Study of a Bounded Jet Flow Considering the Initial Turbulence*

(Experiments with a Nozzle Having Aspect Ratio of 3)

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According to the experimental results about the jet flow issuing from the nozzles contracted on two sides and on four sides into the region bounded by two parallel flat plates, behavior of the flow depends on the nozzle shape and the initial turbulence intensity considerably. The variance and the degree of deformation of the velocity profile between the flat plates and the displacement thickness of boundary layer developed on them are newly introduced in this paper. By using these measures, it has been found that the extent of the secondary flow induced by the flat plates can be estimated quantitatively. Furthermore, the velocity profile of a bounded jet in the streamwise direction is similar to that of a two-dimensional jet approximately regardless of the distance from the bounding plate, except for a flow with small initial turbulence intensity with the nozzle contracted on all sides.

Key Words : Turbulence, Bounded Jet, Nozzle Shape, Nozzle Aspect Ratio, Secondary Flow

1. Introduction

The jet issuing from a rectangular nozzle having small aspect ratio into the region bounded by two parallel flat plates is called a bounded jet and it is encountered in internal flow such as fluidic devices, and the heating or cooling unit between two parallel flat plates, and has been studied by some researchers so far. It is already known that a secondary flow occurs in the bounded jet since the flow is influenced by two parallel flat plates, and that the flow patterns of the bounded jet are different from those of a two-dimensional jet.

Foss and Jones⁽¹⁾ proposed a vortex stretching model to explain the effect of the secondary flow in the bounded jet from their experiments on the flow with a nozzle aspect ratio of 6. They also suggested that the flow appears to be a combination of two flows, that is, flows which are considered to be a two-dimensional jet and a boundary layer flow on the parallel flat plates.

McCabe⁽²⁾ suggested from his experiments on the flow in a nozzle with aspect ratio of 3 that the flow has a three-dimensional nature. A plausible explanation for generation of the secondary flow has been proposed, based on the deformation and stretching of vortex filaments caused by the mean flow.

Holdeman and Foss⁽³⁾ suggested about the structure of the secondary flow, from the results and the qualitative estimation of the vorticity transport equation, that there are near, middle and far field regions in which the secondary motion is initiated, developed and decayed, respectively.

From a different point of view, these vortex stretching models show the interaction between the boundary layer effect in the vicinity of the bounding plates and the free turbulent diffusion as a jet. The flow mechanism of the bounded jet is so complex by this interaction that neither an approximate calculation for such flow nor a method for quantitative estimation of the secondary flow effect using the experimental results has been established yet, so far as the authors are aware.

It has been found from the author's experiments that the bounded jet flow depends considerably not only upon the nozzle aspect ratio but also upon the nozzle shape and the initial turbulence intensity. Therefore, it is necessary to prescribe the nozzle shape and the initial turbulence intensity in order to see the bounded jet. Thus, the excess over the mean value of velocity on the jet center-plane and the excess over the velocity of a two-dimensional jet which is not influenced by the bounding flat plates were evaluated. In order to carry out an approximate calculation of the bounded jet flow further, it is necessary to clarify behavior of the flow affected by these excesses comparing with the result of a two-dimensional flow issuing from a finite width nozzle as reported already⁽⁴⁾.

Since behavior of the two-dimensional jet flow issuing from a finite width nozzle changes with the initial turbulence intensity⁽⁵⁾, behavior of the bounded jet having a relatively small aspect ratio is also likely to change with the initial turbu-

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lence intensity. In the earlier experiments, however, the initial turbulence intensity was not prescribed, so these results are not sufficient. One of the objectives of this study is to carry out the experiments with the initial turbulence intensity prescribed.

Behavior of the bounded jet flow changes with the nozzle shape, as reported by one of the authors⁽⁶⁾ for special initial turbulence intensity. This paper deals with the change in the bounded jet flow behavior which is affected by the initial turbulence intensity and Reynolds number. Thus the initial turbulence intensity and Reynolds number were prescribed and the velocity distributions on the center-plane were measured. By using the excess over the velocity on the jet center-plane and the excess over the velocity of the two-dimensional jet, the degree of deformation and the variance of the velocity distribution were determined. Furthermore, the displacement thickness of the boundary layer developed on the flat plate was determined. By using these measures, it has been suggested that the extent of the secondary flow can be estimated quantitatively. The velocity distribution is measured and compared with the velocity distribution function of a two-dimensional jet, and at the same time, the isotach patterns and the momentum distributions are shown and the effects of the initial turbulence intensity and the nozzle shape on them are investigated.

2. Experimental Apparatus and Procedure

The measuring section of the experimental apparatus and the details of the nozzle shape are shown in Fig.1. With a small nozzle exit, it is difficult to generate a large initial turbulence, because the flow rate is not so large. Therefore, two inner side plates which act as the bounding plates are set in the experimental apparatus having a nozzle aspect ratio of 8 originally. By using such an apparatus, it is easy to get a large initial turbulence even if the effective exit area

is small, since the flow rate from the nozzle is fairly large.

It has been found from the preliminary experiment that the bounded jet flow depends considerably upon the nozzle shape. Therefore, the *EN*-type nozzle, that is, a nozzle which is contracted on two sides, and the *RN*-type nozzle, that is, a nozzle which is contracted on four sides were used in this experiment. The other parts of the apparatus are almost the same as in the previous paper⁽⁵⁾ and omitted here.

Let the origin O be taken at the center of the nozzle exit, the axis of x in the streamwise direction, and the axes of y and z perpendicular to the axis of x , respectively. The x - y plane is defined as the mid-plane of the bounded jet, and the x - z plane as the center-plane. Let U_z be the velocity component in the x -direction on the center-plane and U_0 the velocity on the jet center axis of the bounded jet. Reynolds number is defined for convenience as follows:

$$Re = \frac{2U_0 b_0}{\nu},$$

where U_0 is the velocity at the nozzle exit, $2b_0$ the nozzle width and ν the kinematic viscosity. By using U_0 and the root-mean-square value of the streamwise velocity fluctuation u' at the nozzle exit, the initial turbulence intensity is defined as follows:

$$T_0 = \frac{\sqrt{u'^2}}{U_0}.$$

The method for regulating the initial turbulence intensity is the same as in the previous paper⁽⁵⁾, and omitted here.

3. Experimental Results and Discussions

3.1 Effects of Reynolds number

In the case of the *RN*-type nozzle, the velocity distributions on the center-plane are shown in Fig.2 using the Reynolds number as a parameter. Figures 2(a) and (b) show the distributions at $x/b_0=10$ and 20, respectively, for $T_0=0.009$. In the

Fig.1 Schema of the test section and details of the nozzle

case of a flow having a relatively small Reynolds number, there exist some differences between distributions, but these are small compared with the effects of the initial turbulence intensity as described later. Figures 2(b) and (c) show the experimental results at $x/b_0=20$ using the initial turbulence intensity as a parameter. In this case, the velocity distributions scarcely differ though Reynolds numbers differ, except for small Reynolds number. Therefore, it is found from these figures that the velocity distributions differ scarcely no matter how Reynolds numbers vary in the range of $Re \geq 3.0 \times 10^4$. As the same results were obtained for the nozzle contracted on two sides, the experiments were carried out setting Reynolds number at 3.0×10^4 .

3.2 Effects of initial turbulence intensity

Fig.3 shows the velocity distribution in the center-plane of the jet using the initial turbulence intensity as a parameter. In the case of the RN-type nozzle

Fig.2 Velocity distributions on the center-plane (effects of Reynolds number)

Fig.3 Velocity distributions on the center-plane (effects of initial turbulence intensity)

[Fig.3(a)], dependence of the flow on the initial turbulence intensity is very different in the vicinity of the side plate and in the middle part of the jet. That is, the velocity tends to increase as the initial turbulence intensity increases in the vicinity of the side plate, whereas the velocity tends to decrease for large initial turbulence intensity in the middle part of the jet. This means that the effect of the boundary layer in the vicinity of the side plates weakens relatively and the free turbulent diffusion becomes predominant in the middle part of the jet and the secondary flow becomes as small as in the two-dimensional jet, owing to the increment of the initial turbulence intensity. In the case of the *EN*-type nozzle [Fig.3(b)], as in the case of the *RN*-type nozzle, the velocity distribution flattens as the initial turbulence intensity increases. From these results, it is found that the effect of the initial turbulence intensity is large.

Since it has been found from the above results that the effects of the initial turbulence intensity are so large in the case of the bounded jets, the experiments were carried out at $T_0=0.009$, 0.035 and 0.090 for the *RN*-type nozzle, and at $T_0=0.009$ and 0.090 for the *EN*-type nozzle.

3.3 Velocity distribution and secondary flow in the center-plane of the jet

The velocity distribution on the center-plane of the jet is shown in Figs.4(a), (b) and (c) for the *RN*-type nozzle, and in Figs.5(a) and (b) for the *EN*-type nozzle, respectively, using the downstream location as a parameter. In the case of $T_0=0.009$ which is a relatively small turbulence [Figs.4(a) and 5(a)], the effect of the nozzle shape can be recognized until a relatively downstream location. In the case of $T_0=0.090$ which is a relatively large turbulence [Figs.4(c) and 5(b)], a difference can also be recognized in the velocity distributions.

In the case of the *RN*-type nozzle, there exists a part with gentle slope of the velocity in the vicinity of the side plates, say a shelf, over a wide range downstream from the nozzle exit. This is caused by existence of two distinct parts in the flow, one being the part where the flow is subjected to the effects of the boundary layer near the side plates, and the other the part where the flow is subjected to the effects of the free turbulent diffusion in the middle part of the flow. In this case, therefore, the secondary flow effect is large and this is a peculiar distribution. Such phenomenon can not happen so often at $T_0=0.035$ [Fig.4(b)] and at least at $T_0=0.090$ [Fig.4(c)].

In the case of the *EN*-type nozzle, at $T_0=0.009$, the existence of a shelf cannot be noted and effect of the secondary flow is relatively small. For $T_0=0.090$ [Fig.5(b)], because of the displacement of the boundary layer along the side plates and of the free turbulent diffusion, there is a region where the velocity in the vicinity of the side plate is larger than

Fig.4 Velocity distributions on the center-plane (in the case of *RN*-type nozzle)

Fig.5 Velocity distributions on the center-plane
(in the case of EN-type nozzle)

in the middle part of the jet. But the velocity distribution further downstream from $x/b_0=60$ shows the maximum value in the mid-plane, similarly to the two-dimensional flow between two parallel flat plates. This means that diffusion of the jet becomes large enough and the distance between the side plates is relatively short.

From the above results, it has only been found qualitatively that both development and decay of the secondary flow depend upon the initial turbulence intensity and the nozzle shape. Therefore, in order to estimate the extent of the secondary flow quantitatively, new measures defined as

$$\alpha = \frac{b_0}{H} \int_0^{H/b_0} \left(\frac{U_z}{U_2} - 1 \right)^2 d\left(\frac{z}{b_0}\right) \dots \dots \dots (1)$$

and

$$\beta = \frac{b_0}{H} \int_0^{H/b_0} \left(\frac{U_z}{U_m} - 1 \right)^2 d\left(\frac{z}{b_0}\right) \dots \dots \dots (2)$$

are introduced, where U_2 is the velocity on the center line of a two-dimensional jet and U_m the mean value of U_z on the bounded jet center-plane. The value of α is a measure of the difference from the velocity of a two-dimensional jet which is not influenced by the bounding flat plates. It represents the degree of deformation of the velocity distribution on the bounded jet center-plane from the two-dimensional one, and it is different from the variance of

the velocity distribution. On the other hand, β is a measure of the non-uniformity of the velocity distributions on the center-plane and it is a variance itself. The value of U_2 in Eq.(1) is the center line velocity of a two-dimensional jet issuing from a finite width nozzle⁽⁴⁾⁽⁵⁾ calculated by considering the relations between the initial turbulence intensity and the empirical constant contained in the eddy kinematic viscosity.

The value of U_2 and the velocities on the bounded jet center line obtained by experiments for each initial turbulence intensity are shown in Fig.6. In the zone of established flow, U_2 given by the approximate calculation is indicated by the solid line. In the transitional zone, U_2 is given by interpolation of the experimental results of the two-dimensional jet flow⁽⁵⁾ for each initial turbulence intensity and is shown by the broken line. It can be seen from this figure that the velocities in the case of the EN-type nozzle are not so different from those of the two-dimensional jet flow. This corresponds to the fact that peculiar velocity distributions on the bounded jet center-plane are hardly recognized in the case of the EN-type nozzle.

Using these U_2 's shown in Fig.6 and U_z 's obtained from Figs.4 and 5, the degree of deformation of the velocity distribution α calculated by Eq.(1) is shown in Fig.7.

Fig.6 Center line and center axis velocities of the jets

In the case of the *RN*-type nozzle, for $T_0=0.009$, α begins to increase abruptly at $x/b_0=4$ and shows the maximum value 0.081 at $x/b_0=16$, then it decreases monotonously as x/b_0 increases. For $T_0=0.035$, α shows the maximum value 0.029 at $x/b_0=12$, which is only 36% of the maximum value for $T_0=0.009$, so the extent of the secondary flow for $T_0=0.035$ is smaller than that for $T_0=0.009$. In the case of $T_0=0.090$, α shows the maximum value 0.011 at $x/b_0=10$, which is only 14% of that in the case of $T_0=0.009$. In the last case, α does not decrease monotonously beyond $x/b_0=10$. The neighborhood of the point where α again shows a local maximum value of the velocity distribution shown in Fig.4(c) is not in the middle part of the flow but in the vicinity of the flat plates.

In the case of the *EN*-type nozzle, for $T_0=0.009$, α shows the local maximum value 0.010 at $x/b_0=18$. This value is only 12% of that for the same initial turbulence in the case of the *RN*-type nozzle. It is evident from Figs.4(a) and 5(a) that the flow having the *EN*-type nozzle shows a relatively similar behavior to the two-dimensional jet unlike the flow having the *RN*-type nozzle. Downstream the value of α begins to increase at $x/b_0=50$ again. For $T_0=0.090$, α does not show the maximum value and increases monotonously in the downstream direction.

The variances of the velocity distribution calculated from Eq.(2) are shown in Fig.8. Variation of β and x/b_0 is similar to that of α , that is, the position where β shows the maximum value and the quantitative dependence of values of β upon the initial turbulence intensity and the nozzle shape are almost the same as those of α .

The extent of the secondary flow induced in the bounded jet depends considerably upon the nozzle shape and the initial turbulence intensity and can be estimated quantitatively, introducing the degree of deformation and the variance of the velocity distribution as shown in Figs.7 and 8.

From the above discussions, the extent of the secondary flow is well expressed by introducing the degree of deformation and the variance of the velocity distribution. But it is likely that each measure shows the same value, even though the velocity distributions differ from each other as shown in Figs.4 and 5. In order to consider the shape of the velocity distribution, the displacement thickness

$$\frac{\delta^*}{b_0} = \frac{b_0}{H} \int_0^{H/b_0} \left(1 - \frac{U_z}{U}\right) d\left(\frac{z}{b_0}\right) \dots\dots(3)$$

is introduced, which expresses the interaction between the boundary layer effect in the vicinity of the bounding plates and the free turbulent diffusion as a jet. U_c contained in Eq.(3) is the velocity on the center axis of the bounded jet. The values of δ^*/b_0 are shown in Fig.9. For the *RN*-type nozzle and for the *EN*-type nozzle in the case of $T_0=0.009$, the tendency of variation of δ^*/b_0 with x/b_0 corresponds to that of the degree of deformation and the variance shown in Figs.7 and 8. For the

Fig.7 Degree of deformation of the velocity distribution

Fig.8 Variance of the velocity distribution

Fig.9 Displacement thickness

EN-type nozzle, in the case of $T_0=0.090$, the sign of δ^*/b_0 alters locally. The value of δ^*/b_0 increases gradually at the beginning of the flow and then turns negative in the region about $10 \leq x/b_0 \leq 45$, beyond which δ^*/b_0 is positive and increases as x/b_0 increases.

This change in the value of δ^*/b_0 corresponds to that of the velocity distribution on the bounded jet center-plane

Fig.10 Velocity distributions

Fig.11 Velocity distributions

shown in Fig.5(b). That is, in the region $0 \leq x/b_0 \leq 8$, where δ^*/b_0 has positive values, the velocity distributions are like those in the boundary layer on a flat plate. In the region $10 \leq x/b_0 \leq 45$, where δ^*/b_0 has negative values, the velocity distributions have maximum values not in the middle part of the flow but in the vicinity of the bounding plates. Further, in the region $x/b_0 > 45$ where δ^*/b_0 again turns positive, the maximum velocity occurs on the jet center line.

Thus by introducing the degree of deformation, the variance and the displacement thickness of the velocity distribution on the bounded jet center-plane, the extent of the secondary flow in the bounded jet can be estimated quantitatively.

3.4 Comparison with the velocity distribution of the two-dimensional jet

The effects of the secondary flow have been discussed in the last section, which is limited to the jet center-plane. The velocity distribution in the plane parallel to the jet mid-plane was measured and

compared with the velocity distribution of the two-dimensional jet flow⁽⁴⁾ expressed by the equation

$$f(\eta) = 1 - 6\eta^2 + 8\eta^3 - 3\eta^4, \dots \dots \dots (4)$$

where $\eta = y/b$ using the half width of the jet b . Since it is inadequate to determine the half width directly from the experiment, a half mean width of the jet b_m is used. For this velocity distribution, b_m equals to $2b/5$. The velocity distributions for some initial turbulence intensities are shown in Figs.10 and 11, using z/b_0 as a parameter.

In the case of the RN-type nozzle [Figs.10(a) and (b)], for $T_0 = 0.009$, as pointed out by Holdeman-Foss⁽³⁾, the profile exhibits a cross-over shape with respect to Eq.(4) for small z/b_0 , and as z/b_0 increases, a velocity excess (i.e. tail-excess) occurs. On the other hand, at $z/b_0 = 2.5$ which is nearest to the side plate, neither cross-over nor tail-excess appears, and the profiles are almost the same as the two-dimensional jet. Such cross-over and tail-excess profiles cannot occur in a flow

Fig. 12 Isotach patterns
(in the case of RN-type nozzle)

having T_0 larger than 0.035. In the case of the EN-type nozzle [Figs. 11(a) and (b)], the velocity distribution of the bounded jet may be considered almost the same as that of a two-dimensional jet in spite of the initial turbulence intensity and the depth z .

As mentioned above, the velocity distribution of the bounded jet is almost the same as that of the two-dimensional jet, except in a flow with relatively small initial turbulence intensity and the RN-type nozzle, and this will be useful information to carry out the approximate calculation of the bounded jet flow.

3.5 Isotach pattern and momentum distribution

Figures 12 and 13 show the isotach patterns obtained from the velocity distribution in the planes $z = \text{const}$ for the RN-type and the EN-type nozzles, respectively.

In the case of the RN-type nozzle, the jet width is expanded in the middle part of the jet for $T_0 = 0.009$, whereas the diffusion for the y -direction in the vicinity of the side plates is much smaller than that in the middle part. It is considered that the flow near the side plates transfers to the middle part of the jet. As the initial turbulence intensity becomes large, the width of the jet becomes larger in the vicinity of the side plates than in the middle part. It is likely that this comes from the difference in the influence of the displacement effect of boundary layer on the flow.

In the case of the EN-type nozzle, the turbulent diffusion in the vicinity of the side plates is larger than that in the middle part of the jet regardless of the initial turbulence intensity.

From the above results, it is seen that the flow in the case of the RN-type

Fig. 13 Isotach patterns
(in the case of EN-type nozzle)

Fig. 14 Momentum distributions

nozzle is affected by the initial turbulence intensity; that a flow having small initial turbulence intensity is affected by the nozzle shape; and that these effects correspond to the differences in α and β .

The effect of the secondary flow can not be recognized quantitatively from the isotach patterns shown in Figs. 12 and 13, so the momentum distributions are measured and shown in Fig. 14. In the case of the RN-type nozzle, the momentum of the fluid in the vicinity of the side plates is less than that in the middle part of the flow for $T_0 = 0.009$. For $T_0 = 0.035$, the momentum distribution flattens because of the free turbulent diffusion in the vicinity of the side plate. For $T_0 = 0.090$, the maximum value

of the momentum occurs at $z/b_0=2.5$, not in the middle part of the flow.

In the case of the *EN*-type nozzle, for $T_0=0.009$, flatness of the momentum distribution can be seen in a wide range of the depths (z -direction). This corresponds to the fact that the flow remains two-dimensional. For $T_0=0.090$, the maximum value of the momentum occurs at $z/b_0=2.5$ as in the case of the *RN*-type nozzle, and this value is larger than that for the *RN*-type nozzle.

From the above results, it is understood that the effect of heating or cooling of fluid flow, for example, between two parallel flat plates is improved by using the *EN*-type nozzle as a nozzle shape and a flow having large initial turbulence intensity.

4. Conclusions

According to the experiments on the jet issuing from the nozzles contracted on two sides and on four sides into the region bounded by two parallel flat plates, the effects of the nozzle shape are larger as the initial turbulence intensity is smaller.

Behavior of the flow is almost independent of Reynolds number, defined with a uniform velocity at the nozzle exit and the nozzle width, in the range of $7.5 \times 10^3 \leq Re \leq 7.0 \times 10^4$. But behavior of the flow depends considerably upon the initial turbulence intensity.

From the velocity distribution on the jet center-plane, in the case of a flow having the nozzle contracted on four sides and small initial turbulence intensity, the characteristic flows, namely acceleration in the middle part of the jet and deceleration in the vicinity of the side plates, are recognized. Such phenomena tend to weaken as the initial turbulence intensity increases. In the case of a flow having the nozzle contracted on two sides, as the initial turbulence intensity increases, a peculiar distribution appears, namely the velocity in the vicinity of the side plate is larger than that in the middle part.

In an effort to estimate the extent of the secondary flow quantitatively, which has been estimated only qualitatively so far, the velocity deformation and the variance of the velocity distribution and the displacement thickness of the boundary layer on the jet center-plane are introduced. When using these measures, it has been suggested, behavior of the secondary flow of the bounded jet depends consider-

ably upon the nozzle shape and the initial turbulence intensity. The position where the extent of the secondary flow is maximum moves in the upstream direction and this tendency weakens as the initial turbulence intensity becomes large for a flow from the nozzle contracted on four sides. For a flow from the nozzle contracted on two sides the extent of the secondary flow is relatively small compared with the flow from the nozzle contracted on four sides.

The velocity distributions measured on some planes parallel to the center-plane of the jet were compared with the two-dimensional one. The result shows that for the nozzle contracted on four sides, there are some differences between them for small initial turbulence intensity, on the contrary, it is found that they are almost similar for large initial turbulence intensity and/or for the nozzle contracted on two sides.

It is found also from the isotach pattern that behavior of the bounded jet depends considerably upon the nozzle shape and the initial turbulence intensity. As an attempt to estimate the extent of the secondary flow quantitatively from the isotach pattern, the momentum distributions were measured. By using the result, the effects of the nozzle shape and the initial turbulence intensity on the momentum distribution could be well expressed.

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