Fluid Transient Phenomena Accompanied with Column Separation in Fluid Power Pipeline
(1st Report, On the Horizontal Pipeline Downstream of a Valve Instantaneously Closed)*

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A mathematical model capable of predicting a pressure rise associated with cavity collapse and cavitation duration following a column separation with accuracy enough for practical usage, on the downstream side of a valve instantaneously closed in a fluid power pipeline, was investigated. Particular attention has been paid to assessing the effect of gas released from liquid during the growing phase of the cavity. For that purpose, mechanism of commencement, growth and collapse of gas bubbles and volume of gas released were observed by using a valve made of transparent acrylic resin and a high speed camera. So far as the timing and pressure peak upon collapse of the first cavity are concerned, the predicted results from the present "gas-nonbubbly flow" model, in which the effects of gas released at the separated cavity and of an unsteady shear resistance in pipe flow were considered, agreed with the measured results within 5 per cent at worst.

Key Words : Fluid Power Systems , Fluid Transient , Column Separation , Pressure Rise, Pressure Fluctuation, Transient Cavitation, Air Release, Cavity Collapse

1. Introduction

The transient cavitation produced by power failure in a pump or by rapid closure of a valve has two types. One is known as column separation, which occurs at critical location in a pipeline. The other is referred to as cavitating flow or bubbly flow, in which cavitation bubbles are produced over a considerable length of the pipe.

In regard to the transient phenomena accompanied with column separation, many detailed studies⁽¹⁾⁻⁽⁵⁾ considering even the effect of secondary factor such as a released gas have been done. At present, with respect to the individual system, a mathematical model capable of predicting the timing and the amplitude of pressure rise upon collapse of the first cavity with accuracy enough for practical usage has been developed. Among others, Kranenburg⁽³⁾ studied the fluid transients accompanied with column separation considering all three regions likely to occur, namely, column separation, cavitation and waterhammer, and concluded as follows: (1) Gas release at the separation cavity increases the duration of column separation and slightly increases the cavity collapse pressure, (2) gas release has insignificant influence in case where only cavitating flow occurs, but the influence is considerable in cases where both column separation and cavitating flow occur

simultaneously, to be specific, gas release remarkably decreases the duration of column separation and the cavity collapse pressure because of the energy loss caused by shock waves propagating in the bubble mixture. In addition, he made it clear that the discrepancy between the experimental and theoretical results in Baltzer's⁽⁶⁾ study could be explained by the mathematical model he developed.

Regarding the released gas inside a cavity, Swaffield⁽¹⁾ also achieved reasonable agreement between the computed and measured results about the closure of first cavity behind a closing valve in a short horizontal pipeline, by considering a partial pressure of the gas in a cavity released in accordance with Henry's law from the liquid flowing in a pipe during valve closure. On the other hand, in regard to the effect of the gas released in cavitating flow, Weyler et al.⁽⁴⁾ also stated that the experimental facts, that the actual separation cavity size and pressure rise associated with cavity collapse were smaller than the predicted ones from the classical "vapour-nonbubbly flow" model, could be explained by providing the additional bubble shear stress arising from the nonadiabatic expansion and collapse of gas bubbles dispersed throughout the low pressure column separation flow. Brown⁽²⁾ also obtained better agreement between the computed and measured results when discrete air cavities were assumed along the pipeline. Furthermore, Yamaguchi and Ichikawa⁽⁷⁾ pointed out that the surface tension could not be ignored in the transient cavitation occurring under the condition of low initial velocity, because a mixture of gas bubble and liquid was formed at the separation cavity.

Thus, it may be true that the released

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gas has a significant effect on the fluid transient phenomenon accompanied with column separation. As seen from author's previous work⁽⁶⁾, however, the fluid friction also has a considerable effect on the phenomenon. That is, in the case of column separation occurring at laminar flow for example, there is a large difference between the two calculated values using a one-dimensional steady-state laminar friction and a two-dimensional unsteady-state laminar friction. However in the foregoing studies⁽¹⁾⁻⁽⁷⁾, the steady-state fluid friction was used in both cases of laminar and turbulent flows (although it is due to a compelling reason that the unsteady fluid friction in turbulent flow has not been known in detail⁽¹¹⁾), and the discrepancy between the observed system behavior and the predicted one from the classical "vapour-nonbubbly flow" model was attributed exclusively to the effect of released gas. Furthermore, although the effect of gas on maximum pressure following column separation and cavity duration is reversed⁽⁶⁾ according to whether the gas is released at the separation cavity or in the cavitating flow adjacent to the column separation, a relationship between the mechanism of gas release and the system parameters, such as dimensions of pipeline, fluid properties, operating conditions, etc. has not been known. Therefore, in order to develop a general mathematical model for the fluid transients accompanied with column separation, it is still necessary to collect more experimental results about an individual piping system.

It was the purpose of the present investigation to develop a mathematical model capable of predicting a pressure rise associated with cavity collapse and cavitation duration, with accuracy enough for practical usage, following column separation on the downstream side of a valve suddenly closed in a fluid power pipeline. Particular attention has been paid to assessing the effect of gas released from the fluid during the growing phase of the cavity on the phenomenon. Two types of water-glycol fluids and O/W emulsive fluid (95% water content, the so-called high water base fluid) in addition to the mineral oil commonly used in fluid power system were employed as test fluids.

2. Nomenclature

- A : Pipe cross-sectional area
- c : Wave speed in the pipeline
- D : Pipe bore
- h_f : Pressure drop per unit length due to friction
- L : Pipe length
- N : Number of sections in a pipeline
- p : Pressure
- p' : Partial pressure of released gas within the total volume of cavity
- p_r : Reservoir pressure
- p_v : Fluid vapour pressure
- Re : Reynolds number at initial condition
- t : Time
- Δt : Time increment
- u_0 : Initial flow velocity
- V : Total volume of cavity
- V_g : Quantity of released gas measured

at n.t.p.

x : Distance along the pipeline measured from valve

Δx : Length of pipe section

κ : Polytropic coefficient

ρ : Fluid density

ν : Kinematic viscosity

3. Test Apparatus and Experimental Results

3.1 Test rig and method

The layout of the test rig is shown in Fig.1. The test section consists of a spool-type directional valve, a steel pipe and a reservoir (accumulator). After regulating the initial flow velocity, reservoir pressure and fluid temperature to specified values by a flow control valve, a throttle valve and a heat exchanger, respectively, the valve was suddenly closed by knocking a spool with hammer, and then column separation took place. Pressure recordings were taken at the position of upstream and downstream ends (in addition, the central position in case of pipe of $L \approx 10$ m) of pipeline with semi-conductor pressure transducers. The outputs were once registered

Table 1 Main properties of test fluids

Property Fluid	Density (Kg/m^3)	Kinematic viscosity (mm^2/s)	Percentage of water content %
F-1: Mineral oil	858	51.7	—
F-2: Water-glycol	1045	82.9	42.3
F-3: Water-glycol	1084	86.7	40.0
F-4: O/W emulsion	998	0.983	95.0

Fig.1 Layout of test pipeline and measuring system

Fig.2 Spool-type directional valve made of transparent acrylic resin for visualization test

in the digital memory (feed speed: $50 \mu\text{s}/\text{word}$) and then displayed on the X-Y plotter. The cavity growth and decay sequence was observed by means of a high speed camera, by using a valve made of transparent acrylic resin shown in Fig.2 instead of a steel valve in Fig.1. Dimensions of the directional valve made of steel are same as those of the valve made of acrylic resin indicated in Fig.2, except for the mounting position of pressure transducer (the position in the former valve is 200 mm from the center). Needless to say, the experiments for comparison with the theoretical results were all carried out on the apparatus using the steel valve. The pipes employed were steel pipes for high pressure service of lengths of 2.67 and 10.14 m of 19.1 mm bore. The main properties of the fluids employed are indicated in Table 1. Kinematic viscosity of the fluid was measured by means of a Contraves Co.-made "Rheomat 30", and it was confirmed in advance that all test fluids could be treated as Newtonian fluid at least in the range of shear rates of $0 \sim 300/\text{s}$ in this study.

3.2 Experimental results

3.2.1 Mechanism of cavity growth and decay sequence near the valve downstream face
Figure 3 (a)~(c) illustrates a typical cavity growth and decay sequence

near the valve downstream face, following a sudden valve closure, filmed by a high speed camera. The pressure variations measured simultaneously and shooting times also are indicated in the same figure. The crank-speed is 2200 frames/s in the cases of (a) and (c), and 1700 frames/s in the case of (b). It can be seen from Fig.3 (a) that several small bubbles (①-2) originated from the annular throttling part between spool and sleeve subjected to the largest shear rate are carried downstream (①-3~②) along a stream line, there, a group of bubbles grow up and coalesce to form a large cavity. The separated cavity consists of a gas-liquid two-phase similar to a part of slug flow. Several small gas bubbles are observed in the liquid column adjacent to the cavity grown up to the largest (④), but no gas release is observed in other parts of liquid column so far as the high speed camera can cover. When the separated column is accelerated back towards the valve, the cavity grown up to the largest size collapses into a group of small bubbles at about the same position (⑥-1) and disappears just after rejoining (⑥-2). In the second column separation, microscopic gas bubbles, which are dispersed in the liquid column near the downstream of valve at the time of first rejoining, act as nuclei for cavitation. These microscopic

(a) F-1 (mineral oil), $u_0=0.93\text{m/s}$, $Re=314$, 2200 frames/s

Fig.3 Typical cavity growth and decay sequence near the valve downstream face as filmed by high speed camera

gas bubbles grow in size at about the same position (7) and form the separated part of column (8). This second separated cavity (8) consists of one large bubble close to the valve and a bubbly flow of length of about twice the pipe diameter. This is considerably different in form from that in the first column separation (4), which can be almost considered to be made up of only one cavity. After the column separation has ceased, the gas (considered to be air) of about 120 mm³ volume remains

out of solution at the surface of valve's annular path and in the liquid column adjacent to downstream of valve (11).

Figure 3(b) illustrates a cavity growth and decay sequence in the case of initial flow velocity being relatively low ($R_e \approx 200$), and reveals that the mechanism of development, growth and collapse of a cavity is almost similar to that in case (a), except for the quantitative differences in the number of gas bubbles released and the length of separated cavity. Furthermore,

Fig.3 (continued)

it can be seen from these visualization tests that as the microscopic bubbles commenced at the valve orifice have grown into 3~4mm diameter bubbles in only about 1ms (about 1/25 cavitation duration) after valve closure (Fig. 3 (b), ①-3), the effect of surface tension σ on pressure inside a bubble ($=4\sigma/d$) has already decreased to the order of $2\sim4\times10^{-5}$ MPa (0.16~0.29mmHg). Hence, it may be concluded that the effect of surface tension on the cavity collapse pressures and cavitation durations can be ignored, except for the case where the initial flow velocity is extremely low and hence the cavitation duration is very short.

Figure 3(c) illustrates the result of flow visualization test for the water-glycol working fluid. The quantity of gas remaining out of solution at valve surface after column separation phenomenon is small compared to that in mineral oil, but in regard to the mechanism of incipency, growth and decay of gas, no particular difference between the two fluids is recognized. In an O/W emulsive working fluid, it was impossible to observe with photograph on account of the substance being opaque.

3.2.2 Observed pressure variations

Figures 4 (a) and 5 (a) illustrate the observed pressure variations at valve end and central position of the pipe, respectively, for the test conditions of reservoir pressure being relatively high ($p_r=0.31$ MPa abs.) and pipe length being relatively long ($L=10.14$ m). Thick solid lines in Fig. 6(a)~(e) illustrate the observed pressure variations at valve end for the conditions of reservoir pressure being relatively low ($p_r=0.1$ MPa abs.) and pipe length being relatively short ($L=2.67$ m). The magnified records at low pressure part of pressure variations are shown in Fig. 7, in order to examine the detailed pressure condition within the separated cavity, and thus the effect of released gas within it. In case of mineral oil, the pressure curve is rounded at the

formation and collapse of each cavity and the minimum pressure within a cavity increases gradually from the pressure in vicinity of a vacuum with successive cavity openings. In the cases of water-glycol and O/W emulsive fluids the rounding of pressure curve is relatively small, that is, the pressure changes abruptly at the formation and collapse, and the pressure within the cavity is almost kept at a fluid vapour pressure for the first several successive cavities.

Table 2 shows two wave speeds, c_b and c_d , measured before and after experiment (to be specific, the latter was measured between two continuous peaks just after the column separation had ceased). As can be seen from these results, a remarkable decrease in wave speed after experiment, which was observed clearly in Weyler's experiment⁽⁵⁾, does not arise.

It can be concluded on the basis of the experimental results above that the part of separated cavity and the liquid column have the following characteristics: (1) The separated cavity at downstream face of valve holds a released gas. (2) The effect of surface tension applied on gas bubbles forming the part of separated cavity can be ignored. (3) The length of separated cavity is sufficiently small compared to the length of pipe section. (4) The dynamics of liquid column is scarcely affected by the released gas in it.

4. Considerations

4.1 Theoretical analysis

In this section, a mathematical model for predicting the first pressure rise associated with cavity collapse and cavitation duration of major engineering interest will be investigated. By considering the matters of (1)~(4) above, and assuming a flow in pipe to be two-dimensional, unsteady and

Fig. 4 Measured and predicted pressure variations at valve end ($x=0$)

Fig. 5 Measured and predicted pressure variations at central position of pipeline ($x=L/2$)

(a) F-1 (mineral oil)

(b) F-1 (mineral oil)

(c) F-2 (water-glycol)

(d) F-3 (water-glycol)

(e) F-4 (O/W emulsion)

Fig.6 Measured and predicted pressure variations at valve end ($x=0$)

laminar, the basic differential equations may be derived as follows.

Equation of motion:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) + \frac{\partial p}{\partial x} + h_f = 0 \quad (1)$$

and continuity:

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + \rho c^2 \frac{\partial u}{\partial x} = 0 \quad (2)$$

where h_f denotes pressure drop per unit length due to unsteady laminar resistance which is given by the following expression.

$$h_f = \frac{32\rho\nu}{D^2} u + \frac{16\rho\nu}{D^2} \int_0^t \frac{\partial u(\xi)}{\partial \xi} W(t-\xi) d\xi \quad (3)$$

If the pipe flow can be treated as a one-dimensional, steady and laminar one, the second term of the right side in Eq.(3) can be omitted (refer to the reference (9) for concrete expression of the weighting function $W(t)$). In this research, Eqs.(1) and (2) will be solved numerically by the method of characteristics⁽¹⁰⁾ using specified grid (characteristic line: $dx/dt = \pm c$). By making the following assumptions that a finite quantity of gas (air) V_g is released instantaneously at the downstream face of valve during or just after valve closure, after that there is no occurrence of gas diffusion between vapour and liquid phases, and that the part of separated cavity in gas-liquid two-phase state can be treated as divided into a single gas column and a liquid col-

Table 2 Wave speeds before and after experiment

	Initial flow velocity (m/s)	Wave speed (m/s)		Ratio of C_2 to C_1
		Before: C_1	After: C_2	
F-1	0.82	1380	1088	0.79
	0.44	1376	1245	0.91
	0.25	1392	1310	0.94
F-2	0.83	1471	1271	0.86
	0.40	1463	1402	0.95
F-3	0.82	1480	1263	0.85
	0.38	1471	1350	0.92
F-4	0.29	1430	1330	0.93

Fig.7 Magnified records at low pressure part of pressure variations

um, two boundary conditions of pipeline can be expressed in finite-difference form as follows.

Valve downstream end ($x=0$):

$$\left. \begin{aligned} p_{1,k} &= p_v + p'_k \\ p'_k V_k^e &= p_0 V_0^e = \text{const.} \\ V_k &= V_{k-1} + A(u_{1,k} + u_{1,k-1})\Delta t / 2 \\ u_{1,k} - u_{2,k-1} - (p_{1,k} - p_{2,k-1})/\rho c \\ &+ h_f \Delta t / \rho = 0 \end{aligned} \right\} \dots\dots\dots (4)$$

and reservoir (or accumulator) end ($x=L$):

$$\left. \begin{aligned} p_{n+1,k} &= p_r = \text{const.} \\ u_{n+1,k} - u_{n,k-1} + (p_{n+1,k} - p_{n,k-1})/\rho c \\ &+ h_f \Delta t / \rho = 0 \end{aligned} \right\} \dots\dots\dots (5)$$

In this paper, the mathematical model represented by Eqs. (1)~(5) is referred to as "gas-nonbubbly flow" model using a two-dimensional unsteady laminar friction.

4.2 Comparison of experimental results with calculated results and considerations

Figures 4(b) and 5(b) illustrate the pressure variations calculated from the above-mentioned "gas-nonbubbly flow" model together with its calculated profile (c) from the "vapour-nonbubbly flow" model neglecting the effect of released gas inside a separated cavity and profile (d) from the "vapour-nonbubbly flow" model using a one-dimensional steady laminar friction. The comparison between these various calculated results shows that the profile (b) is the most similar to the observed pressure variations (a). However, the difference between (b) and (c), that is, the effect of released gas within a cavity, is not clearly recognized. The reason is that the effect of the partial pressure of released gas on the driving force applied on a liquid column ($=A(p_{1,k} - p_r)$) is negligibly small, because the reservoir pressure is relatively high in the test of Fig.4 (and Fig.5). In addition, comparison of (c) with (d) shows the agreement achieved by inclusion of the unsteady friction component in Eq.(3).

Figures 6(a) and (b) show the comparison of observed and predicted pressure variations in the case of reservoir pressure being relatively low (close to atmospheric pressure) and the same fluid (mineral oil) being used as in Figs.4 and 5. In this case, the effect of released gas inside a cavity is recognized clearly in calculated results, and the experimental results agree best with the calculated ones from the "gas-nonbubbly flow" model using a two-dimensional unsteady laminar friction. In this investigation also like Swaffield's results⁽⁴⁾, released gas at the separated cavity has an effect to increase the duration of column separation. Regarding the pressure rise upon collapse of the first cavity, however, released gas has an effect to slightly decrease it unlike Swaffield's results. But, this discrepancy can be considered to be merely due to the phase of pressure wave remaining in a liquid column at the time of cavity collapse. Regarding

the gas inside a cavity, Swaffield concluded that it is released in accordance with Henry's law from the volume of fluid that passes through the valve between the time that the downstream pressure falls below atmospheric pressure and final valve closure. But, according to this approach, the volume of gas released becomes zero in such an instantaneously valve closure as in this study. Indeed, Swaffield has pointed out that the actual volume of gas released is within 1.25 to 1.5 times the above-mentioned value, that is, gas release takes place in other part of fluid than where the fluid passing through the valve. Assuming that the gas within a separated cavity is released, in the case of the present test pipeline, mostly from the liquid in annular path of a valve receiving a great agitation during valve closure, the volume of gas released V_g is 80 to 100 mm³ for mineral oil (coefficient of air content α_0 : 8~10%), 10~15 mm³ for water-glycol working fluid (α_0 : 1~1.5%), and 20 mm³ for O/W emulsive working fluid (α_0 : 2%). Those volumes of released gas are closer to the quantities obtained from the visualization tests. Therefore, the volume of gas released in vicinity of the value above was used in

(a) In the column separation on the downstream side of a valve

(b) In the column separation on the upstream side of a valve

Fig.8 Effect of unsteady laminar friction component on pressure peak and cavitation duration

calculation.

Figure 8 (a) and (b) illustrate the predicted pressure variations at valve end and cavity/liquid interface velocities from the "gas-nonbubbly flow" model using a steady laminar friction and the "gas-nonbubbly flow" model using an unsteady laminar friction. It can be seen from these results that the difference between the two calculated values, that is, the effect of an unsteady laminar friction component, is of the same order as the difference due to the presence of the gas released within a separated cavity. The effect of the unsteady laminar friction component will be examined physically in detail further.

First, in the column separation on the downstream side of a valve, the unsteady friction component has an effect of increasing the duration of the subsequent column separation. This can be explained physically as follows. As can be seen from Eq.(3), the unsteady friction component has an effect to reduce the total shear stress h_f in the range of $u > 0$ and $du/dt < 0$ (that is, until the separated column is brought to rest), and an effect to increase it in the range of $u < 0$ and $du/dt < 0$ (that is, until the separated column accelerated back towards the valve collapses the cavity). It is a matter of course that the pressure wave propagating in a liquid column is attenuated more largely by the unsteady friction component. In Swaffield's study⁽⁴⁾ on the column separation occurring at downstream side of a valve, the observed first cavitation duration is still longer than the predicted one, even by utilizing a model that permits gas release at the separated cavity. Based on the above considerations (Fig.8(a)), it is believed that this discrepancy can be explained by the unsteady friction component.

Next, in the column separation on the upstream side of a valve (Fig.8(b)), the effect of unsteady friction component on the first cavitation duration is completely contrary to the case (a), and the pressure rise upon cavity collapse remarkably decreases. The reason is that the rarefaction wave to move the liquid column to the reservoir is attenuated more greatly by the unsteady friction component, and thus the flow velocity at the time of inception of column separation, which affects dominantly the cavitation duration and pressure rise, is reduced considerably. Kranenburg⁽⁵⁾ and Weyler⁽⁶⁾ have explained the difference between theory and experiment in Baltzer's study⁽⁶⁾ dealing with a column separation on the upstream side of a valve by considering only the effect of gas released in a cavitating flow. Although the energy losses due to wall friction in turbulent fluctuating flows have not been known, the effect of the unsteady friction component in turbulent flow may be considered to be same qualitatively as that in laminar flow. Hence, it may be conjectured reasonably that the above-mentioned difference is partly due to the unsteady friction component.

Figure 6(c)~(e) indicate the observed and predicted pressure variations for water-based fluids (provided that the steady-state turbulent resistance was used in case (e)

instead of Eq.(3)). In order to investigate the effect of the liquid vapour pressure, its predicted profile from the "vacuum-nonbubbly flow" model assuming the pressure inside a cavity perfect vacuum is also indicated together with those from other models. It is concluded from these results that for water-based fluid the effect of released gas inside a cavity can be neglected, and thus the "vapour-nonbubbly flow" model can predict both cavitation duration and pressure rise upon collapse of the first cavity.

Lastly, the column separation from the second one on will be considered a little. As can be seen from Fig.6(a)~(e), the growth and collapse of the second cavity in experiment and numerical model are clearly not in agreement, that is, the observed pressure variations show a tendency to reduce the pressure peak and cavitation duration compared to its predicted profile from the present model. In addition, in the present mathematical model treating a liquid column as a non-bubbly flow (that is, neglecting the region of cavitating flow), pressure at grid points except for valve and reservoir ends may fall below vapour pressure of the liquid in calculation. Needless to say, such an event does not actually happen (refer to Fig.5(a)). Regarding the mathematical model for the column separation from second one on, therefore, further investigations including a reasonable improvement of the method of calculation are required.

5. Conclusions

Based on the results of these theoretical and experimental studies, it is concluded that:

(1) In the instantaneous closure of a spool-type directional valve, nearly equal quantity of gas dissolved in the liquid in the annular path of a valve, under heavy agitation during valve closure, is released at the separated cavity in the vicinity of downstream face of the valve.

(2) In the case of mineral oil having a high air content, actual (measured) first cavitation duration is longer and pressure rise upon first cavity collapse is smaller than the predicted one from the classical "vapour-nonbubbly flow" model owing to the partial pressure of the released gas inside a cavity.

(3) So far as the first pressure peak and cavitation duration are concerned, the effects of the energy dissipation due to nonadiabatic behavior of gas bubbles in liquid column and the surface tension applied on gas bubbles forming the part of separated cavity can be ignored, thus, the experimental results can be predicted accurately by the present "gas-nonbubbly flow" model using an unsteady pipe friction. As an exception, in cases of water-based working fluids such as water-glycol and O/W emulsion having a low air content, the "vapour-nonbubbly flow" model using an unsteady pipe friction can be used with accuracy enough for practical usage.

(4) The unsteady friction component has effect of increasing the first cavi-

tion duration in the column separation on the downstream of valve, and of decreasing it in the column separation on the upstream side of valve. In addition, pressure rise upon cavity collapse is also reduced remarkably by it in the latter case.

(5) In the column separation from the second one on, the measured pressure rises on cavity collapse and cavitation duration shows a tendency to become shorter with successive cavity opening than the predicted one from the "gas-nonbubbly flow" model. This tendency is remarkable especially in mineral oil with a high air content. Hence, it may be presumed that this discrepancy is due to the energy dissipation in a cavitating (bubbly) flow formed in front of the cavity/liquid interface (refer to Fig.3(a), ⑧).

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