Analysis of Vibration by Component Mode Synthesis Method*

(Part 5, Forced Vibration II)

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A method is presented to analyze the vibration of a complex mechanical structure by using the natural modes of its components. The structure is divided into some components. The natural modes of each component in the condition of its interface region fixed are calculated by the finite element method. The displacement of the total structure is represented with those natural modes. The equation of motion of the total structure is transformed into the one on a generalized coordinate. The response of the forced vibration is analyzed with these natural modes on the generalized coordinate by using the technique of the modal analysis.

In the present report, the way of analysis of the forced vibration by the authors' proposed method is explained. And as an example of application of this method, the vibration of a continuous centrifuge is analyzed by the method. The calculated results of the machine are compared with the experimental ones.

Key Words: Vibration, Forced Vibration, Natural Frequency, Natural Mode, Numerical Analysis, Component Mode Synthesis Method, Finite Element Method, Generalized Coordinate, Modal Analysis

1. Introduction

Recently, the technology of the vibration analysis has been developed remarkably by using the finite element method (FEM in brief) and big computers. But the vibration of complex and big mechanical structures hardly can be analyzed by FEM. Then, many kinds of the component synthesis methods⁽¹⁾ are being studied by many scientists.

The authors⁽²⁾⁻⁽⁵⁾ proposed "Component Mode Synthesis Method (CMS in brief)" as one of these methods. The process of calculation of CMS is as follows.

A structure is divided into some components which can easily be analyzed by FEM. The interface region of these components is considered as the part of an independent component named "Interface component". Its displacement is represented with a linear combination of the natural modes calculated from the reduced mass and the reduced stiffness matrices of other components.

The displacements of the interior regions of all components except the interface component are represented with the natural modes of the interface component and the constrained natural modes of the interior regions. The equation of the total structure is transformed into the one on a generalized coordinate of these natural modes.

The forced vibration is analyzed with the modal natural modes obtained from the equation on the generalized coordinate by using the technique of the modal analysis.

As an example of application of CMS to real mechanical structures, a continuous centrifuge is analyzed by CMS. The results of the numerical analysis are compared with the experimental ones.

2. Procedure

The analysis of a structure shown in Fig.1 is explained here.

Fig.1 Structure

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The structure is divided into three components which can be analyzed by FEM rather easily. The interior regions of three components (Comp.1, Comp.2 and Comp.3) are called the regions $1a$, $1b$ and $1c$ respectively. The interface region between Comp.1 and Comp.2 is called the region b . The interface region between Comp.1 and Comp.3 is called the region c . The one between Comp.2 and Comp.3 is called the region d .

The displacements of those interface regions are represented with the reduced mass matrices and the reduced stiffness matrices of the three components by Guyan's method⁽⁶⁾.

The static equation of Comp.1 is written as

$$\begin{bmatrix} K_{aa}^{(1)} & K_{ab}^{(1)} & K_{ac}^{(1)} \\ K_{ba}^{(1)} & K_{bb}^{(1)} & K_{bc}^{(1)} \\ K_{ca}^{(1)} & K_{cb}^{(1)} & K_{cc}^{(1)} \end{bmatrix} \begin{Bmatrix} \delta_{1a} \\ \delta_b \\ \delta_c \end{Bmatrix} = \begin{Bmatrix} 0 \\ X \\ Y \end{Bmatrix} \quad \dots\dots\dots (1)$$

where X is the internal force imposed between Comp.1 and Comp.2. Y is the one between Comp.1 and Comp.3. Guyan's method of the static reduction is applied to Comp.1. From the upper part of Eq.(1), we have

$$\{\delta_{1a}\} = -[K_{aa}^{(1)}]^{-1}[K_{ab}^{(1)}]\{\delta_b\} - [K_{aa}^{(1)}]^{-1}[K_{ac}^{(1)}]\{\delta_c\} = [T_{1b}]\{\delta_b\} + [T_{1c}]\{\delta_c\} \quad \dots\dots\dots (2)$$

The internal freedom is eliminated. Using Eq.(2), we have

$$\begin{Bmatrix} \delta_{1a} \\ \delta_b \\ \delta_c \end{Bmatrix} = \begin{bmatrix} T_{1b} & T_{1c} \\ I & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} \delta_b \\ \delta_c \end{Bmatrix} = [R_1] \begin{Bmatrix} \delta_b \\ \delta_c \end{Bmatrix} \quad \dots\dots\dots (3)$$

The mass and the stiffness matrices of Comp.1 are reduced with the matrix $[R_1]$ in Eq.(3).

$$\begin{bmatrix} \bar{M}_{bb}^{(1)} & \bar{M}_{bc}^{(1)} \\ \bar{M}_{cb}^{(1)} & \bar{M}_{cc}^{(1)} \end{bmatrix} = \begin{bmatrix} T_{1b}^T M_{bb}^{(1)} T_{1b} + T_{1b}^T M_{ab}^{(1)} + M_{ba}^{(1)} T_{1b} + M_{bb}^{(1)} & T_{1b}^T M_{bc}^{(1)} T_{1c} + T_{1b}^T M_{ac}^{(1)} + M_{ba}^{(1)} T_{1c} + M_{bc}^{(1)} \\ T_{1c}^T M_{ab}^{(1)} T_{1b} + T_{1c}^T M_{bb}^{(1)} + M_{ca}^{(1)} T_{1b} + M_{cb}^{(1)} & T_{1c}^T M_{bc}^{(1)} T_{1c} + T_{1c}^T M_{ac}^{(1)} + M_{ca}^{(1)} T_{1c} + M_{cc}^{(1)} \end{bmatrix} \quad \dots\dots\dots (4)$$

$$\begin{bmatrix} \bar{K}_{bb}^{(1)} & \bar{K}_{bc}^{(1)} \\ \bar{K}_{cb}^{(1)} & \bar{K}_{cc}^{(1)} \end{bmatrix} = \begin{bmatrix} T_{1b}^T K_{bb}^{(1)} T_{1b} + T_{1b}^T K_{ab}^{(1)} + K_{ba}^{(1)} T_{1b} + K_{bb}^{(1)} & T_{1b}^T K_{bc}^{(1)} T_{1c} + T_{1b}^T K_{ac}^{(1)} + K_{ba}^{(1)} T_{1c} + K_{bc}^{(1)} \\ T_{1c}^T K_{ab}^{(1)} T_{1b} + T_{1c}^T K_{bb}^{(1)} + K_{ca}^{(1)} T_{1b} + K_{cb}^{(1)} & T_{1c}^T K_{bc}^{(1)} T_{1c} + T_{1c}^T K_{ac}^{(1)} + K_{ca}^{(1)} T_{1c} + K_{cc}^{(1)} \end{bmatrix} \quad \dots\dots\dots (5)$$

In the same way as mentioned above, the mass and the stiffness matrices of Comp.2 and Comp.3 are reduced.

Adding these obtained reduced matrices, the equation for the interface regions becomes

$$\left(\begin{bmatrix} \bar{K}_{bb}^{(1)} + \bar{K}_{bb}^{(2)} & \bar{K}_{bc}^{(1)} & \bar{K}_{bd}^{(2)} \\ \bar{K}_{cb}^{(1)} & \bar{K}_{cc}^{(1)} + \bar{K}_{cc}^{(2)} & \bar{K}_{cd}^{(2)} \\ \bar{K}_{db}^{(2)} & \bar{K}_{dc}^{(2)} & \bar{K}_{dd}^{(2)} + \bar{K}_{dd}^{(3)} \end{bmatrix} - \omega^2 \begin{bmatrix} \bar{M}_{bb}^{(1)} + \bar{M}_{bb}^{(2)} & \bar{M}_{bc}^{(1)} & \bar{M}_{bd}^{(2)} \\ \bar{M}_{cb}^{(1)} & \bar{M}_{cc}^{(1)} + \bar{M}_{cc}^{(2)} & \bar{M}_{cd}^{(2)} \\ \bar{M}_{db}^{(2)} & \bar{M}_{dc}^{(2)} & \bar{M}_{dd}^{(2)} + \bar{M}_{dd}^{(3)} \end{bmatrix} \right) \begin{Bmatrix} x_b \\ x_c \\ x_d \end{Bmatrix} = \{0\} \quad \dots\dots\dots (6)$$

Solving Eq.(6), the natural modes are obtained. The displacements of the interface regions are represented with a linear combination of these natural modes.

$$\begin{Bmatrix} \delta_b \\ \delta_c \\ \delta_d \end{Bmatrix} = \begin{bmatrix} \phi_b \\ \phi_c \\ \phi_d \end{bmatrix} \{\xi\} \quad \dots\dots\dots (7)$$

where the matrix $\begin{bmatrix} \phi_b \\ \phi_c \\ \phi_d \end{bmatrix}$ includes the natural modes from the first mode to the k -th one.

$\{\xi\}$ is a coefficient vector.

Next, the way of representing the displacement in the interior regions is considered as follows. The displacements of the interior regions are represented with the influences of the interface regions and the constrained modes of the components.

Considering Comp.1, the influence of the interface regions b , and c is written as Eq.(3). The constrained natural modes of Comp.1 are obtained by solving the following equation

$$([K_{aa}^{(1)}] - \omega^2 [M_{aa}^{(1)}])\{x\} = \{0\} \quad \dots\dots\dots (8)$$

The constrained mode matrix is written as $[\phi_{1a}]$. Then, the displacement of the interior region of Comp.1 is represented as

$$\{\delta_{1a}\} = [T_{1b}]\{\delta_b\} + [T_{1c}]\{\delta_c\} + [\phi_{1a}]\{\xi_1\} \quad \dots\dots\dots (9)$$

Substituting Eq.(7) into Eq.(9), we have

$$\{\delta_{1a}\} = [T_{1b}\phi_b + T_{1c}\phi_c, \phi_{1a}]\begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix} \quad \dots\dots\dots (10)$$

In the same process as mentioned about Comp.1, the displacements of the interior regions of Comp.2 and Comp.3 are represented as

$$\{\delta_{2a}\} = [T_{2b}\phi_b + T_{2a}\phi_a, \phi_{2a}]\begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix} \quad \dots\dots\dots (11)$$

$$\{\delta_{3a}\} = [T_{3c}\phi_c + T_{3a}\phi_a, \phi_{3a}]\begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix} \quad \dots\dots\dots (12)$$

The matrix $[\phi_{2a}]$ includes the first constrained natural mode to the m -th one of Comp.2. It is obtained by solving the equation

$$([K_{aa}^{(2)}] - \omega^2 [M_{aa}^{(2)}])\{x\} = \{0\} \quad \dots\dots\dots (13)$$

The matrix $[\phi_{3a}]$ includes the first constrained natural modes to the n -th one of Comp.3 which are obtained by solving the equation

$$([K_{aa}^{(2)}] - \omega^2 [M_{aa}^{(2)}])\{\chi\} = \{0\} \quad \dots\dots\dots (14)$$

From Eq.(7), Eq.(10), Eq.(11) and Eq.(12), the displacement of the total structure is represented as

$$\begin{Bmatrix} \delta_b \\ \delta_c \\ \delta_d \\ \delta_{1a} \\ \delta_{2a} \\ \delta_{3a} \end{Bmatrix} = \begin{bmatrix} \phi_b & 0 & 0 & 0 \\ \phi_c & 0 & 0 & 0 \\ \phi_d & 0 & 0 & 0 \\ T_{1b}\phi_b + T_{1c}\phi_c & \phi_{1a} & 0 & 0 \\ T_{2b}\phi_b + T_{2c}\phi_c & 0 & \phi_{2a} & 0 \\ T_{3b}\phi_b + T_{3c}\phi_c & 0 & 0 & \phi_{3a} \end{bmatrix} \begin{Bmatrix} \xi_c \\ \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix} = [A]\{\xi\} \quad \dots\dots\dots (15)$$

Since the equation of motion of the total structure is

$$\begin{aligned} & \begin{bmatrix} M_{bb}^{(1)} + M_{bb}^{(2)} & M_{bc}^{(1)} & M_{bd}^{(2)} & M_{ba}^{(1)} & M_{ba}^{(2)} & 0 \\ M_{cb}^{(1)} & M_{cc}^{(1)} + M_{cc}^{(2)} & M_{cd}^{(2)} & M_{ca}^{(1)} & 0 & M_{ca}^{(2)} \\ M_{db}^{(2)} & M_{dc}^{(2)} & M_{dd}^{(2)} + M_{dd}^{(3)} & 0 & M_{da}^{(2)} & M_{da}^{(3)} \\ M_{ab}^{(1)} & M_{ac}^{(1)} & 0 & M_{aa}^{(1)} & 0 & 0 \\ M_{ab}^{(2)} & 0 & M_{ad}^{(2)} & 0 & M_{aa}^{(2)} & 0 \\ 0 & M_{ac}^{(2)} & M_{ad}^{(2)} & 0 & 0 & M_{aa}^{(3)} \end{bmatrix} \begin{Bmatrix} \delta_b \\ \delta_c \\ \delta_d \\ \delta_{1a} \\ \delta_{2a} \\ \delta_{3a} \end{Bmatrix} \\ & + \begin{bmatrix} C_{bb}^{(1)} + C_{bb}^{(2)} & C_{bc}^{(1)} & C_{bd}^{(2)} & C_{ba}^{(1)} & C_{ba}^{(2)} & 0 \\ C_{cb}^{(1)} & C_{cc}^{(1)} + C_{cc}^{(2)} & C_{cd}^{(2)} & C_{ca}^{(1)} & 0 & C_{ca}^{(2)} \\ C_{db}^{(2)} & C_{dc}^{(2)} & C_{dd}^{(2)} + C_{dd}^{(3)} & 0 & C_{da}^{(2)} & C_{da}^{(3)} \\ C_{ab}^{(1)} & C_{ac}^{(1)} & 0 & C_{aa}^{(1)} & 0 & 0 \\ C_{ab}^{(2)} & 0 & C_{ad}^{(2)} & 0 & C_{aa}^{(2)} & 0 \\ 0 & C_{ac}^{(2)} & C_{ad}^{(2)} & 0 & 0 & C_{aa}^{(3)} \end{bmatrix} \begin{Bmatrix} \delta_b \\ \delta_c \\ \delta_d \\ \delta_{1a} \\ \delta_{2a} \\ \delta_{3a} \end{Bmatrix} \\ & + \begin{bmatrix} K_{bb}^{(1)} + K_{bb}^{(2)} & K_{bc}^{(1)} & K_{bd}^{(2)} & K_{ba}^{(1)} & K_{ba}^{(2)} & 0 \\ K_{cb}^{(1)} & K_{cc}^{(1)} + K_{cc}^{(2)} & K_{cd}^{(2)} & K_{ca}^{(1)} & 0 & K_{ca}^{(2)} \\ K_{db}^{(2)} & K_{dc}^{(2)} & K_{dd}^{(2)} + K_{dd}^{(3)} & 0 & K_{da}^{(2)} & K_{da}^{(3)} \\ K_{ab}^{(1)} & K_{ac}^{(1)} & 0 & K_{aa}^{(1)} & 0 & 0 \\ K_{ab}^{(2)} & 0 & K_{ad}^{(2)} & 0 & K_{aa}^{(2)} & 0 \\ 0 & K_{ac}^{(2)} & K_{ad}^{(2)} & 0 & 0 & K_{aa}^{(3)} \end{bmatrix} \begin{Bmatrix} \delta_b \\ \delta_c \\ \delta_d \\ \delta_{1a} \\ \delta_{2a} \\ \delta_{3a} \end{Bmatrix} = [F] \quad \dots\dots\dots (16) \end{aligned}$$

the equation can be transformed into the one on the generalized coordinate of the natural modes by substituting Eq.(15) into Eq.(16) and multiplying by the matrix $[A]^T$.

As the results of this process, the equation of motion of the total structure on the generalized coordinate is written as

$$[\bar{M}]\{\dot{\xi}\} + [\bar{C}]\{\dot{\xi}\} + [\bar{K}]\{\xi\} = [A]^T[F] \quad \dots\dots\dots (17)$$

The degree of freedom of this equation is $(k+l+m+n)$. The degree of freedom of Eq.(17) is usually much less than that of Eq.(16). The natural frequencies and the natural modes on the generalized coordinate are obtained by solving the equation of the eigenvalue problem

$$(\mu^2[\bar{M}] + \mu[\bar{C}] + [\bar{K}])\{\xi_0\} = \{0\} \quad \dots\dots\dots (18)$$

The natural mode $\{\xi_0\}$ is transformed into a natural mode or physical coordinate by using Eq.(15).

The responses of forced vibration are analyzed by the technique of the modal analysis. Ignoring the term of damping in Eq.(18), the natural modes from the first to the p -th are obtained by solving the equation. The matrix including those modes is written as $[\xi_0]$. Then, the vector $\{\xi\}$ is expressed as

$$\{\xi\} = \{\xi_0\}\{d\} \quad \dots\dots\dots (19)$$

where $\{d\}$ is a coefficient vector. Substituting Eq.(19) into Eq.(17) and multiplying by the matrix $[\xi_0]^T$, Eq.(17) becomes

$$(-\omega^2[\xi_0]^T[\bar{M}][\xi_0] + j\omega[\xi_0]^T[\bar{C}][\xi_0] + [\xi_0]^T[\bar{K}][\xi_0])\{d\} = [\xi_0]^T[A]^T[F] \quad \dots\dots\dots (20)$$

Here, $[\xi_0]^T[\bar{M}][\xi_0]$ and $[\xi_0]^T[\bar{K}][\xi_0]$ are the diagonal matrices because of the orthogonality of the natural modes. Then, when the non-diagonal elements of $[\xi_0]^T[\bar{C}][\xi_0]$ can be ignored Eq.(20) is written as

$$\left(-\omega^2 \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_i & \\ & & & m_p \end{bmatrix} + j\omega \begin{bmatrix} c_1 & & & \\ & c_2 & & \\ & & c_i & \\ & & & c_p \end{bmatrix} + \begin{bmatrix} k_1 & & & \\ & k_2 & & \\ & & k_i & \\ & & & k_p \end{bmatrix} \right) \begin{Bmatrix} d_1 \\ d_2 \\ \vdots \\ d_i \\ \vdots \\ d_p \end{Bmatrix} = \begin{Bmatrix} z_1 \\ z_2 \\ \vdots \\ z_i \\ \vdots \\ z_p \end{Bmatrix} \quad \dots\dots\dots (21)$$

The element d_i of the vector $\{d\}$ is represented as

$$d_i = \frac{z_i}{k_i - \omega^2 m_i + j\omega c_i} \quad \dots\dots\dots (22)$$

Substituting the vector $\{d\}$ into Eq.(19), the vector $\{\xi\}$ is obtained. And substituting the vector $\{\xi\}$ into Eq.(15), the displacement of the total structure is obtained.

3. Practical Introduction of Damping

If the damping matrix could be obtained theoretically, the vibration of structures would be able to be analyzed exactly. Here, a few method of practical introduction of damping are explained.

The first one is an application of the proportional damping. The damping matrix is made by using the mass matrix [M] and the stiffness matrix [K].

$$[C] = \alpha[M] + \beta[K] \quad \dots\dots\dots (23)$$

The coefficients α and β are obtained by using the results of "curve fitting" technique.

"Curve fitting" in which damping is assumed to be a proportional damping is a technique for decision of the finest modal parameters Y , $(\phi_s^T \phi_r^T / k_i)$, ζ_i , W and Ω_i from the data obtained by the experiment. The result of the experiment is assumed to be represented as

$$\frac{\delta_s}{F_r} = -\frac{1}{Y\omega^2} + \sum_{i=1}^n \frac{\phi_s^T \phi_i^T}{K_i \left\{ 1 - \left(\frac{\omega}{\Omega_i} \right)^2 + 2j\zeta_i \frac{\omega}{\Omega_i} \right\}} + \frac{1}{W} \quad \dots\dots\dots (24)$$

where

Y is the residual mass

W is the residual stiffness

ϕ_i^T is the i -th natural mode

Ω_i is the i -th natural angular frequency

ζ_i is the i -th modal damping ratio

The coefficients α and β and the modal damping ratio are related as

$$2\zeta_i = \alpha/\Omega_i + \beta\Omega_i \quad \dots\dots\dots (25)$$

The finest α and β are decided with many modal damping ratios ζ_i and the natural frequencies Ω_i by the least square method. Then, Eq. (22) is changed as

$$d_i = \frac{z_i}{k_i \left\{ 1 - \left(\frac{\omega}{\Omega_i} \right)^2 + j\alpha(\omega/\Omega_i) + \beta \right\}} \quad \dots\dots\dots (26)$$

The coefficient α of normal structures is often some number between zero and 100. The coefficient β is often such a number that the diagonal elements of $\beta[K]$ are as large as those of the mass matrix [M].

The second method uses the modal damping ratios ζ_i directly. In this method, Eq. (22) is changed as

$$d_i = \frac{z_i}{K_i \left\{ 1 - \left(\frac{\omega}{\Omega_i} \right)^2 + 2j\zeta_i \frac{\omega}{\Omega_i} \right\}} \quad \dots\dots\dots (27)$$

The third one can be used when the damping mechanisms (for example, oil dampers and oil film in bearings) are set in the structure. The effect of those damping mechanisms will great predominate in the damping of the structure. The values of dampings of those mechanisms can be often obtained theoretically. Then, the damping matrix is made with only those elements, and it is used.

Anyway, since theoretically the damping cannot be calculated exactly now, it will be important to provide some data-bases with regard to the damping from the data of the experiments.

4. An Example

4.1 Modeling of continuous centrifuge

The base of the machine is shown in Fig.2. An exciting force is imposed at the point F in the experiment. The exciting force vector is parallel to Z-coordinate axis. The base is divided into eight components as shown in Fig.3. All the components are divided into triangular elements as shown in Fig.5 to Fig.12. The base is made of these eight components as shown in Fig.4.

The rotor of the continuous centrifuge is shown in Fig.13. The usual operating speed is about 2720rpm. This is simply modeled as shown in Fig.14.

Two big bearing boxes are modeled with the isoparametric elements as shown in Fig.15. The rotor is connected to these bearing boxes by springs whose stiffnesses are shown in Fig.16.

In the experiment, the specimen is excited by a harmonic force imposed by an oil-exciter.

Fig.2 The base of the continuous centrifuge

Fig.3 Division of the base into eight components

Fig.4 Division of the base into finite elements

Fig.5 The first component of the base

Fig.6 The second component of the base

Fig.7 The third component of the base

Fig.8 The 4th component of the base

Fig.9 The 5th component of the base

Fig.10 The 6th component of the base

Fig.11 The 7th component of the base

Fig.12 The 8th component of the base

Fig.13 The rotor

Fig.14 Modeling of the rotor

Fig.15 Modeling of the bearing box

Stiffness of
Spring
 $R_x = 1.0 \times 10^5 \text{ N/mm}$
 $R_y = 1.0 \times 10^3 \text{ N/mm}$
 $R_z = 1.0 \times 10^5 \text{ N/mm}$

Fig.16 Connection of the rotor and the bearing box

4.2 Analysis of the base

The natural frequencies obtained by CMS and the experiment are shown in Table 1. The response (compliance) at the point F of the forced vibration is shown in Fig.17. Damping is ignored in this case. Both analytical natural frequencies and analytical compliances corresponding with those of the experiment as well as CMS can be used practically. The first and the second mode shapes are shown in Fig.18. The mode shapes obtained by CMS correspond with the experimental ones well, too.

It took 1476 seconds of CPU time to calculate 36 natural frequencies and natural modes and the responses at all nodal points from 50Hz to 350Hz at each 0.5Hz step by running a program of CMS made by the authors on HITAC M-200H system.

Table 1 Natural frequencies of the base
Number of adopted modes in CMS
Interface region: 10
Each interior region: 15

order	EXP.	CMS
1	103	104
2	114	123
3	154	160
4	176	172
5	182	183
6	192	186
7	203	195

(Hz)

Fig.17 Transfer function
Measured point: F
Excited point: F

Fig.18 Examples of natural modes
of the base

4.3 The system of the base and the bearing boxes

The system of connecting the base and the bearing boxes is analyzed. In this case, since it is assumed that the stiffness of the bearing boxes can be neglected, only the mass is considered in the calculation. The natural frequencies obtained by CMS and the experiment are shown in Table 2. The results by CMS correspond with those by the experiment well. The natural frequencies of this system are made low in comparison with those in Table 1 because of the influence of the bearing boxes.

Table 2 Natural frequencies of the system
of the base and the bearing boxes
Number of adopted modes in CMS
Interface region: 10
Each interior region: 15

order	EXP.	CMS
1	92	96
2	111	113
3	150	160
4	164	162
5	170	173
6	—	173
7	178	177
8	187	184
9	196	195

(Hz)

4.4 The system of connecting the base, the bearing boxes and the rotor

The natural frequencies of the structural system including the base, the bearing boxes and the rotor are shown in Table 3. The response at the point R (in Fig.4) is shown in Fig.19. The fitting of the calculated result and the experimental one is not so good as that of the result about the base structure (shown in Fig.18). It seems that the degree of the correctness of modeling, the non-linearity in the real structure and the boundary condition influence the result. In the experiment, the magnitude of the exciting force is 980 Newton by an oil-exciter. The modal damping ratios shown in Table 4 are adopted in the calculation by CMS.

The first natural vibration (61Hz by CMS, 66Hz by experiment) is that of the rotor's bearing. The natural frequency of the rotor is 67Hz by the calculation on condition that the base and the bearing boxes are assumed to be rigid. Then, the first natural frequency of the rotor is shifted down by considering the base and the bearing boxes to be elastic. The natural frequencies of the base's first twist mode are 74Hz by experiment and 75Hz by CMS. Those of the first bending mode are 136Hz by experiment and 133 Hz by CMS.

Table 3 Natural frequencies of the system
of the base, the bearing boxes
and the rotor

Number of adopted modes in CMS
Interface region: 10
Each interior region: 15
of. Only natural frequencies
which are certainly detected
are denoted in the column
of EXP.

order	EXP.	CMS
1	66	61
2	72	73
3	74	75
4	88	85
5	102	104
6	112	112
7	136	133
8	—	142
9	155	160
10	173	172
11	—	173
12	—	174

(Hz)

Fig.19 Transfer function
Measured point: R
Excited point: F

Table 4 Model damping ratios adopted in CMS

order	ζ_j
1	1.75×10^{-4}
2	4.60×10^{-4}
3	1.38×10^{-3}
4	2.40×10^{-3}
5	3.50×10^{-3}
6	6.00×10^{-3}
7	1.60×10^{-2}
8	2.00×10^{-2}
9	1.00×10^{-4}
10	6.40×10^{-4}
11	6.40×10^{-4}
12	6.40×10^{-4}

Thus, the natural frequencies of the base change much under the influences of the rotor and the bearing boxes. It will be necessary to analyze the structure like this machine as a total system.

It took 934 seconds of CPU time to obtain 20 natural frequencies and natural modes and the responses at all nodal point from 50Hz to 175Hz at each 0.2Hz step.

5. Conclusion

It is explained how to calculate the forced vibration by "Component Mode Synthesis Method" proposed by the authors. As an example of applications to real mechanical machines, a continuous centrifuge is analyzed by this method. The calculated results agree with the experimental ones. It is obvious that the vibration of those machines must be analyzed as a total system and that the analysis takes rather a short CPU time by the present method.

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