Characteristics of the Flow around Three Circular Cylinders Arranged in Line*

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A detailed experimental investigation was carried out on the characteristics of the flow around three circular cylinders arranged in line. There are three cases concerned with the behavior of the shear layers separated from the first cylinder on the downstream ones: the first is a case without reattachment (W), the second is one with reattachment (R) and the third is one rolling up in the front region of the downstream cylinders (J). The flow patterns were classified according to W, R and J as follows: patterns A(W,W), B'(W,W=J,R), B(R,R), C(R,J), E(R,J=J,J) and D(J,J). The characteristics of these flow patterns were clarified. The results were compared with conventional results on two and four circular cylinders arranged in line.

Key words : Unsteady Flow, Three Circular Cylinders, Strouhal number, Pressure Distributions, Flow Patterns

1. Introduction

The structure of a gas flow around five tubes has been investigated by Ishigai and Nishikawa[1], and the heat transfer around three and four circular cylinders closely spaced in a cross flow of air has been measured by Aiba et al.[2,4]. Ishigai and Nishikawa[1] carried out not only visualization of the flow around the tubes in the range of $1.4 \times 10^3 \leq Re \leq 1.35 \times 10^4$, but also measurements of the pressure distributions and the vortex shedding frequencies behind the tubes. They concluded that no tube expect the last downstream tube has a vortex formation region when $L/d \leq 1.5$, while all tubes shed vortices in tube banks with $L/d \geq 2.0$. Moreover they stated that the pressure distribution changes drastically at $1.5 \leq L/d \leq 2.0$ corresponding to the change in the flow pattern. Aiba et al.[4] stated that there exists a critical Reynolds number Re_c at which the heat transfer and flow around the cylinders change drastically, and that the heat transfer on the rear face of the first cylinder and on the front face of the second one decreases in the region of $Re < Re_c$.

Concerning the mutual interference among the flows around plural bodies, the flow around two cylinders arranged in tandem which is the most simple arrangement of tube banks in line, has been investigated by many workers for a long time. Recently, Zdravkovich[5] reviewed a number of results of investigations in this field. Subsequently, one of the authors carried out detailed experimental investigations on the flow around two circular cylinders with equal diameter[6,7] and with different diameters[8] arranged in tandem. The unclear

flow in the previous reports was clarified and a unique flow pattern which had not been reported at that time was reported in ref.[6,8]. From the above mentioned, the authors investigated the flow around three cylinders arranged in line in more detail. Our attention is focused on the experiments concerning the behavior of a shear layer separated from the first cylinder, occurrence of the unique phenomenon in the case of two cylinders, and an abrupt change in flow pattern of tubes in an in-line bank and its hysteresis. The results obtained were discussed by comparing with the data on two cylinders[6,8] and tubes in an in-line bank[3,4].

Nomenclature

C_D : drag coefficient $= \int_0^\pi C_p \cos \phi d\phi$
 C_p : pressure coefficient $= (p - p_0) / 0.5 \rho u_0^2$
 d : diameter of circular cylinder
 f : vortex shedding frequency, frequency
 L : longitudinal spacing between the axes of cylinders
 p_0 : static pressure at free stream
 $\bar{p}, \bar{p}'$: time-averaged and fluctuating pressures
 Δp : R.M.S. of fluctuating pressure $= (\overline{p'^2})^{1/2}$
 Re : Reynolds number $= u_0 d / \nu$
 Re_c : reattachment Reynolds number
 S : Strouhal number $= fd / u_0$
 u_0 : free stream velocity
 ν : kinematic viscosity of fluid
 ρ : density of fluid
 ϕ : circumferential angle on cylinders
 Subscripts
 1, 2, 3 : first, second and third cylinder

2. Experimental Apparatus and Procedure

The configuration of the three cylinders is shown in Fig. 1. The diameters of these cylinders, d , were 34 mm and the free stream velocity, u_0 , ranged from 5 to 18 m/s. The Reynolds numbers were, therefore,

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in the range $1.09 \times 10^4 \leq Re \leq 3.92 \times 10^4$. And the turbulence intensities of the free stream in a low speed wind tunnel and in a smoke wind tunnel were 0.55 and 0.60 % in this range of Reynolds numbers, respectively. The longitudinal spacing among the centers of the three cylinders, L/d , varied from 1.0 to 4.0. Experimental apparatus and procedure were the same as those described in detail in ref.[6~8]. Both the distributions of the time-averaged pressures and the R.M.S. of fluctuating pressures around these cylinders were measured by a manometer and by a semi-conductor pressure sensor, respectively. The velocity fluctuations around these cylinders were measured using a hot-wire anemometer. The Strouhal numbers of the vortex shedding from the above cylinders were obtained from the frequency analysis of the velocity fluctuations. The flow around the cylinders was visualized using a smoke wind tunnel and the flow on the surface of the cylinders was by an oil-film method.

3. Experimental Results and Discussions

3.1 Vortex shedding frequency

It is clear by previous investigations that the flow around the downstream cylinders is dominated by the behavior of the shear layer separated from the first cylinder. From this view point, the pressure distributions around the second and third cylinders at $Re = 2.18 \times 10^4$, are shown in Fig. 2. Difference in their profiles can be detected. Their profiles, which are associated with the maximum value of pressure coefficient in the front region on the cylinders according to the spacing ratios, are classified into three types. As to the second cylinder, the first type is one without reattachment of the shear layer, that is, the maximum value of C_p at $L/d = 1.18$ is absent. This case will hereinafter be referred as "W". The second type is one with reattachment of the shear layer, that is, in the range $1.32 \leq L/d \leq 2.94$ the value of C_p has maximum at $\phi = 60^\circ \sim 70^\circ$. It is referred to as "R". The third type is one of rolling up of the shear layer in front region of the second cylinder, that is, at $L/d \leq 3.53$ the value of C_p has maximum at $\phi = 0^\circ$. It is referred to as "J". As to the third cylinder, the first type "W" develops at $L/d = 1.18$, the second one "R" at $L/d = 1.32$ and 2.06 and the third one "J" at $L/d \leq 2.94$. Figure 3 shows the pressure distributions around the three cylinders for various spacings at $Re = 2.2 \times 10^4$. If combination of such types for the downstream cylinders is made up, it is possible to read off variation in flow pattern around the two cylinders. In other words, fundamentally four

Fig. 1 Coordinate system and symbols

Fig. 2 Pressure distribution around the second and third cylinders

Fig. 3 Combinations of typical pressure distributions around three cylinders and flow patterns

patterns can be identified: A(W,W), B(R,R), C(R,J) and D(J,J). In addition to this, a pattern E of a bistable flow in between the patterns C and D is identified. Seeing the pattern A at $L/d=1.18$, there is no significant difference in gap pressure between the first and second cylinders and that between the second and third ones. And the gap pressure of the third cylinder is higher than that of the second one. At the pattern B, the shear layers from the first cylinder reattach onto the second and third cylinders alternately, and then a certain difference in gap pressure between the two cylinders comes out. For pattern C, one shear layer reattaches onto the second cylinder and the other shear layer rolls up and forms a separated vortex in front region of the third cylinder. As to the pattern D, the two shear layers roll up and form two separated vortices in the front region of the second cylinder alternately, hence the profiles of the pressure distributions around the second and third cylinders are identical. At the spacing of $L/d=2.06$, the pressure distribution around the third cylinder looks like that of transition to the pattern C, because the distribution on the front face is level with no peak. Such a distribution is similar to that of two cylinders of different diameters with the ratio $d_2/d_1=0.68$ arranged in tandem[8], that is, it has a resemblance to the pressure distribution around the downstream cylinder for $L/d_1=3.0$. It seems to suggest that the shear layer from the first cylinder without reattachment onto the second cylinder reattaches onto the front stagnation region of the third cylinder alternately.

3.2 Drag coefficient

The variations of drag coefficients of the three cylinders with L/d are shown in Fig. 4. The drag coefficients were obtained by integrating the pressure distribution at $Re=2.2 \times 10^4$. The coefficients C_D increase discontinuously at the reattachment spacing $L/d=1.32$ as well as the jumping one $L/d=3.53$. The value of the first cylinder, C_{D1} , decreases gradually with an increase in L/d , and then it increases at about $L/d=2.8$ less than that of the jumping space. The

Fig. 4 Drag coefficients of three cylinders

drag coefficient of two cylinders arranged in tandem does not show such a tendency. The value of the second cylinder, C_{D2} , presents the same tendency as that in the case of the two cylinders. For the spacing less than $L/d=3.53$, the value of C_{D2} is minus and it acts as a thrust: when the shear layer reattaches onto the second cylinder, the value increases stepwise from -0.6 to -0.4 , and then reaches a maximum of -0.2 at $L/d=2.5$. The coefficient C_{D3} of the third cylinder takes any value of 0.0 to 0.15 in the range of $L/d=1.03$ to 1.25 , and increases rapidly at $L/d=1.32$. Then, it increases with an increase in L/d , and beyond $L/d=2.4$ it is nearly equal to 0.6 . For the spacing larger than $L/d=3.53$, it decreases slightly. The combined drag coefficient of three cylinders, C_D , is shown in the figure.

3.3 Strouhal number

Figures 5(a) and (b) indicate the Strouhal number behind the third cylinder. As seen from the figure (a), in the case of three cylinders, the Reynolds numbers have no effect upon the Strouhal number, although for two cylinders[6,7], the Reynolds numbers have direct effects upon the Strouhal numbers of the patterns B and G. As seen from the figure, for $L/d=1.32$ the value of Strouhal number decreases abruptly to half and less at the Reynolds number, Re_c , at which the shear layer separated

(a) effect of Reynolds number

(b) effect of spacing

Fig. 5 Strouhal number

from the first cylinder reattaches onto the downstream one. To avoid complexity owing to overlapping of data, the data obtained at $L/d=3.24$ are not included in the figure (a). As shown in Fig. 5(b), at $L/d=3.24$, there exist two specific Strouhal numbers, that is, the values are nearly equal to those of $L/d=2.94$ and 3.51 . This fact indicates that the flow pattern E is a bi-stable in the transition region between the patterns C and D which occur alternately. Figure 5(b) presents the variation of the average Strouhal number in the range of the above Reynolds numbers with L/d . At $L/d=1.32$ and 1.47 , two Strouhal numbers related to the reattachment of the shear layer from the first cylinder are noticed: the larger one due to the pattern A in the region of $Re < Re_c$ and the smaller one due to the pattern B in the region of $Re \geq Re_c$. The value of the Strouhal number of the pattern A is larger than that of a circular cylinder, and that of the pattern B decreases abruptly at $Re=Re_c$ and increases gradually with an increase in L/d . The value of the Strouhal number of the pattern C remains constant in the spacings $2.35 \leq L/d < 3.24$. At $L/d=3.24$, the pattern is the bistable flow mentioned above and the jumping phenomenon occurs at $L/d=3.38$. The values of the three cylinders are as low as 10~20 % of those of the two cylinders[6] over the whole range $1.0 \leq L/d \leq 4.0$. The flow patterns of the above two cases are summarized in Table 1.

3.4 Fluctuating pressure distribution

Figure 6 shows typical distributions of the R.M.S. of fluctuating pressure around the three cylinders. The profiles of the first and second cylinders vary with an increase in L/d in a similar manner to the case of the two cylinders[6]. As to the first cylinder, the R.M.S. values of the pattern A without reattachment are very small with flat distribution along the circumference of the cylinder and those of patterns B and C with reattachment increase slightly. In particular, the value at $L/d=1.47$ is over twice as large as that of pattern A. For the pattern D beyond the critical spacing, the profile is analogous to that of a single cylinder. As to the second cylinder, the R.M.S. value of the pattern A is small, and the profile indicates a peak at $\phi=70^\circ$ where the pressure gradient is most remarkable. In the pattern B, the R.M.S. values increase drastically and the profile shows three peaks: the maximum peak of them appears slightly upstream of the reattached point of the shear layer from the first cylinder, and the other two peaks at the separation points of the reattachment flow on either sides of the reattachment point. In the pattern C, the peak at the separation point on the front surface disappears. In the pattern D, the peak appears only at $\phi=40^\circ$ where the value is over twice as large as that of the separation point on the first cylinder. On the third cylinder, the R.M.S. value in the patterns B and C increase remarkably. The profiles of the above two patterns show two peaks: one at $\phi=35^\circ$ and the other at $\phi=100^\circ$

corresponding to the separation point of the reattachment flow on the rear surface, then the valley lies at $\phi=75^\circ$ between the two peaks. In the pattern D, the first peak descends, this valley disappears, and then the second peak vanishes. Although this profile is similar to that of the second cylinder, the R.M.S. values reduce to half.

As far as the second cylinder is concerned, the distribution of the R.M.S. values agrees well with that of the two cylinders in the same manner as the pressure distribution shown in Fig. 3. From the results mentioned above, it is clear that

Table 1 A comparison of flow patterns for two and three cylinders

In-line Arrangement	Flow Patterns						
two cylinders	A	B	G	C	D	E, E'	F
three cylinders	A			B	C	E	D

Fig. 6 Fluctuating pressure distributions around three cylinders for various spacing

Fig. 7 Fluctuating pressure distributions around the first cylinder for various Reynolds numbers

the flow around the second cylinder is dominated by the behavior of the shear layer from the first cylinder, and that it is not affected by the third cylinder. It is worth notice that the R.M.S. value of the rear surface of the first cylinder for $L/d=1.47$ in the pattern B is considerably larger than that of the pattern C. Then the effect of the Reynolds number on the R.M.S. of fluctuating pressure was examined. As shown in Fig. 7, the R.M.S. of fluctuating pressure coefficient, $\Delta p/0.5\rho u_0^2$ for $L/d=1.47$ increases with an increase in Re , while that for $L/d=1.76$ is independent of the Reynolds number and nearly constant. Such a tendency can be recognized in the case of the two cylinders[6].

3.5 Flow visualization

Figures 8 and 9 show typical examples of flow pattern visualized using a smoke wind tunnel. Figure 8(a) shows photographs under long exposure, 1sec, and (b) instantaneous photographs. The results obtained show that the shear layer separated from the first cylinder behaves like a type corresponding to the individual flow patterns described in the section 3.1. For the pattern A, the shear layer develops straight and parallels without reattachment onto the downstream cylinders. For the pattern B, the two shear layers reattach periodically onto the second and third cylinders alternately. At the spacing $L/d=3.24$ of the pattern C, the shear layer in the lower side reattaches onto the second cylinder and the one in the upper side rolls up in front region of the third cylinder. For the pattern D, the vortex sheds behind the first cylinder and the downstream cylinders are exposed to a periodic alternate shear flow. Figure 9 shows

that the shear layer separated from the first cylinder reattaches onto the downstream ones with an increase in Reynolds number for $L/d=1.47$ and 1.62 . It is particularly interesting that there exists no pattern in which the shear layer reattaches onto the third cylinder alone. In other words, the existence of an intermediate pattern (W,R) between the patterns A(W,W) and B(R,R) has not been demonstrated. It is likely that when the shear layer of the one side reattaches onto the third cylinder, the one of other side reattaches onto the second cylinder, eventually.

Figure 10 shows typical examples of surface oil-flow pattern on the downstream cylinders. The patterns for $L/d=1.47$ and 1.76 correspond to the respective pressure distributions which are cases of the pattern B for $L/d=1.47$ and 2.06 in Fig. 3. On the rear surface of the first cylinder, the oil-flow pattern does not appear, though it is not included in the figure. The results suggest that the flow behind the first cylinder is different from that of the unique flow pattern on the two circular cylinders in tandem, in which the flow has reattached onto the second cylinder returns to the rear surface of the first cylinder across the centerline of the two cylinders. The oil-flow patterns on the second cylinder present evidence to the existence of a reattachment line of the shear layer and its separation lines. For $L/d=1.47$, the location of reattachment on the third cylinder moves forward compared to the case of the second cylinder, in addition, the line becomes obscure and the reattachment region narrows considerably. On the other hand, for $L/d=1.76$ the above reattachment line which resembles the two-dimensional stagnation flow pattern disappears, and the oil-flow pattern appears only at the downstream

Fig. 8 Flow visualization by smoke wind tunnel : $Re = 1.3 \times 10^4$

side of the point of reattachment. This difference between the flow patterns on the second cylinder for $L/d=1.47$ and 1.76 is closely related to the existence of a sharp peak on the pressure distribution around the third cylinder.

3.6 Reattachment Reynolds number

In the case of small spacing between the cylinders, when the shear layer separated from the first cylinder reattaches onto the second one, the flow characteristics around the downstream ones change drastically; the flow pattern changes from A to B. For example, with an increase in Reynolds number for a spacing, L/d , the pressure distribution around the second cylinder changes abruptly at the specified Reynolds number as shown in Figs. 11(a) and (b). This abrupt change appears clearly on the R.M.S. of fluctuating pressure distribution rather than on the mean pressure distribution as shown in Figs. 12(a) and (b). Such a Reynolds number is hereafter

Fig. 9 Reattachment of shear layer from the first cylinder

Fig. 10 Surface oil-flow patterns on cylinders for pattern B

called a reattachment Reynolds number, Re_c . As shown in Fig. 13, it decreases with an increase in L/d , and the relation between the two is given by

$$Re_c = 5.2 \times 10^4 (L/d)^{-3.8}.$$

On the other hand, for heat transfer around four cylinders closely spaced in tandem, Aiba et al.[3,4] obtained the critical

(a) without reattachment

(b) with reattachment

Fig. 11 Variation of pressure distribution on the second cylinder for various spacing and Reynolds numbers

(a) $L/d = 1.32$

(b) $L/d = 1.47$

Fig. 12 Variation of fluctuating pressure distributions around three cylinders with and without reattachment of shear layer

Reynolds number at which the heat transfer around the four cylinders changes drastically. Their results is expressed by

$$Re_c = 1.14 \times 10^4 (L/d)^{-5.84}.$$

As is obvious from Fig. 13, the value of Re_c for three cylinders agrees well with that for four ones. Aiba et al.[3] stated that the flow pattern changes drastically at Re_c , and the flow between neighboring cylinders becomes very stagnant in the region of $Re < Re_c$. It is likely that this drastic change is caused by the reattachment of the shear layer onto the second one; the flow pattern changes from (W,W,W) to (R,R,R).

3.7 Flow in transition region of flow pattern

Flow in transition region of flow pattern was examined in connection with the reattachment of the shear layer separated from the first cylinder. The power spectrum

Fig. 13 Reattachment Reynolds number

Fig. 14 Power spectrum of velocity fluctuation behind the third cylinder

of fluctuating velocity and the vortex shedding frequency were obtained in two cases; the one in which the free stream velocity, u_0 , gradually increases and the other in which it gradually decreases. Figures 14 and 15 show the variations of the power spectrum and the vortex shedding frequency with u_0 for $L/d=1.32$, respectively. From these figures, a hysteresis can be recognized. But the region of hysteresis is considerably narrow, its width being only 0.2 m/s in velocity. In the transition region, two specified frequencies occur, and the flow is found a bistable. The larger specified frequency is caused by the pattern A without reattachment, while the smaller one is slightly larger than that of the pattern B. It is considered that the smaller one is caused by the pattern (W,R) in which the shear layer separated from the first cylinder reattaches intermittently only onto the third cylinder described in the section 3.5.

The power spectra of fluctuating velocity in the wake of the third cylinder at the critical spacing are shown in Fig. 16. For $L/d=3.09$ and 3.24, the flow is bistable and it does not correspond to the pattern E' which continues for a long time, but corresponds to the pattern E of intermittent jump phenomenon of two cylinders[6].

3.8 Classification of flow patterns

Although classification of the flow patterns was already shown in Table 1,

Fig. 15 Hysteresis of vortex shedding frequency in transition region between patterns A and B

Fig. 16 Power spectrum of velocity fluctuation behind the third cylinder at critical spacing

Table 2 Flow patterns and flow characteristics around three cylinders arranged in line

Flow Pattern	Shear Layer	Range	S	C_{D2}	$\overline{C_D}$
A	(W,W)	$Re < Re_c$	0.23 ~ 0.24	-0.6	0.16
B'	(W,W) & (W,R)	$Re = Re_c, L/d = 1.32$	bistable, hysteresis		
B	(R,R)	$Re \geq Re_c, L/d < 2.21$	0.09 ~ 0.13	-0.4 ~ -0.3	0.3 ~ 0.4
C	(R,J)	$2.21 \leq L/d < 3.24$	0.13	-0.3 ~ -0.1	0.4 ~ 0.5
E	(R,J) & (J,J)	$L/d = 3.24$	bistable: Jump $C \rightarrow D$ and $D \rightarrow C$		
D	(J,J)	$L/d > 3.24$	0.16 ~ 0.18	0.35 ~ 0.42	0.6 ~ 0.7

together with that for two cylinders, we rearrange these patterns by considering the experimental results mentioned above. The combined drag coefficient and the Strouhal number are regarded as physical quantities capable of helping understanding the whole aspect of the flow around plural bodies. As shown in Fig. 17, the correlation between the two can be roughly classified into three regions as follows: the pattern A, the patterns B and C and the pattern D corresponding to the behavior of the shear layer separated from the first cylinder on the second one, that is, W, R, and J, respectively. The relation between $\overline{C_D}$ and S is different from the relation in which S increases with a decrease in $\overline{C_D}$ for a bluff body [9]. The flow patterns which include the bistable pattern B'(W,W $\rightleftharpoons$ W,R) in the range of the reattachment Reynolds numbers and also the bistable pattern E in the range of the critical spacing are classified according to L/d and Re. The results obtained are illustrated graphically in Fig. 18. Their flow characteristics also are summarized in Table 2.

4. Conclusions

Experimental investigations on the flow around three circular cylinders arranged in line were carried out in the range of subcritical Reynolds numbers. The following conclusions can be drawn from the results obtained:

(1) There are three cases concerned with the behavior of the shear layers separated from the first cylinder on the downstream ones; the first is one without reattachment (W), the second is one with reattachment (R) and the third is one of rolling up in front region of the downstream cylinders (J). The flow patterns were classified according to W, R and J as follows; the patterns A(W,W), B'(W,W $\rightleftharpoons$ W,R), B(R,R), C(R,J), E(R,J $\rightleftharpoons$ J,J) and D(J,J).

(2) Flow characteristics of these flow patterns were clarified. The main results are summarized in Table 2. The flows around individual cylinders were analogous to those around two cylinders of equal or different diameters arranged in tandem.

(3) For three cylinders arranged in line, a unique flow related to the Strouhal number and an unstable flow, like the one observed in the case of two cylinders, rarely appeared. An abrupt change of flow pattern of tubes closely spaced in an inline tube bank was also found to depend upon whether the reattachment of the shear layer separated from the first cylinder onto the second one occurs or not.

Fig. 17 Correlation between combined drag coefficient and Strouhal number

Fig. 18 Classification of flow patterns

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