineering applications relating to rocket-nozzle flow, shock waves in a dusty gas, and in supersonic flight⁽¹⁾⁻⁽⁵⁾.

The present paper deals with a supersonic gas-particle flow past blunt bodies. Assuming that the loading ratio (the mass ratio of the particle phase to the gas phase) in the freestream is small enough and that the particle phase does not affect the gas phase, Probst and Fassio⁽⁶⁾, Waldman and Reinecke⁽⁷⁾, and Spurk and Gerber⁽⁸⁾ calculated the particle trajectories in the hypersonic shock layer. Their main interest was to find the collection efficiency defined as the ratio of the number of particles which strike the body to the number of particles which could strike it if their trajectories were straight and unaltered by the vehicle flow field.

Taking into account the interactions, i.e., the drag force and the heat exchange between the gas and particles, Chang⁽⁹⁾ investigated a dusty supersonic flow past blunt bodies using a direct method. His main interest was to clarify the nonequilibrium phenomena in the two phase flow within the shock layer, which is also the purpose of the present paper.

In this paper a numerical method for treating a gas-particle supersonic flow past blunt bodies will be developed. This

method is a so called inverse method⁽¹⁰⁾ in which the shock shape is specified and the flow field and the body shape are to be determined. First, the governing equations for the dusty flow past axisymmetric bodies are described in a cylindrical coordinate system (x, r) . It is assumed that there are no collisions between the particles. The interactions between the gas and particles considered are only the drag force and the heat exchange. Next, to facilitate the integration of the governing equations in the shock layer, the governing equations are transformed into a stream function coordinate system (Ψ, Y) , where Ψ is the stream function for the gas phase, and $Y=r$. The governing equations in the shock layer are further transformed into a normalized stream function coordinate system (ξ, η) , which will be discussed subsequently. The advantages of the present method are : (1) body shape is easily determined ; (2) a relatively low computer storage is required ; and (3) solutions are obtained quickly.

Using the present method, the gas-particle shock layer flows around blunt bodies (nearly spheres) were solved numerically. For experimental purposes, glass micro spheres were chosen as the dusty gas. The numerical calculations were carried out for the following parameters :

$$\begin{aligned} M_\infty &= 2 \\ \bar{d}p &= 1, 2, 5, 10 \mu\text{m} \\ \alpha &= 0, 0.2, 0.5, 1.0 \end{aligned}$$

Numerical results for a pure gas ($\alpha = 0$) were compared with those of Van Dyke and Gordon⁽¹¹⁾.

2. Governing Equations

To formulate the motion of the gas-particle flow in the shock layer, the following assumptions are made : (1) The collisions between individual particles can

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be neglected. (2) The interactions between the gas and particles consist only of the drag force between the gas and particles and a heat exchange between them. (3) The particles are spheres of uniform size. (4) The particle phase flow can be treated as a continuum. Let u and v be the x and r components of velocity (Fig. 1), ρ the density, p the pressure, and T the temperature of the gas, and u_p , v_p , ρ_p and T_p the corresponding values of the particle phase. All lengths are nondimensionalized by the shock nose radius $\bar{R}_s$, velocities by the freestream velocity $\bar{U}_\infty$, densities by the freestream gas density $\bar{\rho}_\infty$, pressure by $\bar{\rho}_\infty \bar{U}_\infty^2$, temperatures by $\bar{U}_\infty^2 / \bar{c}_p$, where $\bar{c}_p$ is the gas specific heat at constant pressure. In this paper a superscript ($\bar{\quad}$) over a quantity means a dimensional quantity, otherwise it is nondimensional. A subscript p means particles.

Under the above assumptions, the governing equations of the steady, axisymmetric, gas-particle flow become^{(2),(9)}. For the gas phase,

$$\frac{\partial}{\partial x}(r\rho u) + \frac{\partial}{\partial r}(r\rho v) = 0 \quad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + F_{px} \quad (2)$$

$$\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial r} = -\frac{\partial p}{\partial r} + F_{pr} \quad (3)$$

$$\rho u c_p \frac{\partial T}{\partial x} + \rho v c_p \frac{\partial T}{\partial r} - u \frac{\partial p}{\partial x} - v \frac{\partial p}{\partial r} = Q_p + (u_p - u)F_{px} + (v_p - v)F_{pr} \quad (4)$$

$$p = \frac{\gamma - 1}{\gamma} \rho T \quad (5)$$

and for the particle phase,

$$\frac{\partial}{\partial x}(r\rho_p u_p) + \frac{\partial}{\partial r}(r\rho_p v_p) = 0 \quad (6)$$

$$\rho_p u_p \frac{\partial u_p}{\partial x} + \rho_p v_p \frac{\partial u_p}{\partial r} = -F_{px} \quad (7)$$

Fig.1 Cylindrical coordinate system (x, r)

$$\rho_p u_p \frac{\partial v_p}{\partial x} + \rho_p v_p \frac{\partial v_p}{\partial r} = -F_{pr} \quad (8)$$

$$\rho_p u_p c_p \frac{\partial T_p}{\partial x} + \rho_p v_p c_p \frac{\partial T_p}{\partial r} = -Q_p \quad (9)$$

where $c_s (= \bar{c}_s / \bar{c}_p)$ is the specific heat of the particles. F_{px} and F_{pr} are the components of the force exerted upon a unit volume of the gas by the particles, and they are described as

$$\left. \begin{aligned} F_{px} &= n(\pi d_p^2) C_D \rho w (u_p - u) / 8 \\ F_{pr} &= n(\pi d_p^2) C_D \rho w (v_p - v) / 8 \end{aligned} \right\} \quad (10)$$

where $w = [(u - u_p)^2 + (v - v_p)^2]^{1/2}$, n is a number of particles in a unit volume, and d_p a particle diameter. C_D is the drag coefficient of a particle, and is assumed as⁽¹³⁾

$$C_D = 0.48 + 28 / Re^{0.85} \quad (11)$$

where Re is the Reynolds number based on the diameter of the particle d_p and the relative velocity w .

The heat transfer between the gas and particles is given by

$$Q_p = n N_u \pi d_p k (T_p - T) \quad (12)$$

where k is the gas thermal conductivity, and Nusselt number is expressed as⁽¹⁴⁾

$$N_u = 2.0 + 0.6 Pr^{1/3} Re^{1/2} \quad (13)$$

where Pr is the Prandtl number. In the present analysis, the Prandtl number is taken at 0.75, and the power law for the viscosity-temperature variation is assumed as

$$\bar{\mu} = 1.71 \times 10^{-4} \times \left(\frac{\bar{T}}{273} \right)^{0.77} \text{ poise} \quad (14)$$

3. Equations of Motion in a Stream Function Coordinate System (Ψ, Y)

Using the von Mises transformation⁽¹²⁾ (15)(16), let us transform the governing equations in a (x, r) coordinate system into those in a (Ψ, Y) coordinate system, where Ψ is a stream function for the gas phase and $Y = r$. Then the governing equations (1)-(9) are transformed as follows: For the gas phase,

$$\Psi_x \frac{\partial u}{\partial \Psi} + \Psi_r \frac{\partial v}{\partial \Psi} + \frac{\partial v}{\partial Y} + \frac{v}{\rho} \frac{\partial \rho}{\partial Y} + \frac{v}{Y} = 0 \quad (15)$$

$$\rho v \frac{\partial u}{\partial Y} = -\Psi_x \frac{\partial p}{\partial \Psi} + F_{px} \quad (16)$$

$$\rho v \frac{\partial v}{\partial Y} = -\frac{\partial p}{\partial Y} - \Psi_r \frac{\partial p}{\partial \Psi} + F_{pr} \quad (17)$$

$$\rho v c_p \frac{\partial T}{\partial Y} - v \frac{\partial p}{\partial Y} = Q_p + (u_p - u)F_{px} + (v_p - v)F_{pr} \quad (18)$$

$$p = \frac{\gamma - 1}{\gamma} \rho T \quad (19)$$

and for the particle phase,

$$\Psi_x \frac{\partial u_p}{\partial \Psi} + \frac{\partial v_p}{\partial Y} + \Psi_r \frac{\partial v_p}{\partial \Psi} + \frac{1}{\rho_p} (u_p \Psi_x + v_p \Psi_r) \times \frac{\partial \rho_p}{\partial \Psi} + \frac{v_p}{\rho_p} \frac{\partial \rho_p}{\partial Y} + \frac{v_p}{Y} = 0 \quad (20)$$

$$\rho_p (u_p \Psi_x + v_p \Psi_r) \frac{\partial u_p}{\partial \Psi} + \rho_p v_p \frac{\partial u_p}{\partial Y} = -F_{px} \quad (21)$$

$$\rho_p (u_p \Psi_x + v_p \Psi_r) \frac{\partial v_p}{\partial \Psi} + \rho_p v_p \frac{\partial v_p}{\partial Y} = -F_{pr} \quad (22)$$

$$\rho_p (u_p \Psi_x + v_p \Psi_r) c_s \frac{\partial T_p}{\partial \Psi} + \rho_p v_p c_s \frac{\partial T_p}{\partial Y} = -Q_p \quad (23)$$

where $\Psi_x = x\rho u$, and $\Psi_r = -\rho v$. Through this transformation, the following relations are obtained:

$$\frac{\partial x}{\partial Y} = \frac{u}{v} \quad (24)$$

$$\frac{\partial x}{\partial \Psi} = -\frac{1}{\rho v} \quad (25)$$

4. Equations of Motion in a Normalized Stream Function Coordinate System (ξ, η)

To facilitate the integration of the governing equations in the shock layer, choose the stream functions ahead of the shock wave as ⁽¹²⁾

$$\Psi = r^2/2 \quad (26\cdot a)$$

$$\Psi_p = r^2/2 \quad (26\cdot b)$$

and introduce further the following independent variables

$$\xi = Y^2/2 \quad (27\cdot a)$$

$$\eta = 2\Psi/Y^2 \quad (27\cdot b)$$

In this (ξ, η) coordinate system, the shock wave surface and the body surface correspond to $\eta = 1.0$ and 0 , respectively. Then the governing equations (15)-(23) become, for the gas phase,

$$\frac{\partial u}{\partial \eta} - 2\xi \frac{\partial x}{\partial \xi} \frac{\partial V}{\partial \eta} + \frac{\eta}{\rho^2} \frac{\partial \rho}{\partial \eta} = \xi \left(\frac{1}{V} \frac{\partial V}{\partial \xi} + \frac{1}{\rho} \frac{\partial \rho}{\partial \xi} \right) + \frac{1}{\rho} \quad (28)$$

$$\eta \frac{\partial u}{\partial \eta} + \frac{\partial p}{\partial \eta} = \xi \frac{\partial u}{\partial \xi} - \frac{F_{px}}{2\rho V} \quad (29)$$

$$\eta \frac{\partial V}{\partial \eta} - \frac{\partial x}{\partial \xi} \frac{\partial p}{\partial \eta} = \xi \frac{\partial V}{\partial \xi} + \frac{1}{2\rho V} \frac{\partial p}{\partial \xi} + \frac{V}{2} - \frac{F_{pr}}{\sqrt{2\xi}} \frac{1}{2\rho V} \quad (30)$$

$$\eta c_p \frac{\partial T}{\partial \eta} - \frac{\eta}{\rho} \frac{\partial p}{\partial \eta} = \xi c_p \frac{\partial T}{\partial \xi} - \frac{\xi}{\rho} \frac{\partial p}{\partial \xi} - \frac{Q_p}{2\rho V} - \frac{u_p - u}{2\rho V} F_{px} - \frac{v_p - v}{2\rho V} F_{pr} \quad (31)$$

$$\frac{1}{p} \frac{\partial p}{\partial \eta} - \frac{1}{\rho} \frac{\partial \rho}{\partial \eta} - \frac{1}{T} \frac{\partial T}{\partial \eta} = 0 \quad (32)$$

and for the particle phase,

$$\frac{\partial u_p}{\partial \eta} - 2\xi \frac{\partial x}{\partial \xi} \frac{\partial V_p}{\partial \eta} + \frac{1}{\rho_p} \left(u_p - 2\xi V_p \frac{\partial x}{\partial \xi} \right) \frac{\partial \rho_p}{\partial \eta} = \xi \left(\frac{1}{V} \frac{\partial V_p}{\partial \xi} + \frac{V_p}{V\rho_p} \frac{\partial \rho_p}{\partial \xi} \right) + \frac{1}{\rho} \frac{V_p}{V} \quad (33)$$

$$\rho_p \left(u_p - 2\xi V_p \frac{\partial x}{\partial \xi} \right) \frac{\partial u_p}{\partial \eta} = \xi \frac{\rho_p V_p}{\rho V} \frac{\partial u_p}{\partial \xi} + \frac{F_{px}}{2\rho V} \quad (34)$$

$$\rho_p \left(u_p - 2\xi V_p \frac{\partial x}{\partial \xi} \right) \frac{\partial V_p}{\partial \eta} = \xi \frac{\rho_p V_p}{\rho V} \frac{\partial V_p}{\partial \xi} + \frac{\rho_p V_p^2}{2\rho V} + \frac{F_{pr}}{\sqrt{2\xi}} \frac{1}{2\rho V} \quad (35)$$

$$\rho_p \left(u_p - 2\xi V_p \frac{\partial x}{\partial \xi} \right) c_s \frac{\partial T_p}{\partial \eta} = \xi \frac{\rho_p V_p}{\rho V} c_s \frac{\partial T_p}{\partial \xi} + \frac{Q_p}{2\rho V} \quad (36)$$

, and the equation (25) becomes

$$\frac{\partial x}{\partial \eta} = -\frac{1}{2\rho V} \quad (37)$$

where, $V = v/Y$, $V_p = v_p/Y$.

For the ten unknown variables $u, V, \rho, T, p, u_p, V_p, \rho_p, T_p$ and x , the ten equations (28)-(37) provide a solution through numerical integration.

5. Initial Conditions

The initial conditions immediately behind the shock wave for the gas are usual jump conditions (Rankine-Hugoniot relations), and for the dust particles under the frozen conditions, that is, the particle flow quantities immediately behind the shock wave are the same as those immediately ahead of the shock wave. Let $x=S(r)$ be an equation of the shock wave shape. Then the flow quantities behind the shock wave are given by ⁽¹²⁾

$$\left. \begin{aligned} u &= 1 - \frac{1-\epsilon}{N^2}, \quad V = \frac{1-\epsilon}{N^2} \frac{S_r}{r}, \quad \rho = \frac{1}{\epsilon} \\ p &= \frac{1}{\gamma M_\infty^2} + \frac{1-\epsilon}{N^2}, \quad T = \frac{\gamma}{\gamma-1} \frac{p}{\rho} \end{aligned} \right\} \quad (38)$$

for the gas phase, and

$$\left. \begin{aligned} u_p &= 1, \quad V_p = 0, \quad \rho_p = \alpha \\ T_p &= \frac{1}{\gamma-1} \frac{1}{M_\infty^2} \end{aligned} \right\} \quad (39)$$

for the particles, where

$$\epsilon = \frac{\gamma-1}{\gamma+1} + \frac{2}{\gamma+1} \frac{N^2}{M_\infty^2}, \quad N^2 = 1 + S_r^2$$

and γ is the specific heat ratio of the gas, $M_\infty = \bar{U}_\infty / a_\infty$ (frozen Mach number), and α the loading ratio in a freestream.

6. Numerical Procedure

The governing equations (28)-(37) are numerically integrated from a point immediately behind the shock wave ($\eta = 1.0$) to the body surface ($\eta = 0$) by a forward (i.e., downstream) integration method⁽¹¹⁾.

The description of the numerical procedure is omitted, because the present numerical procedure is just the same as that of Ref.(11).

The mesh sizes $\Delta\xi$ and $\Delta\eta$ used were $\Delta\xi = 0.005, 0.01$ and $\Delta\eta = 0.1, 0.2$.

7. Numerical Results and Discussions

7.1 Pure-gas case

To check the adequacy of the present numerical results, the flow field around a sphere in a pure gas (i.e., no solid particles) was solved. Any shock wave ahead of a smooth, blunt body may be described by

$$r^2 = 2x - B_s x^2 \quad (40)$$

where B_s is the bluntness of the shock shape. The body shape corresponding to a given shock shape may be approximated by^{(11),(17)-(19)}

$$r^2 = 2R_b(x - \Delta) - B_b(x - \Delta)^2 \quad (41)$$

where R_b is the nose radius, B_b the bluntness of the body, and Δ the shock standoff distance.

Figure 2 shows the given shock wave shape ($B_s = -0.06$) and the calculated body

Fig.2 Shock wave, sonic line and streamlines for a sphere (pure gas case, $M_\infty = 2$, $\gamma = 1.4$)

shape (sphere), sonic line, and streamlines for freestream Mach number $M_\infty = 2$ and $\gamma = 1.4$. For comparison, Van Dyke and Gordon's numerical results⁽¹¹⁾ are shown. The present calculated body shapes agree very well with those of Ref.11. The pressure distribution on the body (sphere) is shown in Fig.4 (in case of $\alpha = 0$). The present results agree very well with those of Van Dyke and Gordon⁽¹¹⁾ in the stagnation region, but the difference between the present results and those of Ref.11 appears near the sonic line on the body. It may be noted that Hamilton⁽¹⁹⁾ has pointed out that the results of Ref.11 underestimate the pressure near the sonic line on the body at low freestream Mach number.

The calculated shock standoff distance Δ , nose radius R_b and bluntness of the body B_b are listed in Table 1 (in the case of $\alpha = 0$). It is seen that the present results coincide with those of Van Dyke and Gordon⁽¹¹⁾ within 2.5%. The process used to determine R_b and B_b is the same as that of Refs. 11 and 17.

7.2 Gas-solid particle case

For experimental purposes, air ($\gamma = 1.4$, gas constant $R = 287 \text{ m}^2/(\text{s}^2\text{K})$) and glass spheres ($\bar{\rho}_s = 2.5 \text{ g/cm}^3$) were used for the gas-solid particle flow. The shock waves described by Eq.(40) are assumed for the gas-solid particle case. The temperature and the pressure of the air ahead of the shock wave are $T_\infty = 300\text{K}$, $p_\infty = 101.3 \text{ kPa}$, the reference length $\bar{R}_s = 1 \text{ cm}$, and the freestream Mach number $M_\infty = 2$ for all the calculations. To investigate the effects of the freestream loading ratio α and the particle diameter $\bar{d}_p$ on the shock layer gas-particle flow, the following range of parameters was used:

$$\alpha = 0, 0.2, 0.5, 1.0$$

$$\bar{d}_p = 1, 2, 5, 10 \text{ } \mu\text{m}$$

Table 1 shows the calculated shock standoff distance Δ , and the nose radius R_b and the bluntness B_b of the calculated body shape for the given shock wave shape (Eq. (40), $B_s = -0.06$). Figure 3 shows the assumed shock shape and calculated body shapes for $\bar{d}_p = 2 \text{ } \mu\text{m}$. From Table 1 and Fig.3, it is seen that the shock standoff distances decrease with the

Table 1 Shock standoff distance Δ , nose radius R_b and bluntness B_b of calculated bodies

$\bar{d}_p$ μm	α	$\Delta = \frac{\bar{\Delta}}{\bar{R}_s}$	$R_b = \frac{\bar{R}_b}{\bar{R}_s}$	$\frac{\bar{\Delta}}{\bar{R}_b}$	B_b
—	0	0.174 (0.170)	0.499 (0.500)	0.349 (0.340)	0.98 (—)
	0.2	0.165	0.532	0.310	0.86
	0.5	0.153	0.574	0.267	0.67
1	1.0	0.136	0.638	0.213	0.52
	0.2	0.171	0.514	0.333	0.93
	0.5	0.167	0.533	0.313	0.81
2	1.0	0.160	0.563	0.284	0.67
	0.2	0.173	0.502	0.345	0.95
	0.5	0.172	0.508	0.339	0.93
5	1.0	0.171	0.517	0.331	0.87
	0.2	0.174	0.500	0.348	0.98
	0.5	0.173	0.503	0.344	0.96
10	1.0	0.173	0.506	0.342	0.93

( ): Van Dyke-Gordon's numerical value⁽¹¹⁾

loading ratio ahead of the shock wave. The reason is that the gas density ρ in the shock layer increases with α (see Fig. 6(a), (b)). As the particle diameter increases, the effect of α on the shock standoff distance becomes weak.

Figure 4 shows the pressure distribution P_b on the body surface shown in Fig.3 for $\bar{d}_p = 2 \mu\text{m}$. P_b increases with α , because the momentum of particles is transferred to the pressure.

Figures 5(a),(b) show the variations of the velocities and the temperatures of the gas and particles along the stagnation streamlines for $\bar{d}_p = 1, 2 \mu\text{m}$, respectively.

In case of $\bar{d}_p = 1 \mu\text{m}$, the gas velocity u decreases gradually along the stagnation streamline and becomes zero at the body surface. However, the particle velocity u_p decreases much more rapidly as the result of the interaction of the particles with the gas and does not reach zero at the body surface. That is, the gas and particle velocities are always in nonequilibrium, and particles collide on body surface with a finite velocity. Figure 5(a) also shows that the gas temperature T increases along a stagnation streamline until it reach a maximum, and then decreases as the loading ratio ahead of the shock α increases from

Fig.3 Assumed shock wave and calculated body shape ($\bar{d}_p = 2 \mu\text{m}$)

Fig.4 Pressure Distributions on body surface ($\bar{d}_p = 2 \mu\text{m}$)

(a) $\bar{d}_p = 1 \mu\text{m}$

(b) $\bar{d}_p = 2 \mu\text{m}$

Fig.5 Variation of gas and particle velocity and temperature along stagnation streamline

0.2 to 1.0. This variation of the gas temperature is explained as follows. Just behind the shock wave, the heat generated by the work done by the particles $(u_p - u)F_{px}$ dominates the heat loss Q_p , and as a result the gas temperature T increases behind the shock wave at first. Near the body surface, the heat loss Q_p dominates the work done $(u_p - u)F_{px}$, and the gas temperature T decreases near the body

surface. When the heat generated by the work done by the particle on the gas is equal to the heat loss by the particle phase, the gas temperature reaches a maximum. The particle temperature T_p increases monotonically because of the heat transfer from the gas phase, but it does not reach the gas temperature T . The gas and particle temperatures are always in nonequilibrium along the stagnation streamline.

The results for $\bar{d}_p = 2 \mu\text{m}$ (Fig. 5(b)) are significantly different from those for $\bar{d}_p = 1 \mu\text{m}$. In this case, the gas temperature

Fig. 6 Variation of gas and particle densities, and gas pressure along stagnation streamline

Fig. 7 Gas and particle streamlines around blunt body

T always increases along the stagnation streamline, and reaches a maximum at the stagnation point. In this paper, the results for $\bar{d}_p = 5, 10 \mu\text{m}$ are not shown (see Ref.20), but the variations of T, T_p , u and u_p are qualitatively same as for $\bar{d}_p = 2 \mu\text{m}$. As $\bar{d}_p$ becomes larger, u_p and T_p indicate the tendency to retain their initial values, that is, they indicate frozen state.

Figures 6(a),(b) show the variation of gas pressure and gas and particles densities along the stagnation streamline. The gas pressure P increases along the stagnation streamline owing to a decrease of momentum of gas and particles. This tendency appears strong as α becomes larger and $\bar{d}_p$ smaller. The gas density increases along the stagnation streamline, this tendency appearing strong as α increases and $\bar{d}_p$ becomes small. The particle phase density ρ_p increases remarkably near the body surface for $\bar{d}_p = 1 \mu\text{m}$ and $\alpha = 1.0$.

Figures 7(a),(b) show the gas and particle streamlines in the shock layer. When $\bar{d}_p = 2 \mu\text{m}$, the particle paths are straight just behind the shock wave, and continue to deflect as they approach the body surface and collide at small impact angle δ as shown in Fig.7(a). As $\bar{d}_p$ increases from 2 to 10 μm , the particle deflection decreases and the particles tend to move in a straight path toward the body surface where they collide at a large impact angle.

This analysis may be valid if the particles break after colliding into much smaller ones to form a thin dusty layer or if they stick to the body surface.

8. Conclusions

An inverse method was developed for treating a gas-particle supersonic flow past axisymmetric blunt bodies. This method is based on two transformations (von Mises and an additional one), which are convenient for determining the shock layer flow fields and the body shapes.

Using the present method, the gas-solid particle flows in the shock layers around bodies (nearly spheres) were solved for freestream Mach number $M_\infty = 2$. The effects of freestream loading ratio α and particle diameter $\bar{d}_p$ on the shock layer thickness, the body surface pressure, the flow quantities along the stagnation streamline and the flow patterns (gas streamlines and particle streamlines) are shown.

Within the limitations of the assumptions adopted in this paper, the present numerical results provide an understanding of a gas-particle supersonic flow past blunt bodies.

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