Studies on Stability of Two-degree-of-freedom Articulated Pipes Conveying Fluid
(The Effect of a Spring Support and a Lumped Mass)*

By Yoshihiko SUGIYAMA**

Stability of a cantilevered system of articulated pipes conveying fluid is studied theoretically and experimentally by considering models having two degrees of freedom. The system is nonconservative and becomes subject to flutter-type or divergence-type instability depending on the system parameters. Firstly the effect of an intermediate lateral spring support on the stability is investigated to show that an additional spring support at the discharge end can be destabilizing. Secondly the combined effect of a spring support and a lumped mass is discussed. It is shown that the stability analysis by neglecting internal damping may result in an erroneous flutter prediction, especially when a big lumped mass is attached to the nonconservative system at its free end. Finally a set of experiments were conducted to verify the theory. Agreement between theory and experiment was satisfactorily good.

Key words: Stability, Buckling, Flutter, Divergence, Spring Support, Lumped Mass, Damping, Nonconservative Systems

1. Introduction

There has been a strong demand for accumulating technical data to assess precisely the strength, stability and vibration characteristics of pipes conveying fluid. The investigation into dynamics of fluid-conveying pipes was pioneered by Benjamin for cantilevered articulated pipes and by Gregory & Paidoussis for continuous tubular cantilevers. They developed the basic theories and conducted quantitative experiments. Thereafter the stability and vibration of pipes conveying fluid have been studied extensively by many researchers and engineers. A mention should be made of a good correlation between stability problems of cantilevered pipes conveying fluid and those of elastic systems subjected to follower forces, as suggested by Herrman. A cantilevered system of pipes conveying fluid provides one physical origin for a tangential follower force. The present author intends to make an experimental verification of nonconservative stability problems, which has been seldom done, by dealing with cantilevered articulated pipes containing a flowing fluid. The effect of a lateral spring support on the stability of a nonconservative elastic system was investigated analytically by the author and his collaborators. However there was given no experimental background to the theoretical prediction.

* Received 6th May, 1983.
** Associate Professor, Faculty of Engineering, Tottori University, Koyama-cho, Tottori-shi.

It is then one of the intended aims of the present paper to supply an experimental backing to the earlier paper by the author and others. The present paper is a continuation of the earlier papers by the author and his collaborators on the stability of articulated pipes conveying fluid and its attention is brought to focus upon the effect of an intermediate lateral spring support and a lumped mass as well. The latter may be accompanied by the former.

2. Statement of the Problem

Figure 1 shows a mathematical model of articulated pipes containing a flowing fluid. The model is composed of two rigid pipes and two massless viscoelastic joints. Each pipe has length l and mass per unit length m. It is assumed that two joints have the same restoring moment coefficient k and viscous damping coefficient c.

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![Mathematical model of two-degree-of-freedom articulated pipes conveying fluid.](image-url)
Small motion of the model is allowed in the horizontal \( x, y \) plane and may be described by generalized coordinates \( \varphi_1 \) and \( \varphi_2 \). A spring support of spring constant \( K \) and mass \( M \) is attached to the second pipe \( l_\ell \) apart from the second joint. An incompressive fluid flows through the pipes at a constant velocity \( U \) relative to pipe wall. Mass of the flowing fluid contained in the unit length of pipe is denoted by \( m_\ell \).

The effect of a lateral spring support having a negligible mass will be discussed first. Then the combined effect of a spring and a concentrated mass will be considered. Theoretical prediction must be checked by experiment later.

3. Theory

3.1 Basic equations

It is assumed that the motion of the pipes is small. The fundamental theories of dynamics of articulated pipes conveying fluid have been presented by Benjamin \(^1\) and in a more general form by Paloussia & Dekniss \(^2\). According to Benjamin's formulation, \(^1\) the equations for a small motion of the model in Fig. 1 are written in the dimensionless forms:

\[
\begin{align*}
\frac{d^2 \phi_1}{U^2} + \frac{1}{2} \phi_2 + \alpha (\phi_1 + \eta \phi_2) + \beta \phi_1 (\phi_1 + 2 \phi_2) + \\
\gamma (2 \phi_1 - \phi_2) + (2 \phi_1 - \phi_2) + \kappa (\phi_1 + \eta \phi_2) + \\
u^2 (\phi_1 - \phi_2) = 0 , \\
\frac{d^2 \phi_2}{U^2} + \phi_1 + \alpha (\phi_1 + \eta \phi_2) + \beta \phi_1 (\phi_1 + 2 \phi_2) + \\
\gamma (2 \phi_1 - \phi_2) + (2 \phi_1 - \phi_2) + \kappa (\phi_1 + \eta \phi_2) + \\
\nu^2 (\phi_1 - \phi_2) = 0 .
\end{align*}
\]

with dimensionless parameters

\[
\alpha = \frac{K}{(m + m_\ell) l} , \quad \beta = \frac{m_\ell}{m + m_\ell} ,
\]

\[
\gamma = \frac{\alpha}{l U / (m + m_\ell) l}, \quad \kappa = \frac{\alpha^2}{K l}, \quad \eta = \frac{l_\ell}{l},
\]

\[
u = U / \sqrt{\frac{m_\ell}{K l}} , \quad \tau = \frac{U^2}{l / (m + m_\ell) l}.
\]

In Eqs. (1) \( \dot{} \) means differentiation with respect to the dimensionless time \( \tau \).

The solution to Eqs. (1) is now cast in the form

\[
\varphi_1(\tau) = A e^{\lambda \tau}, \quad ( \tau = 1, 2 ). \quad \cdots \tag{4}
\]

Substituting Eq. (4) into Eqs. (1) and applying the condition for existence of a non-trivial solution, one obtains the characteristic equation

\[
a_1 \lambda^6 + a_2 \lambda^5 + a_3 \lambda^4 + a_4 \lambda^3 + a_5 \lambda^2 + a_6 \lambda = 0 , \quad \cdots \tag{5}
\]

where

\[
a_1 = \frac{7}{36} + \frac{1}{3} \eta + \frac{4}{3} \eta^2 , \\
a_2 = 3 \gamma + (1 + 2 \eta + 2 \eta^2) \alpha \gamma + \frac{3}{2} \alpha \beta \eta \gamma \alpha \beta \eta \gamma , \\
a_3 = 3 + (1 + 2 \eta + 2 \eta^2) \alpha + \left( \frac{5}{3} - \eta \right) \kappa , \\
+ (1 - \eta) \kappa - \frac{5}{3} \alpha \beta \eta \gamma \alpha \beta \eta \gamma + \frac{5}{3} \alpha \beta \eta \gamma \alpha \beta \eta \gamma , \\
a_4 = 2 \gamma + (1 + 2 \eta + 2 \eta^2) \kappa \gamma + (5 - \eta) \kappa \gamma \alpha \beta \eta \gamma \alpha \beta \eta \gamma , \\
a_5 = 1 + (1 + 2 \eta + 2 \eta^2) \kappa = (1 + \eta) \kappa \gamma \alpha \beta \eta \gamma \alpha \beta \eta \gamma .
\]

3.2 Stability analysis

The root of Eq. (5) may be expressed by a complex eigenvalue

\[
\lambda = \sigma \pm j \omega , \quad \cdots \tag{7}
\]

where \( \omega \) is a dimensionless oscillatory frequency and \( \sigma \) a dimensionless damping. \( j \) is the imaginary unit.

The system is stable when \( \sigma < 0 \). The system is dynamically unstable when \( \sigma > 0 \) and \( \omega = 0 \), while it is statically unstable when \( \sigma = 0 \) and \( \omega = 0 \).

The boundary of dynamic instability (flutter) is determined by the Routh-Hurwitz condition

\[
a_3 a_1 a_3 - a_2 a_4 - a_1 a_5 = 0 , \quad \cdots \tag{8}
\]

The limit for static instability (divergence) is given by the condition of vanishing characteristic root

\[
a_2 = 0 . \quad \cdots \tag{9}
\]

4. Effect of a Spring Support

4.1 Theoretical prediction

Figure 2 shows the dimensionless critical flow velocity \( \varphi \) as a function of the dimensionless spring constant \( \kappa \) in case where a lateral elastic support is attached.

\[
\text{Fig. 2 Effect of spring constant}(\alpha = 0, \quad \gamma = 0.001, \quad \eta = 1.0).
\]
to the pipe at the discharge end. Since the flutter and divergence limits depend considerably on the magnitude of spring constant only in the vicinity of critical spring constant at which transition of instability mechanism takes place from flutter to divergence, the \( \kappa \) range of order of 0.1 to 10 is considered.

It is interesting to note that the stability map of Fig. 2 bears a good similarity to the corresponding maps given by Chen\(^{13}\) for a continuous tubeular cantilever and by Sugiyama and others\(^{12}\) for elastic systems subjected to a tangential follower force. This implies that if an experimental check is made of the present stability map, then the corresponding maps can find, though indirectly, their experimental backing.

Figure 3 shows the dimensionless critical flow velocity \( \frac{U_c}{\rho} \) as a function of the dimensionless position \( \eta \) for pipes with \( \beta = 0.5 \) and \( \gamma = 0.001 \).

When \( \kappa = 0.1 \), the system loses its stability only through flutter regardless of the position of a support. The flutter velocity increases slightly as the position shifts from the joint side to the tip side. Thus, a weak spring support at the tip end (\( \eta = 1.0 \)) may be stabilizing. The broken line for \( \kappa = 0.1 \) gives the latent divergence limit.

When \( \kappa = 1.0 \), the system becomes subject to either flutter or divergence depending on support position. It is seen that the support at the discharge end results in the lowest critical velocity.

When \( \kappa = 10 \), divergence is predominant for the most support position. It is confirmed that a strong spring support at the free end leads to the greatest destabilization. On the other hand, the highest stabilizing effect occurs at the critical position at which transition of instability mechanism takes place. Chen\(^{13}\) and Sugiyama et al.\(^{12}\) have dealt with nonconservative systems having a spring support at their free ends. However, it is now shown that there is a justification for an engineer to add a spring support (of effective spring constant) to nonconservative systems at their free ends, only if he intends to destabilize the system.

4.2 Experiment
4.2.1 Outline of experiment

The aim of the experiment was to check theoretical prediction. The outline of the experimental apparatus and measurements involved may be found in earlier papers.\(^{11,17}\)

An elastic springs, U-shaped formed wires were made of piano wires with different diameters and dimensions. The non-dimensional spring constant \( \kappa \) was set in the range of 0.1 to 30 with appropriate spacings.

The test pipes were basically the same ones as used in the earlier works.\(^{15,17}\) Only the joints were newly furnished with fresh rubber sleeves. A careful adjustment was made of the sleeves to make the two joints have an equal elastic coefficient. The measured value of \( \kappa \) was 9.8 N.cm/rad. The
measured damping coefficient was in its dimensionless form γ=0.0074. Flowing fluid through the pipes was water.

4.2.2 Experimental results

The results of experiment about the effect of spring constant are plotted in Fig. 4 and compared with the theoretical curves. Fig. 5 shows a comparison between the theoretical and experimental results about the effect of support position. Curves were plotted from Eqs. (8) and (9) using the measured quantities of the test pipes. A good agreement exists between theory and experiment. Thus the conclusions in Section 4.1 are now experimentally verified.

5. Combined Effect of a Spring Support and a Lumped Mass

5.1 Theoretical prediction

Figure 6 shows the dimensionless critical flow velocity $\frac{\omega r}{c}$ as a function of the dimensionless spring constant $K$, when the mass of a spring is taken into account. It is noticed in Fig. 6 that there are solid and dotted lines for flutter boundaries. The solid curves indicate flutter velocities obtained by taking a small damping into account, while the dotted ones do so by neglecting the damping. It is interesting to see that there exists a considerable discrepancy between the solid curves and dotted ones as the mass parameter $\alpha$ increases to be $\alpha>1$. This stabilizing effect of a small damping, in conjunction with a big added mass, was first found in author's earlier paper (10).

It is seen that transition of instability mechanism takes place as the dimensionless spring constant $K$ increases. The transition eventually results in a sudden stabilization of the system due to a strong spring support. This implies that when a spring support at the tip end is accompanied by a considerable mass (say, $\alpha>1$) only a spring with a sufficiently high spring constant can be stabilizing.

It is then concluded that a combination of the dimensionless spring constant $K$ of the order of 1 to 10 and the dimensionless mass $\alpha$ of the same order is a subject of interest from a view point of stability theory. The combination range of $K$ and $\alpha$ will be considered in the following experiment.

Stability map in Fig. 7 shows the combined effect of a spring and a lumped mass when they are attached to the second pipe at its mid span. In contrast to Fig. 6, Fig. 7 comprises only solid curves, which were obtained by considering a small damping. This is because flutter boundaries by neglecting damping are very close to those by taking damping into account. That is there is no considerable effect of a small damping on flutter limit when a big lumped mass is located at the mid span of the second pipe, instead of the tip.

Now let us discuss in detail the relationship between the critical points A, B and C as well as the points E and F in Fig. 6.

Figure 8 shows the variation of natural frequencies of the pipes containing fluid of vanishing flow velocity with an increasing spring constant. A concentrated mass is involved in the form of dimensionless mass parameter $\alpha$. The points D and A denote the first and second eigen-frequencies of the pipe-system having a strong

![Fig. 6 Combined effect of a spring support and a lumped mass ($\beta=0.25$, $\eta=1.0$)](image1)

![Fig. 7 Combined effect of a spring support and a lumped mass ($\beta=0.25$, $\gamma=0.001$, $\eta=0.5$)](image2)

![Fig. 8 Variation of eigen-frequencies with an increasing spring constant ($\beta=0.25$, $\gamma=0.001$, $\eta=1.0$)](image3)
spring ($\kappa=10^3$) and a big lumped mass ($\omega=10^3$) at the free end. The points D and A in Fig. 8 correspond to the starting points D and A of characteristic branches in Fig. 9. The dotted lines in Fig. 9 denote the branches for the pipe-system without damping, and the solid ones do those with damping. The flutter instability occurs on the second branch. The dotted second branch leaves the point A on the imaginary axis for the stable plane at one time with a very slight increase in flow velocity. Soon after, however, the branch comes back to the axis and finally crosses it at $u_{cr}=0.04$. If a small damping is present, the starting point A on the imaginary axis shifts to a new point $A'$ in the stable plane. The fundamental shape of the branch is, however, not disturbed before and after the introduction of damping. Thus the small shift of the starting point due to the small damping results in a considerable stabilization of the present system. The first branch is connected with divergence, crossing the origin O (that is C in Fig. 9) at the divergence velocity $u_{cr}=1.361$, which corresponds to the divergence limit C in Fig. 6.

Figure 10 shows the influence of magnitude of damping upon flutter velocity. Flutter points E and F in Fig. 10 correspond to the limiting points E and F in Fig. 6. It is seen from Fig. 10 that flutter velocity becomes sensitive to a smaller damping as the dimensionless mass parameter becomes bigger, than say 10.

5.2 Experimental results

The experimental results about the combined effect of a spring support and a lumped mass are plotted in Fig. 11, together with the theoretical curves. It is seen that agreement between theory and experiment is good.

Unstable configurations of the test pipes are shown in Fig. 12. The part (a) of Fig. 12 demonstrates a typical fluttering motion of the pipes when a big lumped mass is attached at the discharge end. The part (b) shows a static buckling mode of the pipe.

![Fig.10 Effect of internal damping($\beta=0.25$, $\kappa=0$, $\eta=1.0$).](image)

![Fig.11 Comparison between theoretical and experimental results on the combined effect of a spring support and a lumped mass($\beta=0.299$, $\gamma=0.0074$, $\eta=0.94$).](image)

(a) Flutter-type instability($\kappa=0.937$).

(b) Divergence-type instability($\kappa=153$).

![Fig.12 Observation of unstable configurations($\alpha=18.5$, $\beta=0.299$, $\gamma=0.0074$).](image)
of the pipe-system when it is supported by a strong spring at its free end.

6. Conclusions

The effect of a spring as well as a lumped mass on the stability of nonconservative systems conveying fluid has been investigated theoretically and experimentally. The theory has been verified by the experiment. As for the effect of a lateral spring, the following conclusions can be drawn:

(1). The addition of a spring support to an initially otherwise unstiffened nonconservative system may have a destabilizing effect.

(2). The spring support attached closer to the tip end of a cantilevered nonconservative system may be more destabilizing. A strong spring support at the tip end among others may result in the greatest destabilization.

As to the combined effect of a spring and a lumped mass:

(3). The introduction of a lumped mass in addition to a spring support tends to make the system susceptible to flutter instability at a lowered critical velocity.

(4). When a spring support has a considerable mass, say 1 kg, only the spring having a sufficiently high spring constant can be stabilizing.

(5). Theoretical prediction by neglecting the damping may yield an erroneous flutter value, particularly when a big lumped mass is attached to a cantilevered nonconservative system at its tip end.

References


