Bending Fatigue Strength of Spur Gears in Low-cycle Range*

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This paper presents a study on bending fatigue strength of spur gears in low-cycle range.

The calculating methods for adjacent-tooth gaps and the circumferential load, under which adjacent-tooth contacts occur due to the deflections of meshing teeth, are introduced. The validity of these calculating methods is confirmed by carrying out a static loading test. The characteristics of the bending fatigue strength of spur gears in low-cycle range are clarified to a considerable extent by carrying out a bending fatigue test.

Key Words : Gear, Fatigue, Low-cycle, Bending Strength, Gap, Pressure Angle, Contact Ratio, Deflection Limit

1. Introduction

Power transmitting gears are usually designed for semipermanent life. There are, however, some cases where the gears may be designed for a finite life and where they must be exchanged periodically because of the limits in weight, space and so on. The bending fatigue strength of these gears must be considered in low-cycle range. Studies on bending fatigue strength of spur gears in high-cycle range have been published, but those in low-cycle range (number of load cycles $N \leq 10^5$) are only a few⁽¹⁾.

In the present paper, the bending fatigue strength of spur gears in low-cycle range is investigated. When the meshing positions of spur gear pairs with geometric contact ratio of $1 < \epsilon < 2$ lie in the range of single tooth pair contact, gaps (adjacent-tooth gaps) exist between adjacent teeth meshing with each other and they decrease with an increase in applied load due to the deflections of meshing teeth. When the applied load increases still further, the gaps become zero and the contacts of adjacent teeth may occur. A calculating method for adjacent-tooth gaps was introduced and the gaps at each meshing position were calculated for various spur gear pairs. According to these calculated results, the meshing position where the circumferential load, under

which the contacts of adjacent teeth occur, becomes maximum (worst loading position) and the average gap at the position (worst gap) are determined, and the transmitted load (circumferential load) under which contact ratio becomes 2 is calculated. The validity of these calculating methods has been confirmed by carrying out a static loading test with gear pair meshing at the worst loading position and the characteristics of the bending fatigue strength of spur gears in low-cycle range have been clarified to a considerable extent by carrying out a bending fatigue test.

Furthermore a design method of the gear pair with contact ratio of $\epsilon \geq 2$ under an application of load is also indicated.

2. Calculating method

The calculations of adjacent-tooth gaps and the circumferential load under which the contacts of adjacent teeth occur due to deflections of meshing teeth were made on standard spur gear pairs of pressure angle $\alpha_0 = 20^\circ$ without undercut, whose geometric contact ratios were within the range $1 < \epsilon < 2$ ⁽²⁾. When the meshing positions of these gear pairs lie in the range of single tooth pair contact, adjacent-tooth gaps exist between two teeth P1 and G1 and between two teeth P2 and G2 as

Fig.1 Meshing condition

* Received 26th October, 1982.

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shown in Fig.1. These gaps decrease with an increase in applied load due to the deflections of meshing teeth. When the applied load increases still further, the gaps become zero and the contacts of adjacent teeth may occur. Since the transmitted load is shared by double or triple tooth pairs when a circumferential load, under which the contacts of adjacent teeth occur, is transmitted, it might be considered that the bending fatigue life becomes longer in this loading condition than in the case where the contacts of adjacent teeth are assumed not to occur. The calculating method of adjacent-tooth gaps is as follows.

The distance on line of action from pitch point to meshing position is taken as l . l is negative or positive when meshing position lies in the root or tip side of driving gear respectively. Using the symbols as shown in Fig.2, the following relations are obtained at each meshing position.

$$\left. \begin{aligned} \theta_1 &= \tan^{-1}\{l \cos \alpha / (m z_1 / 2 + l \sin \alpha)\} \\ \theta_2 &= \tan^{-1}\{l \cos \alpha / (m z_2 / 2 + l \sin \alpha)\} \end{aligned} \right\} \quad (1)$$

where m : module, z_1, z_2 : numbers of teeth of driving and driven gears. The gap g_1 between two teeth P1 and G1 and g_2 between two teeth P2 and G2 are equal to lengths of arcs K_1L_1, K_2L_2 respectively and expressed as follows.

$$g_1 = R_1 \psi_1, \quad g_2 = R_2 \psi_2 \quad (2)$$

where

$$R_1 = (m/2) \sqrt{(z_1 + z_2)^2 - 2(z_1 + z_2)(z_1 + 2) + (z_1 + 2)^2}$$

$$R_2 = (m/2) \sqrt{(z_1 + z_2)^2 - 2(z_1 + z_2)(z_2 + 2) + (z_2 + 2)^2}$$

$$\psi_1 = 2\pi/z_2 + \theta_2 + \text{inv} \alpha_{t1} - \text{inv} \alpha_2 - \sin^{-1}\{m(z_1 + 2)\sin \psi_1 / 2R_1\}$$

$$\psi_2 = 2\pi/z_1 - \theta_1 + \text{inv} \alpha_{t2} - \text{inv} \alpha_1 - \sin^{-1}\{m(z_2 + 2)\sin \psi_2 / 2R_2\}$$

$$\psi_1 = 2\pi/z_1 + \theta_1 + \text{inv} \alpha_1 - \text{inv} \alpha_{K1}, \quad \psi_2 = 2\pi/z_2 - \theta_2 + \text{inv} \alpha_2 - \text{inv} \alpha_{K2}$$

and α_1, α_2 : pressure angles at meshing position of driving and driven gears, α_{K1}, α_{K2} : pressure angles at tooth tip of driving and driven gears, α_{t1}, α_{t2} : pressure angles at positions L_1, L_2 .

The average gap at each meshing position is defined by

$$g_a = (g_1 + g_2)/2 \quad (3)$$

The circumferential load under which contact ratio becomes 2 is calculated by using the following equations.

$$\left. \begin{aligned} W &= F_t / \cos \alpha, \quad \delta_1(W) = (m z_1 \sin \beta_1) / 2 \cos(\alpha_1 - \beta_1) \\ \delta_2(W) &= (m z_2 \sin \beta_2) / 2 \cos(\alpha_2 - \beta_2), \quad g_d = R_1 \beta_2 + R_2 \beta_1 \end{aligned} \right\} \quad (4)$$

where F_t : circumferential load; W : normal tooth load corresponding to F_t ; $\delta_1(W), \delta_2(W)$: normal tooth deflections⁽³⁾ of driving and driven gears due to W ; β_1, β_2 : rotational angles of driving and driven gears due to $\delta_1(W), \delta_2(W)$; g_d : circular deflection corresponding to $\delta_1(W), \delta_2(W)$. The meshing position, where circumferential load under which the contacts of adjacent teeth occur becomes maximum, is defined here as the worst loading position in low-cycle range and the average gap at the position, as the worst gap. The contact ratio becomes 2 when the circumferential load under which g_a is equal to g_d at worst loading position is transmitted.

3. Experimental procedure and apparatus

3.1 Test gears

The main dimensions of test gear pairs used in this experiment are module $m = 4$, pressure angle $\alpha_0 = 20^\circ$, gear ratio $i = z_2/z_1 = 30/30$ and face width $b = 10$ mm. Test gears are made of S45C normalized steel and are hobbled to an accuracy of 4th class in JIS. The shapes and dimensions of test gears are shown in Fig.3.

Fig.2 Dimensions for calculation of gap

3.2 Static loading test

In order to determine the circumferential load under which the contact ratio becomes 2, a static loading test was carried out with test gear pair meshing at the pitch point (worst loading position). The root stresses were measured by a strain gage. The strain gage was pasted at the root fillet in tooth height direction at the middle of the face width on loaded side of driving gear as shown in Fig.4.

Figure 5 shows the static loading testing machine used in this experiment. The center distance is variable and the meshing positions can be determined by measuring the distance between two specific gear teeth with a reading microscope. Test gear pair can be loaded at arbitrary meshing positions with a manual oil pump by fixing a driven gear. The applied load value is detected from the readings of the strain gages pasted at the shaft. A backlash can be set by changing the center distance, and a circumferential backlash $c_0 = 160 \mu\text{m}$, which is equal to the minimum value specified in JIS, is selected in this experiment.

3.3 Bending fatigue test

Bending fatigue tests were carried out with test gear pairs meshing at the pitch point. In order to examine the effects of adjacent-tooth contacts on bending fatigue strength, bending fatigue tests were carried out also for the single tooth pair contact by cutting off the adjacent teeth of driving gear.

The bending fatigue testing machine used is of hydraulic type consisting of a fuel injection pump and its driving apparatus, a pressure controller, a loading apparatus and a gear supporting apparatus

Fig.3 Test gear

Fig.4 Position of strain gage

as shown in Fig.5. The frequency of load applications is about 500 c/min.

4. Calculated and experimental results and discussions

4.1 Calculated results of gaps and tooth deflections

Figure 6 shows the adjacent-tooth gaps g_1 , g_2 , g_a and circular deflection g_d calculated at each meshing position for gear pair of module $m = 4$, pressure angle $\alpha_0 = 20^\circ$, gear ratio $i = z_2/z_1 = 30/30$ and face width $b = 10 \text{ mm}$. Points A and B on the abscissa indicate the inner and outer points of single tooth pair contact of driving gear respectively and $l = -2.05 \text{ mm}$ at point A, $l = 2.05 \text{ mm}$ at point B. It is seen from this figure that $g_1 = g_2$ and the smaller one of g_1 and g_2 takes maximum value at pitch point contact. g_d in Fig.6 is calculated for the circumferential load $F_t = 7.7 \text{ kN}$, under which $g_a = g_d$ at pitch point contact. g_d is almost constant at each meshing position and takes minimum value at pitch point contact. When $F_t = 7.7 \text{ kN}$ is transmitted in this gear pair, double or triple tooth pairs contact occurs at each meshing position. Therefore the worst loading position comes at pitch point and the worst gap is equal to the average gap at the position in this gear pair. The cases of $z_2 > z_1$ are examined as follows.

Figure 7 shows the adjacent-tooth gaps g_1 , g_2 , g_a and circular deflection g_d calculated at each meshing position for gear pair of $m = 4$, $\alpha_0 = 20^\circ$, $i = z_2/z_1 = 65/18$ and $b = 10 \text{ mm}$. In Fig.7 $l = -2.78 \text{ mm}$ at point A and $l = 1.20 \text{ mm}$ at point B. It is seen from this figure that $g_1 > g_2$ at pitch point contact and that the worst loading position shifts a little toward the point A from pitch point ($l = -0.343 \text{ mm}$). g_d is calculated for the circumferential load $F_t = 9.0 \text{ kN}$, under which $g_a = g_d$ at pitch point contact. Even when $F_t = 9.0 \text{ kN}$ is transmitted in this gear pair, the range (C~D in Fig.7) of single tooth pair contact remains.

4.2 Worst loading positions and worst gaps for various gear pairs

Figure 8 shows the relations between gear ratio i and length ratio $l_r = |l_w|/l_0$

Fig.5 Static loading and bending fatigue testing machine

($|l_w|$: length on line of action from pitch point to worst loading position, l_0 : length of action) for $z_1 = 18, 30, 48$. It is seen from Figs. 7, 8 that the worst loading position moves from pitch point toward inner point of single tooth pair contact with an increasing gear ratio. The length $|l_w|$ does not always increase with an increasing z_1 . The values of l_r for $i = 7$ are 0.027, 0.031, 0.025 for $z_1 = 18, 30, 48$ respectively. Hence the worst loading position is considered to lie in the close vicinity to pitch point.

Figure 9 shows the relations between gear ratio i and gap ratio $g_r = (g_w - g_{ap}) / g_{ap}$ (g_w : worst gap, g_{ap} : average gap at pitch point contact). Since the value of g_r for $z_1 = 18$ is positive, $g_w > g_{ap}$ and the range of single tooth pair contact remains the same as is the case with Fig. 7 (C~D) for the circumferential load, under

which $g_{ap} = g_d$ at pitch point contact. When g_r takes maximum value ($g_r = 0.05$), the range of single tooth pair contact becomes maximum and is only 0.8% of length of action. Hence $g_w \approx g_{ap}$ for $z_1 = 18$. The value of g_r for $z_1 = 30$ is positive in the range $1 \leq i < 1.4$ and negative in the range $i > 1.4$. The case of $g_r > 0$ is considered to be the same as in the case of $z_1 = 18$. In the case of $g_r < 0$, $g_w < g_{ap}$ and the double or triple tooth pairs contact occurs for the circumferential load, under which $g_{ap} = g_d$ at pitch point contact. The value of g_r for $z_1 = 48$ is negative and this is considered to be the same as in the case of $z_1 = 30$ and $i > 1.4$. Since the worst loading position lies in the close vicinity to pitch point and the

Fig. 6 Relations between gaps and tooth deflection at each meshing position ($z_2/z_1 = 30/30$)

Fig. 7 Relations between gaps and tooth deflection at each meshing position ($z_2/z_1 = 65/18$)

Fig. 8 Relation between gear ratio and worst loading position

Fig. 9 Relation between gear ratio and worst gap

worst gap is nearly equal to the average gap at pitch point contact, the following calculations are carried out for pitch point contact.

4.3 Calculated results of worst gap for various gear pairs

Figure 10 shows the worst gaps g_w calculated for various gear pairs of module $m=1$, pressure angle $\alpha_0=20^\circ$. Numbers of teeth z_1, z_2 are taken larger than 17 to exclude the undercut gear. Since the gap is proportional to module, g_w for gear pairs of different modules can be obtained by multiplying the values in Fig.10 by module. It is seen from Fig.10 that g_w decreases with an increasing z_1, z_2 .

4.4 Effects of backlash on worst gap

Figure 11 shows the relations between the circumferential backlash ($c_0 = 160 \sim 640 \mu\text{m}$) specified in JIS and the worst gap g_w , the circumferential load F_t for which contact ratio becomes 2 for the gear pair of $m=4, \alpha_0=20^\circ, i=z_2/z_1=30/30, b=10 \text{ mm}$. It is seen from this figure that g_w and F_t increase with an increasing c_0 .

4.5 Static loading test results

Figure 12 shows the measured results of root stresses for the test gear pair shown in Fig.3. The curve has discontinuity (T) near $F_t = 9.8 \text{ kN}$. This indicates that the adjacent-tooth contacts occur due to the deflections of meshing teeth. The calculated circumferential load, for which the contact ratio becomes 2, obtained from the results in Fig.11 for $c_0 = 160 \mu\text{m}$ is 9.9 kN and agrees well with the measured

Fig.10 Worst gaps for various gear pairs

one. Hence the calculating method for the gap used in this investigation is found valid.

4.6 Effects of adjacent-tooth contacts on bending fatigue strength

In order to examine the effects of adjacent-tooth contacts on bending fatigue strength, two kinds of bending fatigue tests were carried out.

Test I : The driving gear (loading gear) with adjacent teeth is used and single or triple tooth pairs contact occurs depending on applied load.

Test II : The driving gear (loading gear) without adjacent teeth is used and only single tooth pair contact occurs regardless of applied load.

Fig.11 Relations between backlash and worst gap, circumferential load for triple tooth pairs contact

Fig.12 Static loading test results

Figure 13 shows the bending fatigue test results of test I and test II. Since the bending fatigue-limit-load ($F_t = 8.8$ kN) for test I is smaller than $F_t = 9.8$ kN, for which the triple tooth pairs contact begins (Fig.12), only single tooth pair contact occurs for $F_t = 8.8$ kN and the limit load for test I becomes equal to that for test II. As the triple tooth pairs contact occurs in the range $F_t \geq 9.8$ kN, the fatigue life in test I shows a tendency to be longer in this range than that in test II. This tendency is more remarkable in the range $F_t > 14$ kN. This might be due to the shared load of adjacent teeth increasing with an increase in the circumferential load. The broken line in Fig.13 is an S-N curve for the outer point of single tooth pair contact converted from the results (solid line) of test II by using the root stress factor⁽⁴⁾. Since adjacent-tooth contacts do not occur under $F_t = 8.8$ kN, the bending fatigue-limit-load of this gear pair might be considered from Fig.13 to be $F_t = 7.9$ kN.

4.7 Design method of the gear pair with contact ratio of $\epsilon \geq 2$ under an application of load

Maximum deflection without tooth breakage for the circumferential load corresponding to the number of load cycles to failure N is defined as the normal tooth deflection limit, and that for $N = 1$ is called the static deflection limit δ_s and that for $N = 10^7$ the bending fatigue deflection limit δ_f . The circumferential loads F_s and F_f correspond to δ_s and δ_f respectively. F_t for $N = 10^3$ obtained by the extrapolation from the results of test II in Fig.13 is used here as F_s . The breaking stresses $\sigma_{ns} = 1760$ MPa, $\sigma_{nf} = 550$ MPa are obtained from $F_s = 28$ kN, $F_f = 8.8$ kN (Fig.13) by using the root stress factor⁽⁴⁾. These stresses are available for various gears made of S45C normalized steel. The normal deflections δ_{ns} and δ_{nf} are obtained by using σ_{ns} , σ_{nf} , the root

Fig.13 Bending fatigue test results

stress factor⁽⁴⁾ and the tooth deflection formulas⁽³⁾. The circular deflection limits δ_{dns} and δ_{dnf} are obtained from δ_{ns} and δ_{nf} by using Eq.(4) and shown in Fig. 14. The circular deflection limits δ_{dcs} and δ_{dcf} for casehardened gears are obtained in the same manner as in the case of the S45C steel gears. The breaking stresses for casehardened gears are $\sigma_{cs} = 2450$ MPa and $\sigma_{cf} = 980$ MPa⁽⁵⁾. δ_{dcs} and δ_{dcf} are shown in Fig.14.

Figure 15 shows the relation between the breaking stress σ and the circular deflection limit δ_d for the gear pairs of $m = 4$. The contact ratio of a gear pair may exceed 2 due to an application of load when g_w is smaller than g_d . The solid line in Fig.15 indicates the boundary where the contact ratio of a gear pair may exceed 2 due to an application of load.

Figure 16 shows the relations between the breaking stress and the circular deflection limit for various gear pairs.

Fig.14 Relation between worst gap and circular deflection limit

Fig.15 Relation between breaking stress and circular deflection limit

Fig.16 Relation between breaking stress and circular deflection limit for various gear pairs

It is seen from this figure that the inclination of solid line increases with an increasing module. This figure can be used to design the gear pair the contact ratio of which exceeds 2 due to an application of load.

5. Conclusions

The main results obtained from this investigation are summarized as follows.

- (1) A calculating method for adjacent-tooth gaps of spur gear pairs with geometric contact ratio of $1 < \epsilon < 2$ is introduced.
- (2) The meshing position where the circumferential load, under which adjacent-tooth contacts occur, becomes maximum is defined here as the worst loading position in low-cycle range and the average gap at the position, as the worst gap.
- (3) In the standard spur gear pairs of

pressure angle $\alpha_0 = 20^\circ$, the worst loading position lies in the close vicinity to pitch point and the worst gap is nearly equal to the average gap at the position.

(4) The calculated circumferential load for which adjacent-tooth contacts occur agrees well with the measured one.

(5) The worst gap for various standard spur gear pairs of $\alpha_0 = 20^\circ$ is calculated. It decreases with an increasing number of teeth in pinion and gear.

(6) When the contacts of adjacent teeth to meshing teeth occur, the bending fatigue life becomes longer than that in the case where the adjacent-tooth contacts are assumed not to occur.

(7) A design method for the gear pair with contact ratio of $\epsilon \geq 2$ under an application of load is proposed.

Acknowledgement

The authors wish to thank Mr. C. Namba of Department of Mechanical Engineering of Tottori University, Mr. K. Araki of Koyo Co., Ltd. and Mr. M. Mizune of Tottori Sanyo Electric Co., Ltd. for their assistance in performing this investigation.

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