Sudden Expansion of Gas-solid Two-phase Flow
in Vertical Downward Flow

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Gas-solid two phase flow in sudden expansion of a vertical downward flow is experimentally examined by using different expansion ratios and different solid particles. The result shows that additional expansion loss takes a negative value when the flux Richardson number exceeds 0.0015 as in horizontal pipes under the condition that the velocity of solid particles is larger than that of air.

Key Words: Turbulence, Multiphase Flow, Solid Particles, Sudden Expansion, Flux Richardson Number, Downward Flow

1. Introduction

In the previous paper (1) we examined an onset condition of a reverse transfer of momentum from particles to gas by using three different pipelines with different particles in the sudden expansion of a horizontal pipeline, and confirmed that the pressure recovery and the additional expansion loss coefficient are correlated to the flux Richardson number and that an effect of addition of particles occurs when the number exceeds about 1.5x10^{-3}. Furthermore, the velocity profile of the suspension is developed without taking flat shape after the expansion interface when the effect occurs. However, a solid loading ratio was set at low values to avoid particles deposition. In this work, we examine the effect of gravity including the effect of physical property and of expansion ratio, on the onset point of the reverse transfer of momentum by using three different pipelines with three different particles in the sudden expansion of a vertically downward flow, where the solid loading ratio can be made sufficiently larger. Additionally, we discuss the result in terms of the particle velocity and air velocity in the expansion.

2. Nomenclature

\( P_{max} \): maximum pressure in the downstream pipe
\( \Delta P_g \): pressure recovery = \( P_{max} - P_{1} \)
\( \Delta E \): nondimensional pressure recovery = \( \frac{\Delta P_g}{(1/2) \rho_1 U_1^2} \)
\( Re \): Reynolds number = \( UD/\nu \)
\( Rf \): flux Richardson number
\( \tau^* \): relaxation time of particle
\( u_s \): superficial air velocity = \( \frac{M}{Ap(1-c)} \)
\( a \): solid loading ratio
\( u_i \): terminal velocity of particle
\( \dot{g} \): superficial particle velocity = \( \frac{M}{Ap} \)
\( v \): particle velocity
\( w \): characteristic fluctuation velocity of air
\( X \): force on air per unit volume by particles
\( \delta \): diameter ratio = \( D_1/D_2 \)
\( \xi \): expansion loss coefficient, additional expansion loss coefficient
\( \rho \): air density
\( \sigma \): particle material density
\( \nu \): kinematic viscosity of air

Subscripts

a: air
p: particles
1: upstream pipe
2: downstream pipe
r: radial component
\( \theta \): azimuthal component
z: axial component

3. Experimental Setup and Procedure

Figure 1 shows a schematic diagram of experimental setup. The pipelines consisted of transparent acryl pipes placed vertically. The length and inside diameter of smaller pipe were 2.0m and 20.0mm respectively. The length of larger pipes was 3.0m and their inside diameters were 30.0mm, 40.2mm and 49.4mm. The particles were added to the air flow, which was produced by a blower, with the feeder at the upstream end.
of smaller pipe, and were separated from the air stream by the separator. The air flow rate was measured after the separator and was released to the atmosphere through the blower.

Fig.1 Schematic diagram of experimental setup

We used a calibrated quadrant flow nozzle to measure the air flow rate. The mass flow rate of particles was determined by measuring the time taken for a measured weight of particles to be discharged. The static pressure distribution along the pipe was obtained by using a digital pressure gauge. Static pressure taps were bored at 8 points in the smaller pipe and 12 points in the larger pipe. Table 1 shows the characteristics of particles. The range of pipe Reynolds number Re, as referred to the smaller pipe, are (1.9-7.6)×10^4 for β=0.67, (1.8-7.6)×10^4 for β=0.498, and (1.1-7.7)×10^4 for β=0.405.

| PVC powder | 102 | 1330 | 3.16×10^7 |
| Glass beads | 280 | 2480 | 2.14×10^7 |
| Iron beads | 760 | 7860 | 1.11 |

4. Theoretical Discussion

The particles used in this work correspond to the coarse particles whose relaxation time is larger than the characteristic-time of air flow at the expansion interface. In this case, turbulent fluctuation of particle phase can be neglected and turbulence of gas phase will be suppressed by the action of particles which dissipate turbulent fluctuation of gas phase. Thus, the loss brought about by the Reynolds stress of gas phase will decrease. Furthermore, the momentum transfer from particles to gas is expected from a reversal of mean flow velocities between two phases. Now, we consider the onset condition from the viewpoint of energy balance of gas phase turbulence.

In general, production of turbulence energy by Reynolds stress and mean shear flow (gain), SP, nearly balances with dissipation of turbulence energy (loss) due to molecular viscosity. If the particles are added, an extra production of turbulence energy by the particles, EP, appears. The flux Richardson number, RF, is defined as
\[ RF = EP/SP \]
When EP is the loss, it means that the turbulence is weakened by the additives. Our hypothesis is that the onset of an increase in the pressure recovery due to the addition of particles occurs at a certain value of RF, which brings about a negative additional expansion loss coefficient.

If we assume that the force on air per unit volume by the particles X follows Stokes' law, the ith component is given as follows:
\[ X^i = \frac{ac}{\tau^p(1-c)(u^i - v^i)} \]  \hspace{1cm} (1)

Furthermore, if we assume 1-c=1, EP and SP are given as follows:
\[ EP = -\frac{1}{(1-c)z} \left[ \begin{array}{ccc} (u^2-u)+ & (u^1-v)+ & (u^z-z) \\ (u^2-u)+ & (u^1-v)+ & (u^z-z) \\ \end{array} \right] - c^i v^i \log \]
\[ SP = \frac{1}{(1-c)z} \left[ \begin{array}{ccc} (u^2-u)+ & (u^1-v)+ & (u^z-z) \\ (u^2-u)+ & (u^1-v)+ & (u^z-z) \\ \end{array} \right] - c^i v^i \log \]  \hspace{1cm} (2)

Thus, if the turbulent fluctuation of particles can be neglected, the gravity term \(-c^i v^i \log\) is also eliminated in EP. Then, EP reduces to the same expression as that for a horizontal flow, that is,
\[ EP = \frac{ac^i v^i}{\tau^x} \]  \hspace{1cm} (3)

Under the same assumption also it is reduced to the same expression as that for horizontal flow, that is,
\[ SP = \frac{ac^i v^i}{(1+c)u_1/D_2} \]  \hspace{1cm} (3)

Therefore, considering \(ac^i v^i\), RF is estimated in the same expression as that for horizontal flow, that is,
\[ RF = \frac{(1+c)u_1}{D_2} \]  \hspace{1cm} (3)

If X follows Newton's law, Xi becomes
\[ Xi = \frac{ac^i v^i}{\tau^x} \]  \hspace{1cm} (7)

Where we put free falling \(u^i = v^i \gamma^f\) and 1-c=1. Furthermore, \(u^i_{R_i}\) is the relative velocity between two phases and is given as
\[ u^i_{R_i} = (u^i - v^i) \gamma^f \]  \hspace{1cm} (7)

If we assume that \(\gamma^f = \gamma^f(\theta, \phi)\),
and the turbulent fluctuation of particles can be neglected as in the case of Stokes' law, Eq. (2) becomes

\[ EP = \frac{20c}{\tau_s} \]  

where \( u = \frac{u}{\tau_s} \). Thus, in this case, flux Richardson number becomes

\[ RF = \frac{2m_2}{(1+\beta)u_1\tau_s} \]  

This is about twice Eq. (6).

After all, the gravity term can be neglected in the cases of both Stokes' law and Newton's law, and it is considered that the turbulent fluctuation of air flow is suppressed by adding the particles regardless of magnitude of velocities of both phases.

5. Experimental Results and Discussion

5.1 Pressure recovery and expansion loss coefficient of one phase flow

From the measurement of pipe friction coefficient for air it was shown that the pipe was hydraulically smooth. A ratio of pressure recovery for air flow, \( \Delta P_E \) to Bordc-Carnot's theoretical value \( \Delta P_{BC} \) decreases with an increase in \( Re \); according to the result of the previous reports.\(^{(1,2)}\) Moreover, the relation between a ratio of \( \frac{\Delta P}{\Delta P_{BC}} \) and \( Re \) has the same tendency as in the previous reports, where the pressure difference is defined as that between a maximum pressure recovery and a pressure at the sudden expansion interface as in the previous reports.\(^{(1,2)}\)

5.2 Static pressure distribution of gas-solid two-phase flow

Figure 2 and 3 show examples of static pressure distribution along the pipe of a gas-solid two-phase flow. The data were obtained under the same flow condition of the air flow alone. The reason why the static pressure decreases as a whole with an increase in the solid loading ratio is that the air flow rate decreases with an increase in the solid loading ratio. In the upstream pipe, the static pressure gradient decreases near the expansion interface, which is caused by the influence of particles acceleration. In the downstream pipe, with an increasing solid loading ratio, the maximum of static pressure disappears and the pressure distribution becomes flat downstream and, in particular for gas beads, there appears a monotomous pressure increase. This means that the particles velocity \( v \) is already larger than the air velocity \( u \) at the position of the maximum pressure, and then a momentum transfer from particles to gas occurs. The same tendency was observed when P.V.C powder was added. On the other hand, in the case of iron beads, the pressure gradient was scarcely different from the case of the air alone as about lm after the expansion interface in spite of an increased solid loading ratio and after that the gradient showed a slight increase, where \( \beta \) was 0.405 and the decrease in air velocity was the largest. The reason is that as far as about this position \( v \) is still smaller than \( u \) or is little larger than it. In the case of ex-
pansion ratio 0.667 when the decrease in air velocity is the lowest at the expansion, the pressure rise was not observed with iron beads suspension. The range of solid loading ratios used for the figure after Fig. 4 was such that until the distribution becomes flat, for example, in the range of Reynolds numbers shown in Fig. 2 we applied the data of which loading ratio was smaller than 0.5. This range of loading ratios was dependent on Re. The overall range of m is 0<m<2.4 for P.V.C. powder, 0<m<1.4 for glass beads, and 0<m<3.2 for iron beads. For reference, we quote the range of m in our horizontal pipeline in the previous reports, that is, 0<m<0.4 for P.V.C. powder, and 0<m<1.1 for glass beads.

5.3 Pressure recovery of gas-solid two phase flow
According to the previous reports, the increase of the nondimensional pressure recovery $\Delta P_E$ of suspension against that for air alone was well represented by using flux Richardson number $R_f$, and it was found that $R_f$ at the onset point of the increase is about $1.5 \times 10^{-3}$.

![Fig. 4 $\Delta P_E$ vs. $R_f$ ($\beta=0.667$)](image)

![Fig. 5 $\Delta P_E$ vs. $R_f$ ($\beta=0.498$)](image)

![Fig. 6 $\Delta P_E$ vs. $R_f$ ($\beta=0.405$)](image)

Figure 4 to 6 show $\Delta P_E$ vs. $R_f$ for different diameter ratios, where $R_f$ is determined by Eq. (6). For P.V.C. powder and glass beads, when $R_f$ exceeds about $2.0 \times 10^{-3}$ the effect of addition of particles appears and the pressure recovery begins to increase over that of air alone. On the other hand, in the case of iron beads, when $R_f$ exceeds about $4.0 \times 10^{-4}$ most of data begin to decrease conversely and it is seen that $\Delta P_E$ is not a function of $R_f$ alone.

Further discussion is given in the next section.

5.4 Additional expansion loss coefficient of gas-solid two phase flow
In the same way as in the previous paper, we consider the data by the additional pressure loss coefficient $\zeta_p$ to define the onset point clearly where $\zeta_p$ is defined by

$$ P_1 + \frac{1}{2} \rho U_1^2 = P_2 + \frac{1}{2} \rho U_2^2 + \zeta_p \frac{1}{2} \rho U_2^2 $$

where we use as $\zeta_p$ the value for air alone.

![Figures 7 to 9 show $\zeta_p$ vs. $R_f$](image)

where $R_f$ is defined by Eq. (6). When $\zeta_p$ is negative, the pressure recovery is essentially larger than that of air alone. For P.V.C. powder and glass beads, the value of $R_f$ at which $\zeta_p$ becomes negative is about $1.5 \times 10^{-3}$ which is independent of the diameter ratio $\beta$. This value is the same as in the case of horizontal pipeline which supports our assumption, made to introduce $R_f$, that the gravity term can be neglected. For a given $R_f$, the degree of decrease in $\zeta_p$ increases with an increasing $\beta$.

It seems that the attached eddies at the expansion section decrease with an increasing $\beta$, and that a part of momentum transfer from particles to gas flow is dissipated by the attached eddies. Thus, with an increase in $\beta$, the net momentum transfer to the gas stream increases, since the momentum dissipated by the attached eddies becomes small.

Next, we consider the case of iron beads suspension, where most values of $\zeta_p$ become larger than that of air alone when $R_f$ exceeds about $3 \times 10^{-4}$. We neglect the turbulent fluctuation of the particles and consider the direction of momentum transfer between two phases in the expansion section. If $U-V$, it is likely that the momentum of the mean flow of gas phase is transferred to the mean flow of particle phase and vice versa.

As above-mentioned, the direction of momentum transfer between the turbulent...
magnitude of mean velocities of both phases. Therefore, it is likely that \( \Delta p \) in the case of 
\( U>V \) is larger than that in the case of \( U<V \).

Moreover, according to a simple calculation of velocity of a single particle being dropped 
in a vertical downward flow of uniform air stream under an air 
drag\(^{(3)}\), the particle is still in 
acceleration at the expansion interface and its velocity is 
smaller than that when \( \zeta_p \) is 
larger than that of air alone.

In the cases of P.V.C. powder and 
glass beads, the velocity of 
particles is larger than that of 
air in nearly all the conditions.

From these it seems that in 
the most cases of iron beads \( U \) is 
larger than \( V \) at the expansion 
interface and that a fraction of 
\( \Delta p \) is so large that \( \Delta p \) is 
independent of the decrease in 
\( \Delta a \). This is the main reason why 
\( \zeta_p \) is larger than that of air 
alone. Figure 10 shows \( \zeta_p \) vs. \( Rf \) 
with \( Re_1 \) as a parameter for iron 
beads suspension is \( \beta=0.667 \).

From this figure it is found that 
\( \zeta_p \) increases with an increasing 
\( Rf \) at \( Re_1 \). According to the above-
mentioned calculation, the velocity 
difference between two phases increases with \( Re_1 \). Thus, 
it is inferred that the total 
pressure loss increases as the 
momentum transfer from gas to 
particles increases. Moreover, 
it follows that the reason why \( \zeta_p \) 
becomes larger with an increasing 
\( \beta \) is that the velocity difference 
between two phases becomes larger, 
since the decreases of air ve-
locity becomes small with an 
increasing \( \beta \). Therefore, in 
the case of iron beads, the effect of 
addition is so large that the be-
havior of suspension flow can not 
be determined by \( Rf \) alone, which 
means the gas flow of suspension 
is very different from that of 
air alone.

In addition, when \( \zeta_p \) is nega-
tive, according to the above 
calculation, the velocities of two 
phases at the expansion interface 
are nearly equal and the particle velocities 
are larger than that of air for the last 
three points in \( Rf \) in Fig. 10. The value at 
which \( \zeta_p \) becomes negative is about \( 1.5\times10^{-3} \), 
which is different from that at which \( \zeta_p \) be-
comes positive.

From the above-mentioned, it is seen that 
\( V>U \) at the expansion interface 
is necessary, at least, as the condition for \( \zeta_p \) 
being negative. Therefore, also for iron
beads, if $V > U$ at that interface, $\gamma_p$ will be
negative. Conversely for P.V.C. powder and
glass beads, if $U > V$ at that interface, $\gamma_p$
will become positive as just as in the case
of iron beads. If $U > V$ at the expansion
interface, it is difficult for $\gamma_p$ to
correlate with $RF$ alone even if Stokes' law
or Newton's law is applied.

6. Conclusions

By using three different diameter
ratios and particles of three different free
falling velocities, we examined how the
gravity affects the reverse transfer of
momentum from particle phase to gas phase at
the expansion interface, and obtained the
following results.
(1) For coarse particles as those in this
experiment, the inertia effect can not be
neglected but the gravity effect can be

neglected.
(2) The condition in that the pressure re-
covery of suspension becomes larger than
that of air alone or that the addition-
al expansion loss coefficient becomes nega-
tive is that at least the particle velocity
is larger than the air velocity at the ex-
pansion interface. The onset condition in
this case is given by the same value of the
flux Richardson number in the horizontal
pipeline.

References

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