A Study on Transmission Characteristics of Toothed Belt Drives*

(4th report, Transmission Error at Normal and Reverse Revolution under a State of Initial Tension)

By

Masanori KAGOTANI**, Toshio AIDA***

Tomio KOYAMA⁺, Susumu SATO⁺⁺, Takeshi HOSHIRO⁺⁺⁺

In toothed belt drives under a state of initial tension in the static region, the generating mechanisms of a transmission error when both of pulleys are revolved reversely from a stable meshing state between belt and when both of pulleys are in normal revolution are theoretically and experimentally discussed.

The magnitude of an initial tension, a difference of the pitch difference between both pulleys and a backlash between belt tooth and pulley tooth have great effect on the transmission error, and some methods to reduce the transmission error are discussed.

Key Words : Machine Element, Toothed Belt, Transmission Error, Initial Tension, Normal and Reverse Revolution

1. Introduction

Recently, with advances in electromechanics, toothed belt drives have come to be used increasingly at the positions where correct synchronization of rotation or correct point setting is required.

Considering the function of the apparatus, there are many occasions in which belts are used in normal and reverse revolutions. In the case of drive of serial printer head, position control of robots for industries, transmission error at normal and reverse revolutions of the driving pulley is a problem. Use of pulleys in normal and reverse revolutions is expected to increase, but a study on transmission error under normal and reverse revolutions has not been made yet.

In this paper, the transmission error in the case where pulleys make a reverse revolution from the normal revolution under initial tension in static regions was theoretically investigated. The effect of initial tension, pitch difference and backlash on the transmission error at the reverse revolution was also discussed.

2. Theoretical Analysis

As mentioned in the first report (1), when a toothed belt is set in an arbitrary initial tension and a pulley is made to rotate in a certain direction, a meshing state between belt and pulley teeth gradually changes, until at last a stable meshing state is attained. The transmission error at reverse revolution of the pulley is discussed in the case when the pulleys are made to rotate in a reverse direction to this stable meshing state.

Figure 1 shows the meshing state of belt teeth and pulley teeth at normal revolution and reverse revolution. Dotted lines indicate the meshing state at normal revolution, and thick solid lines indicate the stable meshing state after the pulleys are rotated. Points (b) and (d) indicate the beginning of meshing, when the pulley 1 is making normal revolution in the direction of the dotted arrow, but when the pulley 1 is made to rotate in the direction of the solid arrow, points (a) and (c) indicate the beginning of meshing. In other words, the contact state between the belt teeth and the pulley teeth at (a) and (c) makes a gradual change from the beginning of meshing to the end of meshing, until at last it becomes

Fig.1 Meshing state in normal and reverse revolution

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** Lecturer, Osaka Industrial University, 3-1-1, Nakagaito, Daito-shi, Osaka.

*** President, Osaka Industrial University, + Associate Professor, Osaka Institute of Technology, 5-16-1, Omiya, Asahi-ku, Osaka.

++ Professor, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto.

+++ Director, General Technical Manager, Mitsubishi Belting Ltd., 4-1-21, Hamazoe-dori, Nagata-ku, Kobe.

stable. At (b) and (d), the contact state becomes stable in an opposite way. In this way, when the pulley makes a reverse revolution, the transmission error is produced in both pulleys until the contact state at the beginning and the end of meshing is interchanged.

First, the contact amount between the belt teeth and the pulley teeth in the process of both pulleys being rotated reversely is discussed. In order to simplify the problem, it is supposed that the contact amount changes every one pitch revolution of the pulley.

Figure 2 shows the contact state between the belt teeth and the pulley teeth at the beginning and the end of meshing for one round of the belt by straight lines. Fig.2(i) indicates that the contact state between the belt teeth and the pulley teeth is kept stable when the belt is rotated normally in the direction of the arrow. Fig.2(ii) shows the contact state when the pulleys shift by $x_{p(j)}$ on the pitch line, and the pulley 1 is rotated reversely in the direction of the arrow by $\theta_{(1)}$ from the state (i). Where, the teeth numbers of the belt are expressed by (j, k) , and as shown in Fig.1, $j=1$ indicates the pulley 1, and $j=2$ indicate the pulley 2. $k=1$ indicates the beginning of meshing at the reverse revolution, and $k=n$, the end of meshing. Here, n is an arbitrary positive integer.

As shown in Fig.2, at the normal revolution, the contact amounts between the belt teeth and the pulley teeth are $\lambda_{(j,1)}$ and $\lambda_{(j,n)}$ at the beginning and the end of meshing respectively at first. From this meshing condition, the pulleys are rotated reversely, and by shifting the pulley 1 by $x_{p(1)}$ the contact amounts become $\lambda_{r(j,1)}$, $\lambda_{r(j,n)}$; and suppose the pulley 2 shifts by $x_{p(2)}$, then the difference of the amount of shifting between the pulleys 1 and 2 is given by

$$x_{p(2)} - x_{p(1)} = \{\lambda_{r(2,1)} - \lambda_{(2,1)} + \lambda_{r(2,n)} - \lambda_{(2,n)}\} / 2 - \{\lambda_{r(1,1)} - \lambda_{(1,1)} + \lambda_{r(1,n)} - \lambda_{(1,n)}\} / 2$$

where, the sign of the amount of contact at normal and reverse revolutions of the pulleys is taken as follows: the plus sign is in the case that the belt tooth flank on the opposite side to the belt running direction at the reverse revolution contacts the pulley tooth flank, and the minus sign is in the case that the belt tooth does not contact the pulley tooth flank. At the beginning and the end of meshing in reverse

Fig.2 Contact state for one round of belt

revolution the amount of belt shifting becomes equal, if the bearing friction, the moment of inertia, the sliding slip between the belt bottom land and the pulley tip land during the revolution of pulleys are neglected. Consequently, the difference of the amount of the movement between the pulleys 1 and 2 is from the above equation given by

$$x_{p(2)} - x_{p(1)} = \{\lambda_{r(2,1)} - \lambda_{(2,1)}\} - \{\lambda_{r(1,1)} - \lambda_{(1,1)}\} \quad \dots\dots\dots (1)$$

From this equation, it can be said that a transmission error is decided by the contact amount at the beginning of meshing of the pulleys 1 and 2. In Eq.(1), the first term of the right side indicates the amount of movement of the belt from the contact state in normal revolution when the pulley 2 makes a reverse revolution by the angle $\theta_{(2)}$, and the second term indicates that when the pulley 1 makes a reverse revolution. When this amount of movement is to be expressed in $x_{b(j)}$ from Eq.(1), $x_{b(j)}$ is given by

$$x_{b(j)} = \lambda_{r(j,1)} - \lambda_{(j,1)} \quad \dots\dots\dots (2)$$

Next discussion is made on the amount of movement of the belt when a pulley makes a reverse revolution by a distance of one pitch. Suppose the pulley makes a movement by a distance of the pulley pitch $t_{p(j)}$ on the pitch line, and then the amount of movement of the belt is a distance of $t_{p(j)}$ at the beginning and the end of meshing. The belt pitch t_b is extended by the initial tension T_i , and the belt pitch at that time is expressed in t_b' , the belt tooth makes a movement by the distance of $t_b' - t_{p(j)}$ from the position of the belt teeth in normal revolution. The amount of the movement of the belt becomes zero, when $t_b' = t_{p(j)}$, and if the belt tension at that time is expressed in $T_{c(j)}$, $T_{c(j)}$ is given as

$$T_{c(j)} = -\frac{SE}{t_b} \Delta t_{(j)} \quad \dots\dots\dots (3)$$

where, S indicates a cross sectional area, and E indicates a modulus of elasticity, $\Delta t_{(j)}$ indicates a pitch difference; and it is expressed in plus when $t_b > t_{p(j)}$, and in minus when $t_b < t_{p(j)}$. The amount of movement of the belt $\Delta x_{b(j)}$, when the pulley makes a reverse revolution by one pitch is expressed as follows with $T_{c(j)}$.

$$\Delta x_{b(j)} = \frac{t_b}{SE} \{T_i - T_{c(j)}\} \quad \dots\dots\dots (4)$$

The maximum amount of movement of the belt $x_{b(j)\max}$ from the beginning of the reverse revolution of the pulley until the contact state becomes stable, is given by

$$x_{b(j)\max} = \pm \{\lambda_{(j,1)} + \lambda_{(j,n)} + b_{(j)}\} \quad \dots\dots\dots (5)$$

where, when $j=1$, the sign of the right side is plus, and when $j=2$, it is minus, and $b_{(j)}$ indicates a backlash.

From Eqs.(4) and (5), the angle of revolution of the pulley $\theta_{s(j)}$ until the contact state of the pulley 1 and the pulley 2 becomes stable is given by

$$\theta_{s(j)} = \left| \frac{SE(\lambda_{G(j)} + \lambda_{G(n)} + b_{t(j)})}{t_b(T_i - T_{c(j)})} \right| \frac{2\pi}{z_{(j)}} \dots\dots\dots (6)$$

where, $z_{(j)}$ indicates the number of pulley teeth.

The transmission error $\Delta\theta_r$ in reverse revolution is expressed as an angle of lead or lag of the pulley 2 generated by the revolution of the pulley 1, and $\theta_{(1)}$ is generated within the limit of $0 < \theta_{(1)} < \theta_{s(j)}$. After getting the limit $\theta_{(1)} > \theta_{s(j)}$, the contact state of the pulleys 1 and 2 remains stable, consequently $\Delta\theta_r$ is not generated. Within the limit until the contact state of either the pulley 1 or 2 gets stable, $\Delta\theta_r$ in the case that the pulley 1 makes a reverse revolution by the angle $\theta_{(1)}$ is given by

$$\Delta\theta_r = \frac{z_{(1)}\theta_{(1)}}{2\pi} \frac{t_b}{SE} (T_{c(1)} - T_{c(2)}) / R_{c(2)} \dots\dots\dots (7)$$

where, $R_{c(2)}$ indicates the pitch radius of the pulley 2, and a plus $\Delta\theta_r$ means that the pulley 2 goes forward to the angle of the revolution of the pulley 1, and a minus $\Delta\theta_r$ means that it goes backward. After either of the pulleys reaches the stable contact state, $\Delta\theta_r$ is determined by which is larger, $\theta_{s(1)}$ or $\theta_{s(2)}$ in Eq.(6), and is given by

$$\left. \begin{array}{l} \text{(I) in the case } \theta_{s(1)} > \theta_{s(2)} \\ \Delta\theta_r = \left[x_{b(2)\max} - \frac{z_{(1)}\theta_{(1)}}{2\pi} \frac{t_b}{SE} (T_i - T_{c(1)}) \right] / R_{c(2)} \\ \text{(II) in the case } \theta_{s(1)} < \theta_{s(2)} \\ \Delta\theta_r = \left[x_{b(1)\max} + \frac{z_{(1)}\theta_{(1)}}{2\pi} \frac{t_b}{SE} (T_i - T_{c(2)}) \right] / R_{c(2)} \end{array} \right\} \dots\dots\dots (8)$$

The contact state of either the pulley 1 or the pulley 2 is kept stable already before the beginning of the reverse revolution; $\Delta\theta_{r(j)}$ is given by the following equation, if the transmission error at the side where the contact state is not stable is shown in $\Delta\theta_{r(j)}$,

$$\Delta\theta_{r(j)} = \pm \frac{z_{(1)}\theta_{(1)}}{2\pi} \frac{t_b}{SE} (T_i - T_{c(j)}) / R_{c(2)} \dots\dots\dots (9)$$

where, the sign of the right side is minus when $j=1$, and plus when $j=2$.

3. Comparative Study between Experimental and Calculated Results

In this chapter, the effect of initial tension, pitch difference, backlash on the transmission error, when the pulley is made to rotate reversely from the stable contact state at the normal revolution is experimented with the aid of the theoretical analysis in Chapter 2. And the experimental results are compared with the calculated results.

3.1 Experimental apparatus and procedure of experiment

The same experimental apparatus mentioned in the 3rd report⁽²⁾ was used. Rotary magnetic encoders were attached to each of the shafts. Pulse signals from the encoders were loaded on RAM of a micro-computer continuously through the gate circuit and the interface. And the relative transmission error between both shafts was calculated. As the theoretical analysis in the static region was made, the number of revolutions of the pulley 1 was set at 0.0109 s^{-1} , and the gate time for loading the data in to the micro-computer was set at 5 seconds. As the dividing angle of the rotary encoder used in this experiment is $14.54 \text{ } \mu\text{rad}$, the number of pulses while the gate is open is about 23,500. In order to measure $\Delta\theta_r$, the following procedure was taken in the experiment: (1) The initial tension is applied. (2) The pulley 1 is made to rotate until the stable contact state is reached. (3) Then the belt tension is measured. (4) After that, the pulley 1 is made to rotate reversely and at the same time loading of data in to the micro-computer is started. In order to measure $\Delta\theta_{r(j)}$ in the case that the contact state of the pulley 1 or the pulley 2 is stable in reverse revolution, the pulley 1 is set, and a wire rope is wound around the shaft of the pulley 2 in the direction of clockwise or counterclockwise, then a mass is hung down at the other end of the rope. By force of the mass a sliding slip occurs, which changes the contact state of either pulley. Then the mass is taken away and the experiment is finished.

Table 1 indicates the main dimensions of pulleys used in this experiment. Each pulley has 18 teeth, Nos.4 and 5 are the pulleys on market and Nos.1-3 are the trial manufactured pulleys. $\Delta t_{(1)}$ of Nos.1 and 2 was made as equal as possible, and in order to get $\Delta t_{(1)} = \Delta t_{(2)}$, it is necessary to get an equal outside diameter for both pulleys. This pair of hobbled pulleys are divided into two parts, after their outside surface is ground. $b_{t(j)}$ of Nos.1-3 is smaller than that of the pulleys on market by about 0.01mm, as the outside surface is ground. Two kinds of 270L polychloroprene rubber belts ($t_b = 9.525\text{mm}$, number of teeth $z_b = 72$) are used. One has $b = 9.5\text{mm}$ width and the other 19mm. $\Delta t_{(j)}$ are the calculated results in the case that the belt is wound lightly around the pulley.

3.2 Effect of initial tension on transmission error

It is very rare that the pulleys on the market have the same outside diameter, as the tolerance of the outside diameter is $+0.10-0\text{mm}$ in the case that $z_{(j)} = 18$ teeth⁽³⁾. It means that the pitch

Table 1 Dimensions of pulleys

No.	Unit:mm				
	Trial manufacture pulleys			Pulleys on market	
$\Delta t_{(j)}$	1	2	3	4	5
$2R_{c(j)}$	53.787	53.790	53.765	53.870	53.848
$b_{t(j)}$	0.45	0.45	0.04	0.46	0.05

difference between the two pulleys is usually different. So in the experiment about the effect of T_i on $\Delta\theta$, the pulleys Nos.2 and 4 in Table 1 with a belt of $b=19\text{mm}$ were used.

$\Delta\theta$ is considered to be largely influenced by the contact state between the belt teeth and the pulley teeth at the beginning and the end of meshing in normal revolution. So, the numerical calculations on contact state were done first. The values in Table 1 were used in the calculation, the value of the compliance of the belt tooth being $f_{(j,k)}=2.55\mu\text{m/N}$, the product of sectional area of load-carrying cords of belt and the modulus of elasticity of belt being $SE=147\text{kN}$, the coefficient of friction between the belt and the pulley being $\mu=0.35$.

Figure 3 shows the effect of the initial tension T_i on the contact amount $\lambda_{(j,k)}$. The solid line and the dotted line indicate the contact amounts $\lambda_{(1,1)}$, $\lambda_{(1,n)}$ at the beginning and the end of meshing in the case of the stable contact state of normal revolution, and the dot-dashed line and the double-dots-dashed line indicate $\lambda_{(2,1)}$ and $\lambda_{(2,n)}$ on the pulley 2 respectively. As $b_{(1,1)} \approx 0.46\text{mm}$, the belt teeth and the pulley teeth do not contact within the limit of the contact amount from 0 to -0.46mm . When the contact amount is plus, the belt teeth flanks at the opposite side from the running direction contact the pulley teeth flanks, and when the contact amount is less than -0.46mm , the belt teeth flanks of the running direction side make contact with pulley teeth flanks. As the contact direction between the belt teeth and the pulley teeth changes depending on the range of initial tensions T_i , the lines of the contact amount change rapidly at $T_{c(1)}$, $(1), (4)$. And at $T_{c(1)}$, $t_{p(1)}$ is equal to t_b . This $T_{c(1)}$ becomes larger as the value of $\Delta t_{(1)}$ becomes larger in minus. As $\Delta t_{(1)}$ of the pulleys No.1 and No.2 are different, the contact direction of the pulleys No.1 and No.2 differs according to the conditions of $T_i < T_{c(1)}$, $T_{c(1)} < T_i < T_{c(2)}$ and $T_i > T_{c(2)}$, as shown in the figure. As the pulley is made to rotate reversely from such a contact state, $\Delta\theta$ suffers large influence from T_i .

Figure 4 shows the experimental results about the transmission error. $\circ$ indi-

Fig.3 Contact state in normal revolution

cates $\Delta\theta$ in normal revolution, and $\bullet$ indicates $\Delta\theta_r$ in reverse revolution. $\circ$ indicates $\Delta\theta_{r(1)}$ in the case that the contact state of the pulley 2 is already stable before the beginning of the reverse revolution, and the contact state of only the pulley 1 changes. And $\bullet$ indicates $\Delta\theta_{r(2)}$ in the case that the contact state of only the pulley 2 changes. In both experimental results, the transmission error changes with a period of almost $8\pi\text{rad}$. This is $\Delta\theta$ with a period of one round of the belt, because of $z_{(1)}=18$ teeth, $z_b=72$ teeth⁽²⁾. As the pulley has a little eccentricity, $\Delta\theta$ with the period of one round of the pulley, is obtained, but this $\Delta\theta$ does not appear in Fig.4, because the order of abscissa is big. But $\Delta\theta$ with the period of one pitch of the pulley did not occur theoretically as well as experimentally⁽⁵⁾.

In this experimental results, with an increases of $\theta_{(1)}$, $\Delta\theta$ keeps almost constant, while $\Delta\theta_r$ appears clearly. $\Delta\theta_r$ increases to the point A in the figure, then decreases from point A to B, and becomes stable around zero. This tendency is studies from $\Delta\theta_{r(1)}$, $\Delta\theta_{r(1)}$ and $\Delta\theta_{r(2)}$ are compared first. $\Delta\theta_{r(1)}$ is stable in plus, and $\Delta\theta_{r(2)}$ is stable in minus. This is explained by the fact that the belt moving direction until the contact state becomes stable coincides with the belt running direction, because $\lambda_{(j,k)}$ is plus as shown in Fig.3, when $T_i=197\text{N}$ $\Delta\theta_{r(1)}$ becomes stable faster than $\Delta\theta_{r(2)}$, as $T_i < T_{c(2)}$, $T_{c(1)} > T_{c(2)}$. As is evident from the experimental results of $\Delta\theta_{r(1)}$, $\Delta\theta_r$ appears as the sum of $\Delta\theta_{r(1)}$ and $\Delta\theta_{r(2)}$. In other words, $\Delta\theta_r$, up to the point A, is within the limits until the contact state of the pulley 1 becomes stable, and between A and B, it is within the limits until only the pulley 2 becomes stable.

As experimental results coincide with the calculated ones well, the theoretical analysis mentioned in Chapter 2 is considered to be valid.

Fig.4 Relation between $\theta_{(1)}$ and $\Delta\theta, \Delta\theta_r, \Delta\theta_{r(1)}$

Figure 5 shows the relation between $\theta_{(1)}$ and $\Delta\theta_r$, using T_i as a parameter. In the experimental results, the first incline of each $\Delta\theta_r$ is positive and almost equal independently of T_i . This is explained by the fact that the sign and the incline of $\Delta\theta_r$ are determined by the magnitudes of $T_{c(1)}$ and $T_{c(2)}$ and their difference. The tendency of $\Delta\theta_r$ after that differs according to the magnitude of T_i . $\Delta\theta_r$, marked $\bigcirc, \bullet, \bigcirc$ reduces and then becomes stable, while $\Delta\theta_r$, marked $\ominus$ and $\oplus$ increases and then becomes stable. As $T_{c(1)} \approx 370$ N, $T_{c(2)} \approx 240$ N from Fig. 3, $\Delta\theta_{r(1)}$ is plus when $T_i < 370$ N, and minus when $T_i > 370$ N. $\Delta\theta_{r(2)}$ has the same tendency with $T_i \approx 240$ N as a boundary. When $T_i < 240$ N, the values of $\Delta\theta_{r(2)}$ appear in both plus and minus, and $\Delta\theta_r$ decreases. When $240 \text{ N} < T_i < 370 \text{ N}$, both of $\Delta\theta_{r(2)}$ are plus, and $\Delta\theta_r$ increases. When $T_i > 370 \text{ N}$, $\Delta\theta_r$ has the same tendency as when $T_i < 240 \text{ N}$ in the experimental results.

Figure 6 shows the effect of T_i on the maximum value $\Delta\theta_{r\max}$ of $\Delta\theta_r$, and on the angle of revolution of the pulley 1, $\theta_{(1)\max}$, until $\Delta\theta_r$ becomes stable. $\Delta\theta_{r\max}$ and $\theta_{(1)\max}$ differ according to the magnitudes of $T_{c(1)}$, T_i etc., and they are given by

- (I) in the case $T_{c(1)} > T_{c(2)}$
- (1) when $\theta_{s(1)} > \theta_{s(2)}$, $T_i < T_{c(1)}$
 $\Delta\theta_{r\max} = \Delta\theta_{x(1)} + \Delta\theta_{x(2)}$, $\theta_{(1)\max} = \theta_{s(1)}$
 - (2) when $\theta_{s(1)} > \theta_{s(2)}$, $T_i > T_{c(1)}$
 $\Delta\theta_{r\max} = \Delta\theta_{x(2)} - \beta_{(1)}\theta_{s(2)}$, $\theta_{(1)\max} = \theta_{s(1)}$
 - (3) when $\theta_{s(1)} < \theta_{s(2)}$, $T_i < T_{c(2)}$
 $\Delta\theta_{r\max} = \Delta\theta_{x(1)} + \beta_{(2)}\theta_{s(1)}$, $\theta_{(1)\max} = \theta_{s(2)}$
 - (4) when $\theta_{s(1)} < \theta_{s(2)}$, $T_i > T_{c(2)}$
 $\Delta\theta_{r\max} = \Delta\theta_{x(1)} + \Delta\theta_{x(2)}$, $\theta_{(1)\max} = \theta_{s(2)}$
- (II) in the case $T_{c(1)} < T_{c(2)}$
- (1) when $\theta_{s(1)} > \theta_{s(2)}$, $T_i < T_{c(1)}$
 $\Delta\theta_{r\max} = \Delta\theta_{x(2)} - \beta_{(1)}\theta_{s(2)}$, $\theta_{(1)\max} = \theta_{s(1)}$
 - (2) when $\theta_{s(1)} > \theta_{s(2)}$, $T_i > T_{c(1)}$
 $\Delta\theta_{r\max} = \Delta\theta_{x(1)} + \Delta\theta_{x(2)}$, $\theta_{(1)\max} = \theta_{s(1)}$
 - (3) when $\theta_{s(1)} < \theta_{s(2)}$, $T_i < T_{c(2)}$
 $\Delta\theta_{r\max} = \Delta\theta_{x(1)} + \Delta\theta_{x(2)}$, $\theta_{(1)\max} = \theta_{s(2)}$
 - (4) when $\theta_{s(1)} < \theta_{s(2)}$, $T_i > T_{c(2)}$
 $\Delta\theta_{r\max} = \Delta\theta_{x(1)} + \beta_{(2)}\theta_{s(1)}$, $\theta_{(1)\max} = \theta_{s(2)}$
-(10)

where, $\beta_{(j)} = \Delta x_{b(j)} / R_{c(2)}$, $\Delta\theta_{x(j)} = x_{b(j)\max} / R_{c(2)}$.

In the figure, the plotted points are

Fig. 5 Relation between $\theta_{(1)}$ and $\Delta\theta_r$ using T_i as a parameter

experimental results, the mark $\bigcirc$ indicates the case that $\Delta\theta_r$ becomes stable around zero, the mark $\bullet$ indicates the case when $\Delta\theta_r$ becomes stable at a value which keeps increasing. In both experimental and calculated results, $\theta_{(1)\max}$ increases abruptly around $T_i = 240 \text{ N}$ and 370 N . This is because these T_i are around $T_{c(1)}$, and the coiled T_i comes to $T_{c(1)}$, $\theta_{(1)}$, until $\Delta\theta_{r(1)}$ becomes stable, becomes the larger. The theoretical value is discontinuity at the point C and D (■ mark). The discontinuity at the point C is caused by the fact that $\Delta\theta_{r(2)}$ becomes zero and only $\Delta\theta_{r(1)}$ has an effect on $\Delta\theta_r$. At the point D, it is caused by the fact that $\Delta\theta_{r(1)}$ becomes zero. $\Delta\theta_{r\max}$ also is discontinuous at $T_{c(1)}$, because the positions where $\Delta\theta_r$ becomes stable differ according to the limit of $T_{c(1)}$. As is clear from the figure, $\theta_{(1)\max}$ and $\Delta\theta_{r\max}$ become maximum around $T_i = T_{c(1)}$. $\Delta\theta$ with a period of one pitch of the pulley has less value, if T_i is set around $T_{c(1)}$ (2), (5). But it takes long time for $\Delta\theta_r$ in reverse revolution to reach a stable contact state, if T_i is set around $T_{c(1)}$.

3.3 Effect of pitch difference on transmission error

As mentioned in Chapter 3.2, $\Delta\theta_r$ is greatly affected by $T_{c(1)}$. It is considered that $\Delta\theta_r$ occurs by a difference in the pitch difference between the two pulleys. To confirm this, the experiments were done in the case of $\Delta t_{(1)} \approx \Delta t_{(2)}$, using the pulleys No. 1 and No. 2.

Figure 7 shows the experimental results about the transmission error in the case of $\Delta t_{(1)} \approx \Delta t_{(2)}$. Except for $\Delta t_{(1)}$, the same experimental conditions as those of Fig. 4 were adopted. As was made clear from the experimental results, $\Delta\theta$ in normal

Fig. 6 Effect of T_i on $\theta_{(1)\max}$ and $\Delta\theta_{r\max}$

revolution and $\Delta\theta$, in reverse revolution have little difference, though $\Delta\theta_{r(1)}$ occurs, and $\Delta\theta_r$ does not occur when the pitch difference between the two pulleys is equal. In the case of Fig.4, the difference of the outside diameter between the two pulleys is 0.08mm, while in the case of Fig.7 it is 0.003mm. From above fact, in order to minimize $\Delta\theta_r$ in reverse revolution, the difference of the outside diameters between the two pulleys should be kept in micron order.

Figure 8 shows the calculated results about the effect of $\Delta t_{(1)}$ on $\theta_{(1)\max}$ and $\Delta\theta_{r\max}$. The solid lines indicate $\theta_{(1)\max}$, and the dotted line $\Delta\theta_{r\max}$. Here, $T_i=460\text{N}$ is kept constant, and $\Delta t_{(1)}$ of only the pulley 1 is changed. $\theta_{(1)\max}$ becomes infinite around $\Delta t_{(1)}=-0.03\text{mm}$. This is explained by the fact that $\Delta t_{(1)}$ is about -0.03mm , when $T_i=460\text{N}=T_{c(1)}$. So, the discontinuity at the point D in the figure occurs for the same reason mentioned in Fig.6. Within the limit of $\Delta t_{(1)}<-0.045\text{mm}$ and $\Delta t_{(1)}>-0.015\text{mm}$, $\theta_{(1)\max}$ is constant. As $\Delta t_{(2)}=-0.0152\text{mm}$ is kept constant, $\Delta\theta_{r(2)}$ does not change even if

$\Delta t_{(1)}$ changes. So, within the above mentioned limit of $\Delta t_{(1)}$, $\Delta\theta_{r(2)}$ becomes stable later than $\Delta\theta_{r(1)}$, and $\theta_{(1)\max}$ becomes constant. In the case that $\Delta t_{(1)}=-0.0152\text{mm}$, $-\Delta\theta_{r(1)}=\Delta\theta_{r(2)}$ is obtained, $\Delta\theta_r$ does not occur, and $\theta_{(1)\max}=0$ is obtained, which causes the discontinuity. But if $\Delta t_{(1)}$ becomes larger in the direction of minus even a little, $|\Delta\theta_{r(1)}|>\Delta\theta_{r(2)}$ is obtained and $\theta_{(1)\max}$ becomes continuous.

$\Delta\theta_{r\max}$ occurs in plus when $\Delta t_{(1)}<-0.0152\text{mm}$, and in minus when $\Delta t_{(1)}>-0.0152\text{mm}$. This is caused by the fact that the magnitudes of $T_{c(1)}$ and $T_{c(2)}$ interchange with $\Delta t_{(1)}=-0.0152\text{mm}$ as a boundary. $\Delta\theta_r$ gets stable around zero in the case of $\Delta t_{(1)}>-0.03\text{mm}$, but in the case of $\Delta t_{(1)}<-0.03\text{mm}$, $\Delta\theta_r$ gets stable after its increase. This is because the contact direction in normal revolution in the former case is the same for both the pulleys No.1 and No.2, and in the latter the contact direction is different.

As is clear from Figs.7 and 8, in order to make $\theta_{(1)\max}$ and $\Delta\theta_{r\max}$ as small as possible, it is necessary to make the difference of the pitch difference smaller. And if T_i is fixed, it is necessary to avoid $\Delta t_{(1)}$ which comes at $T_i \approx T_{c(1)}$ as it takes long time until $\Delta\theta_r$ becomes stable.

3.4 Effect of belt width and backlash on transmission error

Figure 9 shows the relation between T_i and $\theta_{(1)\max}$, using b as a parameter. The experimental conditions are the same with those of Fig.6 except b . The plotted points indicate the experimental results for $b=9.5\text{mm}$, $\circ$ mark indicates the case in which the stable position of $\Delta\theta_r$ is close to zero and $\bullet$ mark does the case in which the stable position of $\Delta\theta_r$ is set keeping its increases. Solid lines indicate the calculated results for $b=9.5\text{mm}$ and dotted lines the results for $b=19\text{mm}$. When b becomes a half, we got $f_{(1,k)}=5.1\mu\text{m/N}$ and $SE=73\text{kN}$. As is clear from the experimental results, $T_{c(1)}$ becomes a half, if b is made a half. As b becomes smaller, the limit of T_i where $\Delta\theta_r$ becomes large decreases. If the pulleys available on the market have various $\Delta t_{(1)}$, favorable results are obtained if b is made smaller, as the range of T_i for decrease of $\Delta\theta_r$ becomes wider. But in this case, SE becomes also smaller, which easily causes an

Fig.7 Relation between $\theta_{(1)}$ and $\Delta\theta$, at $\Delta t_{(1)} \approx \Delta t_{(2)}$

Fig.8 Effect of $\Delta t_{(1)}$ on $\theta_{(1)\max}$ and $\Delta\theta_{r\max}$

Fig.9 Effect of b on $\theta_{(1)\max}$

interference of the belt teeth with the pulley teeth at the incomplete meshing region. So small b is considered to have bad effect on belt life and $\Delta\theta$ (5),(6).

Figure 10 shows the relation between $\theta_{(1)}$ and $\Delta\theta$, taking $b_{(1)}$ as a parameter. $\bigcirc$ mark indicates experimental results with $b_{(1)} \approx 0.05$ mm, the pulleys being Nos.3 and 5, and the belt width $b=19$ mm. $\bullet$ mark does the experimental results for $b_{(1)} \approx 0.46$ mm. The experimental conditions are the same with the case at Fig.4 except T_i . The pulley No.3 was made a little smaller by means of grinding its outside diameters in order to close to the difference of the pitch difference in the case of $\bullet$ mark. Solid lines indicate the calculated results for $b_{(1)} \approx 0.05$ mm, and dotted lines for $b_{(1)} \approx 0.46$ mm. In the experimental results for $b_{(1)} \approx 0.05$ mm and $b_{(1)} \approx 0.46$ mm, it is shown that there is some difference in the change of $\Delta\theta$, after it becomes stable after the reverse revolution. This change as a period of one round of the pulley, as the amount of eccentricity of the pulleys is different between the two experiments. Comparing the experimental results for $b_{(1)} \approx 0.05$ mm and for $b_{(1)} \approx 0.46$ mm, the maximum value of $\Delta\theta$, and the angle of revolution $\theta_{(1)}$ of the pulley 1 when of $\Delta\theta$, becomes stable are smaller than those for 0.46mm, though in the case of $b_{(1)} \approx 0.05$ mm, the pitch difference between the two pulleys is larger, and T_i is set close to $T_{c(1)}$.

Fig.10 Relation between $b_{(1)}$ and $\Delta\theta$.

Fig.11 Effect of $b_{(1)}$ on $\theta_{(1)\max}$ and $\Delta\theta_{r\max}$

Figure 11 shows the calculated results about the effect of $b_{(1)}$ on $\Delta\theta_{r\max}$ and $\theta_{(1)\max}$. Here, $b_{(1)}$ of both pulleys change equally. The solid line indicates $\theta_{(1)\max}$, and the dotted line $\Delta\theta_{r\max}$. When $b_{(1)}$ is larger, $\theta_{(1)\max}$ and $\Delta\theta_{r\max}$ increase linearly. Considering the load distribution at $T_i=460$ N used in the calculation, all the $\lambda_{(j,n)}$ in normal revolution contact in the same direction except one or two teeth at the beginning of meshing of the pulley 2. So, in Eq.(5), as the change of $b_{(1)}$ hardly affects $\lambda_{(j,1)}$ and $\lambda_{(j,n)}$, and as only $b_{(1)}$ affects $\Delta\theta_r$, $\theta_{(1)\max}$ and $\Delta\theta_{r\max}$ have such tendencies as shown in Fig.11. And as $\lambda_{(j,1)}$ and $\lambda_{(j,n)}$ do not become zero, $\theta_{(1)\max}$ and $\Delta\theta_{r\max}$ do not become zero either, even if $b_{(1)}=0$. As is clear from the results as shown in Figs.10 and 11, it is much better to make $b_{(1)}$ small, as $\Delta\theta_r$ also becomes small in reverse revolution.

4. Conclusions

A theoretical analysis on the transmission error when the pulley changes its revolution from the normal revolution to the reverse one in the static region and on the applied initial tension was made, and the calculated and the experimental results were compared. Methods to reduce the transmission error in reverse revolution were sought. As the result, the following conclusions were obtained:

(1) The transmission error in reverse revolution occurs due to a difference in pitch difference between two pulleys. That is, if the difference between outside diameters of the two pulleys is controlled to micron order, the transmission error can be a negligible one.

(2) The initial tension has great effect on the transmission error. To reduce the transmission error, it is necessary to avoid applying it in the tension range in which the contact direction between the belt tooth flank and the pulley tooth flank is reversed.

(3) In the case that the outside diameters of the two pulleys have a large production error, the smaller the belt width, the narrower the tension range becomes in which a great transmission error occurs.

(4) Smaller backlash causes less transmission error.

(5) The experimental results and the calculated ones on above mentioned terms coincide comparatively well, which confirm the effectiveness of the theoretical analysis.

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