An Optimum Design Method for the Dual Dynamic Damper and its Effectiveness*

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In order to eliminate the drawbacks of a conventional dynamic absorber whose damping performance is very sensitive to the variation of parameters, use of a dual dynamic absorber is proposed in this paper.

To reduce the transmissibility across a main vibration system at resonance, the optimum design procedure of the dual dynamic absorber is described, and optimum values for its tuning and damping parameters are specified in graphical forms and by formulas for the designing. Utilizing the design formulas, the effectiveness of the dual dynamic absorber in reducing the transmissibility at resonance is studied and shown to be considerably greater than that of the conventional dynamic absorber.

Also, it is shown that by use of the dual absorber, it is possible to avoid a decrease of damping performance with the change of the natural frequency of the main vibration system and of damping in its absorbers.

Key Words: Vibration, Dynamic Absorber, Optimum Design, Damping Performance

1. Introduction.

A large number of problems of vibration in a machine system come from the insufficiency of the inner damping effect which exists in the machine system itself. In order to increase the inner damping effect, various dampers have been developed and their optimum adjustment conditions have been reported. Above all, a dynamic damper, which is one of the typical passive-type absorbers, is very popular for this purpose. This damper is composed of the elements such as a mass, a spring and a damper. This damper is useful to increase the damping effect if it is suitably applied, and its optimum adjustment condition is established⁽¹⁾. However, a trouble in using the dynamic damper singly is that the vibration damping effect is easily affected by the changes in parameters such as the natural frequency of the main vibration system and the damping coefficient of the damper system, and so on. Consequently, it is said that the reliability lacks in a dynamic damper as applied singly.

On the other hand, an application of the dual dynamic damper system as a viscous damping element of a vibration isolator such as the handle of a hand-operated vibration tool which requires lightness in

weight, compactness in size and a higher vibration-controlling effect is presented⁽²⁾, and the data for the optimum design is reported⁽³⁾. In these presentations, the following became clear: 1) a dual dynamic damper system whose total weight of masses is equivalent to the weight of mass of a single dynamic damper is more useful in a limited space; 2) a dual dynamic damper system is free from faults of a single dynamic damper mentioned above and reduces the transmissibility at resonance peak.

The purpose of this report is as follows:

- 1) to present the optimum design data in graphical form and to derive practical equations for the optimum design by analyzing the optimum adjustment condition of the dual dynamic damper system in detail, and to confirm the effectiveness of the equations;
- 2) to present the effect of connecting two dynamic dampers on both the fluctuation of the natural frequency and the change of damping coefficient in the main vibration system; and
- 3) to present an adjusting method of the dual dynamic damper system allowing for the possible change in the natural frequency and the data for the method.

Nomenclature

x_{st} : Static displacement
 M : Mass of main vibration system kg
 K : Spring stiffness of main vibration system N/m
 Ω : Natural frequency of main vibration system rad/s
 m_1 : Mass of first dynamic damper kg

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- k_1 : Spring stiffness of first dynamic damper N/m
 c_1 : Damping coefficient of first dynamic damper Ns/m
 m_2 : Mass of second dynamic damper kg
 k_2 : Spring stiffness of second dynamic damper N/m
 c_2 : Damping coefficient of second dynamic damper Ns/m
 μ_1 : Mass ratio of first dynamic damper to main vibration system
 γ_1 : Natural frequency ratio of first dynamic damper to main vibration system
 ζ_1 : Damping factor of first dynamic damper
 ω_{d1} : Natural frequency of first dynamic damper rad/s
 μ_2 : Mass ratio of second dynamic damper to main vibration system
 γ_2 : Natural frequency ratio of second dynamic damper to main vibration system
 ζ_2 : Damping factor of second dynamic damper
 ω_{d2} : Natural frequency of second dynamic damper rad/s

2. Theoretical Analysis.

2.1 Displacement transmissibility.

The vibrating system as shown in Fig. 1 is taken as the object of analysis in this paper. The spring whose stiffness is K is attached to the mass M of the main vibration system, and two dynamic dampers, each of which consists of a damper, a spring and a mass m , are connected to the mass M in parallel. When an input force f is applied to the mass M , let x , x_1 and x_2 be the displacements of the masses M , m_1 and m_2 respectively. After Laplace transformation of the system equations which are obtained through the analysis of the vibration model based on Fig. 1, the vibration can be represented by a signal flow diagram as shown in Fig. 2, where S is Laplace operator. The displacement transmissibility x/x_{st} can be expressed by the following equation, where x_{st} is the static displacement which is defined as the ratio of F_0 to the total spring stiffness K (that is, $x_{st} = F_0/K$):

Fig. 1 Schematic view of a vibration system with dual dynamic dampers.

$$\frac{x}{x_{st}}(j\omega) = \frac{R_N + jI_N}{R_D + jI_D} = A e^{j\alpha} \quad (1)$$

, where

$$A = \frac{\sqrt{R_N^2 + I_N^2}}{\sqrt{R_D^2 + I_D^2}}, \quad \alpha = \tan^{-1} \frac{I_N}{R_N} - \tan^{-1} \frac{I_D}{R_D}$$

For the sake of generality, the following nondimensional factors are introduced:

$$\lambda = \frac{\omega}{\sqrt{K/M}}, \quad \mu_1 = \frac{m_1}{M}, \quad \mu_2 = \frac{m_2}{M},$$

$$\gamma_1 = \frac{\sqrt{K/M}}{\sqrt{k_1/m_1}}, \quad \gamma_2 = \frac{\sqrt{K/M}}{\sqrt{k_2/m_2}},$$

$$\zeta_1 = \frac{c_1}{2\sqrt{m_1 k_1}}, \quad \zeta_2 = \frac{c_2}{2\sqrt{m_2 k_2}}$$

Using these nondimensional factors mentioned above, R_N , I_N , R_D and I_D can be expressed as follows:

$$R_N = \gamma_1^2 \gamma_2^2 \lambda^4 - (\gamma_1^2 + \gamma_2^2 + 4\gamma_1 \gamma_2 \zeta_1 \zeta_2) \lambda^2 + 1 \quad (2)$$

$$I_N = -2\gamma_1 \gamma_2 (\gamma_1 \zeta_2 + \gamma_2 \zeta_1) \lambda^3 + 2(\gamma_1 \zeta_1 + \gamma_2 \zeta_2) \lambda \quad (3)$$

$$R_D = -\gamma_1^2 \gamma_2^2 \lambda^6 + \{\gamma_1^2 \gamma_2^2 + \gamma_1^2 + \gamma_2^2 + \gamma_1^2 \mu_2 + \gamma_2^2 \mu_1 + 4\gamma_1 \gamma_2 \zeta_1 \zeta_2 (1 + \mu_1 + \mu_2)\} \lambda^4 - \{\gamma_1^2 + \gamma_2^2 + 4\gamma_1 \gamma_2 \zeta_1 \zeta_2 + (1 + \mu_1 + \mu_2)\} \lambda^2 + 1 \quad (4)$$

$$I_D = 2\gamma_1 \gamma_2 \{\gamma_1 \zeta_2 (1 + \mu_2) + \gamma_2 \zeta_1 (1 + \mu_1)\} \lambda^5 - 2\{\gamma_1 \gamma_2 (\gamma_1 \zeta_2 + \gamma_2 \zeta_1) + (\gamma_1 \zeta_1 + \gamma_2 \zeta_2) \times (1 + \mu_1 + \mu_2)\} \lambda^3 + 2(\gamma_1 \zeta_1 + \gamma_2 \zeta_2) \lambda \quad (5)$$

2.2 The optimum adjustment condition for the dual dynamic damper.

In order to acquire the optimum adjustment condition for the dual dynamic damper, let us choose routinely three adjustment conditions as follows;

- $\zeta_1 = 0$, $\zeta_2 = \infty$, ii) $\zeta_1 = \infty$, $\zeta_2 = 0$, iii) $\zeta_1 = \infty$, $\zeta_2 = \infty$

The displacement transmissibilities in each combination of ζ_1 and ζ_2 can be expressed by the following equations (6) to (8).

- in the case of $\zeta_1 = 0$ and $\zeta_2 = \infty$

$$\left| \frac{x}{x_{st}}(\lambda) \right| = \frac{-(\gamma_1^2 \lambda^2 - 1)}{(1 + \mu_2) \gamma_1^2 \lambda^4 - \{\gamma_1^2 + (1 + \mu_1 + \mu_2)\} \lambda^2 + 1} \quad (6)$$

- in the case of $\zeta_1 = \infty$ and $\zeta_2 = 0$

$$\left| \frac{x}{x_{st}}(\lambda) \right| = \frac{-(\gamma_2^2 \lambda^2 - 1)}{(1 + \mu_1) \gamma_2^2 \lambda^4 - \{\gamma_2^2 + (1 + \mu_1 + \mu_2)\} \lambda^2 + 1} \quad (7)$$

Fig. 2 Signal flow diagram.

iii) in the case of $\zeta_1 = \infty$ and $\zeta_2 = \infty$

$$\left| \frac{r}{x_{st}}(\lambda) \right| = \frac{-1}{(1+\mu_1+\mu_2)\lambda^2-1} \dots\dots\dots(8)$$

Fig.3 shows three displacement transmissibility curves for the equations (6) to (8) in the case of $\mu_1 = \mu_2 = \mu'$. There are seven intersections which are common to any two curves of equations (6) to (8), and the three points of them are referred to as P, Q and T, as shown in Fig. 3. As clarified in the previous report, these three points P, Q and T have the same properties as the two intersection points which determine the optimum adjustment condition for a single dynamic damper. Using the properties of these three points, the optimum adjustment condition for the dual dynamic damper can be acquired:

(1) by making the three intersection points P, Q and T locate at the same height, and

(2) by making a displacement transmissibility curve pass through the places in the vicinity of the points P, Q and T, and furthermore making the curve have maximum values in the vicinities of these three points.

The first item corresponds to the optimum tuning condition for a single dynamic damper. The second item corresponds to the condition which satisfies the optimum damping coefficient.

2.3 The optimum adjusting equations and the optimum designing chart for the dual dynamic damper.

In the case of $\mu_1 = \mu_2 = \mu'$, the equations which make the points P, Q and T locate at the same height can be obtained by the following steps:

Step 1. Calculate the values of the exciting frequency ratios λ_P , λ_Q and λ_T at the points P, Q and T.

i) The case of the point P:
The point P is an intersection of the equations (6) and (8). If the phases of these

two curves are taken into consideration, the value of λ_P is given as one of the roots of the equation (9).

$$\begin{aligned} & \frac{-(\gamma^2\lambda^2-1)}{(1+\mu')\gamma^2\lambda^4 - \{\gamma^2 + (1+2\mu')\}\lambda^2 + 1} \\ & = \frac{-1}{(1+2\mu')\lambda^2-1} \dots\dots\dots(9) \end{aligned}$$

According to this equation, it is clear that these two curves intersect each other at two points. But the intersection P is a point having a smaller value of λ_S . Therefore, the exciting frequency ratio λ_P can be expressed as follows.

$$\lambda_P = \left[\frac{1}{(2+3\mu')\gamma^2} \{\gamma^2 + (1+2\mu') - \sqrt{\gamma^4 - 2(1+\mu')\gamma^2 + (1+2\mu')^2}\} \right]^{1/2} \dots\dots\dots(10)$$

ii) The case of the point Q:

This point is one of the intersections of the equations (7) and (10). If the phases of these two curves are taken into consideration, the point Q is an intersection which has a larger value of λ_S , where λ_S is the root of the following equation.

$$\begin{aligned} & \frac{-(\gamma^2\lambda^2-1)}{(1+\mu')\gamma^2\lambda^4 - \{\gamma^2 + (1+2\mu')\}\lambda^2 + 1} \\ & = -\frac{1}{(1+2\mu')\lambda^2-1} \dots\dots\dots(11) \end{aligned}$$

Therefore,

$$\lambda_Q = \left[\frac{1}{(2+3\mu')\gamma^2} \{\gamma^2 + (1+2\mu') + \sqrt{\gamma^4 - 2(1+\mu')\gamma^2 + (1+2\mu')^2}\} \right]^{1/2} \dots\dots\dots(12)$$

iii) The case of the point T:

This point is one of the intersections of the equations (6) and (7). In consideration of the phases, the following equation can be derived.

$$\begin{aligned} & \frac{-(\gamma^2\lambda^2-1)}{(1+\mu')\gamma^2\lambda^4 - \{\gamma^2 + (1+2\mu')\}\lambda^2 + 1} \\ & = -\frac{-(\gamma^2\lambda^2-1)}{(1+\mu')\gamma^2\lambda^4 - \{\gamma^2 + (1+2\mu')\}\lambda^2 + 1} \dots\dots\dots(13) \end{aligned}$$

After the arrangement of the equation (13) for λ , it can be expressed as a sixth order equation of λ as follows.

$$\lambda^6 + A_1\lambda^4 + A_2\lambda^2 + A_3 = 0 \dots\dots\dots(14)$$

where,

$$\begin{cases} A_1 = -\frac{(2+3\mu')(\gamma^2 + \gamma_2^2) + 2\gamma_1^2\gamma_2^2}{2(1+\mu')\gamma_1^2\gamma_2^2} \\ A_2 = \frac{(1+2\mu') + \gamma_1^2 + \gamma_2^2}{(1+\mu')\gamma_1^2\gamma_2^2} \\ A_3 = -\frac{1}{(1+\mu')\gamma_1^2\gamma_2^2} \end{cases}$$

By this equation, it is evident that there are three intersections in all. The intermediate intersection among them is the desired point T, and the root for this point will be denoted as λ_T .

Step 2. Obtain the conditional equation for making the intersection points P, Q and T locate at the same height.

(1) The conditional equation to make the intersection points P and Q locate at the same height.

Fig.3 Three significant intersection points P, Q and T.

As the points P and Q are on the same curve of the equation (8), the desired conditional equation can be obtained if the value of the equation (8) for λ being λ_P is equated to that for λ being λ_Q . That is,

$$\frac{-1}{(1+2\mu')\lambda_P^2-1} = \frac{-1}{(1+2\mu')\lambda_Q^2-1}$$

This equation therefore becomes as follows:

$$\lambda_P^2 + \lambda_Q^2 = 2/(1+2\mu') \dots \dots \dots (15)$$

Arrange the equation (15) by using the equations (10) and (12), and the following equation will be derived.

$$(1+2\mu')^2(\gamma_1^2 + \gamma_2^2) = 2(1+\mu')\gamma_1^2\gamma_2^2 \dots \dots \dots (16)$$

This equation (16) is the desired conditional equation which makes the points P and Q locate at the same height.

(2) The conditional equation which makes the points P and T locate at the same height.

The point P is on the curve of the equation (8), and the point T is on the curve of the equation (6). Thus, the desired conditional equation can be derived if the value of the equation (8) for λ being λ_P is equated to the value of the equation (6) for λ being λ_T . Therefore,

$$\frac{-1}{(1+2\mu')\lambda_P^2-1} = \frac{-(\gamma_1^2\lambda_T^2-1)}{(1+\mu')\gamma_1^2\lambda_T^2 - \{\gamma_1^2 + (1+2\mu')\}\lambda_T^2 + 1} \dots \dots \dots (17)$$

This equation (17) is the desired conditional equation which makes the points P and T locate at the same height.

Step 3. Solve the simultaneous equations (16) and (17) in order to obtain the conditional equations which make the points P, Q and T locate at the same height. These simultaneous equations are expressed only by μ' , γ_1 and γ_2 , and they are equivalent to the tuning condition of the dual dynamic dampers used in parallel. Therefore, in order to adjust the dual dynamic damper in the optimum tuning, first choose the mass ratio μ' , and next solve the simultaneous equations for γ_1 and γ_2 . Fig. 4(a) is a flow chart for computing the optimum tuning values of γ_1 and γ_2 .

The method to satisfy the second optimum adjustment condition, that is, the method to make the displacement transmissibility curve pass near the points, P, Q, T and take a maximum value in the vicinity of each point is as follows. It can be attained by giving ζ_1 and ζ_2 the optimum values. In order to calculate the condition exactly, it is the best way to differentiate the equation (1) with respect to λ and obtain the maxima, but it is so complicated. That, an easier method to obtain the optimum damping coefficients, ζ_1 and ζ_2 will be described in this chapter.

The steps in the method are as follows. These steps use the property of the point T, such that, the point T under the optimum condition exists in the vicinity where the middle peak of the curve becomes the lowest when ζ_1 and ζ_2 are varied from 0 to ∞ .⁽³⁾

Step 1. Make the points P, Q and T locate at the same height by giving optimum values to γ_1 and γ_2 .

Step 2. Change the values of ζ_1 and ζ_2 in order to minimize the distance between the point T and the middle peak of the displacement transmissibility curve.

Step 3. Make fine adjustments of ζ_1 and ζ_2 such that the values of the three peaks of the curves have the same height. Fig. 4(b) is a flow chart for computing the optimum values of ζ_1 and ζ_2 .

The optimum adjusting design data for the dual dynamic damper which were computed with the program of Fig. 4 are shown in Fig. 5. In Fig. 5, the optimum frequency ratios γ_1 and γ_2 , and the optimum damping factors ζ_1 and ζ_2 are illustrated with the abscissa μ' . And the maximum displacement transmissibility x/x_{st} in the optimum adjustment values γ_1 , γ_2 , ζ_1 and ζ_2

Fig. 4 Flow chart to compute the optimum adjusting values.

Fig. 5 Optimum adjusting design chart for the dual dynamic damper.

ζ_2 is also illustrated in Fig. 5. Fig. 6(b) shows the displacement transmissibility curves drawn by a computer with the optimum adjustment values for the dual dynamic damper from Fig. 5. The displacement transmissibility curves for a single dynamic damper are shown in Fig. 6(a) for the sake of comparison. Unlike the single damper, it is evident that the dual dynamic damper can hold the maximum displacement transmissibility at a lower level than the single dynamic damper.

2.4 A proposal of the approximate equations for practical use.

In order to adjust the dual dynamic damper in the optimum condition, it is necessary to read the optimum adjustment values γ_1 , γ_2 , ζ_1 and ζ_2 which correspond to a mass ratio μ' from Fig. 5. However, it is inconvenient and moreover inaccurate to read the data from Fig. 5. Therefore, the authors would propose the following approximate equations (18) to (22) instead of the characteristic curves in Fig. 5. These equations may be more practical, because there is only the value μ' in each equation.

$$\frac{1}{\gamma_1} = \frac{1}{2.48}(\mu' + 0.13)^{-0.434} \quad (18)$$

$$\frac{1}{\gamma_2} = -0.72\mu' + 1.04 \quad (19)$$

$$\zeta_1 = \left(\frac{\mu'}{45}\right)^{0.07} - 0.505 \quad (20)$$

$$\zeta_2 = \left(\frac{\mu'}{5}\right)^{0.377} - 0.038 \quad (21)$$

$$\left.\frac{x}{x_{st}}\right|_{\max} = 1.36(\mu' - 0.017)^{-0.328} \quad (22)$$

The values which were calculated by the approximate equations (18) to (22) are plotted in Fig. 5 for comparison. It is seen that the plots of these approximate equations and the characteristic curves in Fig. 5 resemble each other very much in the range of $\mu' > 0.02$. The displacement transmissibility curves based on the approximate equations are shown in Fig. 7 for comparison with Fig. 6. The two resemble each other very much. Thus, it is confirmed that these approximate equations are useful enough for practical application.

3. An Effect of Connecting Two Dynamic Damper.

Generally, one of significant drawbacks in using a dynamic damper singly is that the vibration effect decreases very much because of being easily affected by the change of vibration-system specifications with the change of the using condition or the using circumstance. The change of the natural frequency of a main vibration system and that of the damping coefficient of an absorber are good examples. In order to eliminate these drawbacks, the influences of these changes on the vibration of the system in the case of the dual

(a) The case of a single dynamic damper

(b) The case of a dual dynamic damper

Fig. 6 Displacement transmissibility curves based on the values of Fig. 5 with the mass ratio as a parameter.

Fig. 7 Displacement transmissibility curves based on the approximate equations with the mass ratio as a parameter.

dynamic damper will be discussed in this chapter.

3.1 An effect on the change of the natural frequency.

In the case of a single dynamic damper, the optimum natural frequency ratio γ_{opt} is expressed as $\sqrt{K/M} / \sqrt{K_{opt}/m}$ ($=\Omega/\omega_d$).

From this formula, it is evident that the action to compensate for the decrease in vibration controlling effect due to a change in the natural frequency Ω of the main vibration system can not be expected. On the other hand, the optimum tuning points of the dual dynamic damper are decentralized to two points, that is, the first optimum natural frequency ratios γ_{1opt} ($=\sqrt{K/M}/\sqrt{k_{1opt}/m_1}$) exists on the lower frequency side of the natural frequency Ω , while, the second one γ_{2opt} ($=\sqrt{K/M}/\sqrt{k_{2opt}/m_2}$) exists on the higher-frequency side of Ω . Consequently, if the natural frequency Ω changes, it can be expected to lessen the decrease of the vibration controlling effect by an effective controlling of the damper which is closer to the side of Ω changing. The following is a test of this effect.

Fig. 8 shows the displacement transmissibility curves using Ω as a parameter in the optimum-adjusted vibration system with a single dynamic damper. Fig. 9 shows the displacement transmissibility curves using Ω as a parameter in the system with the dual dynamic damper. Both Fig. 8 and Fig. 9 are drawn by the computer, and the value of Ω is represented by the value of the spring stiffness K of the main vibra-

Fig. 8 Change in the pattern of a displacement transmissibility curve according to the change of K .
(the case of a single dynamic damper)

Fig. 9 Change in the pattern of a displacement transmissibility curve according to the change of K .
(the case of a dual dynamic damper)

tion system which corresponds to Ω .

Fig. 10 shows the relation of the vibration of spring stiffness K to the change of the maximum of each curve in Fig. 8 and in Fig. 9. In the case of a single dynamic damper, this relation is calculated as $\mu = 2\mu'$, where μ' is the mass ratio. According to Fig. 10, it is clear that two dynamic dampers connected, that is, a dual dynamic damper, is superior to a single damper in the following points:

- (1) the former is almost independent of the change of the natural frequency Ω and
- (2) the former is effective for reducing the decrease of the vibration controlling effect.

For example, in the case of $\mu' = 0.15$, when it is desirable that the maximum displacement transmissibility ratio be controlled to within 3, the admissible range of the change ratios of the spring stiff-

Fig. 10 Change of the maximum value of a displacement transmissibility curve for the change ratio of K .

Fig. 11 Change in the pattern of a displacement transmissibility curve based on the adjusting method allowing for the change of the natural frequency with K as a parameter.

ness K in the case of a single dynamic damper is restricted to the range of -7% to 7% (as to Ω , it is -3.5% to 3.5%), but in the case of the dual dynamic damper, the admissible range of K becomes wider, namely, it is in the range of -9.5% to 13.5% (as to Ω , it is -5% to 6.5%). And, when the admissible range of the change ratios of the spring stiffness K is restricted to the range of -2% to 2% (as to Ω , it is -1% to 1%), the maximum displacement transmissibility ratio can be held at 2.66 in the case of the dual dynamic damper, but in the case of the single dynamic damper, the ratio amounts to 2.85.

3.2 A proposal of an adjusting method allowing for the possible change in the natural frequency.

As mentioned above, it is considered due to an effect of tuning point being decentralized to two points that a system consisting of two dynamic dampers is less affected than a single damper by the change in the natural frequency. Then, using this property, in this section the authors propose an adjusting method for the dual dynamic damper allowing for the change in the natural frequency. This adjusting method is to shift the natural frequencies of each dynamic damper, which is a part of the dual dynamic damper, from its own optimum adjusting point. That is, adjustment is done merely by shifting ω_{d1opt} a little lower (namely, making γ_{1opt} a little larger) and by shifting ω_{d2opt} a little higher (namely, making γ_{2opt} a little smaller); where ω_{d1opt} and ω_{d2opt} are the optimum natural frequencies of each damper which consists of a dual dynamic damper. Fig. 11 shows the case of $\mu' = 0.15$. Fig. 11 is a graph showing a displacement transmissibility curve for the exciting frequency ratio λ using K as a parameter instead of Ω under the following adjustment conditions: (1) the damping factors ζ_1 and ζ_2 are taken as the optimum values; (2) the natural frequency ratio of the first damper γ_1 is increased by 5% over its optimum natural frequency ratio γ_{1opt} ; and (3) that of the second damper γ_2 is decreased by 5% over its optimum natural frequency ratio γ_{2opt} . From Fig. 11, it can be seen that the middle peak of the displacement transmissibility curve marked with 'x1' locates higher on account of γ_1 and γ_2 being shifted from their optimum tuning points, that is, the damping effect decreases a little. However, it is clear that even if the spring stiffness K changes, the increase in the maximum value of the transmissibility ratio is relatively small. Thus, it is clear from Fig. 11 that the stable range of the damping effect is wider than in Fig. 9.

Fig. 12 shows the change in the maximum values of the transmissibility with the parameters of the adjusting ratios of γ_1 and γ_2 according as the spring stiffness K changes. Fig. 12 is illustrated for

comparison with Fig. 10. In this case, the adjusting ratios of γ_1 and γ_2 are changed in 1% step from 0% to 10%. From this figure, it is seen that in spite of the maximum value of a displacement transmiss-

Fig.12 Change of the maximum value of a displacement transmissibility curve for the change ratio of K .

Fig.13 Relation between the tuning ratio of γ and the maximum value of each curve in Fig.12 with μ' as a parameter.

Fig.14 Relation between the tuning ratio of γ and the change ratio of K with μ' as a parameter.

sibility curve increasing, the more the adjusting ratios of γ_1 and γ_2 , the wider the admissible range of the spring stiffnesses K becomes. Fig. 13 and Fig. 14 show the change in these admissible ranges.

In Fig. 13, whose abscissa is the adjusting ratio of γ and ordinate is the maximum transmissibility, the relation of the maximum value of the middle peak of a characteristic curve in Fig. 12, which is indicated as B in Fig. 13, to the adjusting ratio of γ is shown. In Fig. 14, whose abscissa is the adjusting ratio of γ and ordinate is the change ratio of the spring stiffness K , the relation of the change ratio of K in the places, which take the same value of the point B (see Fig. 13) and are indicated as B_1 and B_2 in Fig. 14, to the adjusting ratio of γ is shown with the mass ratio μ' taken as a parameter.

By using these characteristic charts of Fig. 13 and Fig. 14, it is possible to adjust easily the dual dynamic damper allowing for the change in the natural frequency. For example, in the case of $\mu'=0.15$, if the maximum transmissibility ratio is permitted up to 1.2 times of the optimum-adjusted value, the adjusting ratio of γ , say 4.2 %, can be obtained from Fig. 13 as the abscissa of the point of intersection of the characteristic curve in the case of $\mu'=0.15$ and the horizontal line drawn at the indicated ordinate, say 1.2. Therefore, in order to adjust the natural frequency ratios γ_1 and γ_2 to 1.2, multiply the ratio 0.958 by γ_1 and 1.042 by γ_2 , where 0.958 is the decimal form of $(100-4.2)\%$ and 1.042 is that of $(100+4.2)\%$, and it can be seen from Fig. 14 that the ratio of spring stiffnesses K is permitted to change in the range of -18% to 24% .

Fig. 15 shows the change of the characteristic curve in the case of change ratio of γ being 5 % when the damping factors ζ_1 and ζ_2 in Fig. 12 are varied from -8% to 8% . As seen from Fig. 15, it may

Fig. 15 Influence of the change of ζ on the maximum value of a displacement transmissibility in the adjustment allowing for the change of natural frequency.

be thought that the adjustment allowing for the change in the natural frequency yields a desirable result even if ζ_1 and ζ_2 , the damping factors, remain at their own optimum values without fine adjustment.

3.3 An influence of the change of the damping coefficient C .

In a vibration system, one of the elements which are easily affected by the working environment is the damping element, that is, a damper. Particularly, an oil-type damper is easily affected by the change of the temperature. So, in this section, the influence of the change of the damping coefficient C will be discussed.

Fig. 16 shows the influence of the change of the damping coefficient C on the displacement transmissibility in the case of a single dynamic damper. Fig. 17 shows the influence of the change of C on the transmissibility in the case of the dual dynamic damper. In Fig. 16 and Fig. 17, the damping factor ζ is used instead of C .

Fig. 16 Influence of the change of ζ on a displacement transmissibility curve.
(the case of a single dynamic damper)

Fig. 17 Influence of the change of ζ on a displacement transmissibility curve.
(the case of a dual dynamic damper)

Fig.18 Relation between the change ratio of ζ and each maximum value of a displacement transmissibility curve in Fig.16 and Fig.17.

Fig. 18 shows the relation of the changing ratio of ζ to the maximum value of the displacement transmissibility curve. The broken line is for the single damper and the solid line for the dual dynamic damper.

From these figures, it is evident that the dual dynamic damper can decrease the influence of the change of the damping coefficient C unlike the single damper.

4. Summary and Conclusions.

In this paper, the conditional equations for the optimum adjustment which were derived by analyzing the optimum adjusting conditions for the dual dynamic damper in the previous report⁽³⁾, the flow chart of computer program to calculate the optimum adjustment values from the equations, the practical approximate equations

with only one parameter μ' , the method of the optimum adjustment, and the charts for the optimum adjustment are presented.

From the analysis described above, the following are concluded:

(1) The dual dynamic damper can reduce the decrease of the vibration control effect, which is a drawback to an ordinary dynamic damper, caused by the change of the natural frequency in the main vibration system.

(2) A system with two dampers connected, that is, a dual dynamic damper, enables an adjustment allowing for the change of the natural frequency in the main vibration system.

(3) The influence of the change of the damping coefficient C is less in the dual dynamic damper than in the single damper.

The authors will be pleased if this paper can be of any help to the future studies in this field.

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