Study on the Mechanism of Noise Generation from
Railway Wheel and its Countermeasures
(6th Report, Noise reduction by a Damped Wheel*)

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Vibration and noise of a damped wheel are investigated theoretically and experimentally. A dynamic damper composed of a rubber and a steel thin annular plate is attached to the web of a conventional wheel. In the theoretical analysis, the rubber is assumed to be a spring with viscous damping. The results show that the vibration and noise can be considerably reduced by the dynamic damper. When the fundamental frequency of the dynamic damper becomes equal to the natural frequency of the wheel, the vibration is remarkably decreased. In order to have an effective reduction of the vibration and noise, the rubber should have a large loss factor and the damper should be attached to the outer side of the wheel.

Key Words : Train, Wheel, Vibration, Noise, Dynamic Damper

1. Introduction

Main source of train noise is vibration of running devices which is caused by an interaction between wheels and rails¹⁾. When a train is running in passage of curves, flexural vibration of a wheel, especially in normal modes with nodal diameters, is a dominant noise source²⁾.

In order to reduce the noise, we have to make the vibration of normal modes with nodal diameters small. This vibration reduction can be achieved by a dynamic vibration damper which is made of a rubber and a steel thin annular plate and is attached to the web of the conventional wheel. This damped wheel has a sufficient structural strength for actual use. Its effect on noise reduction was proved by some experiments making use of actual railways^{3,4)}. A theoretical study on the damped wheel was carried out by Nishimura et al.⁵⁾ on the assumption that the wheel is a cantilever. However, there is a big difference between a wheel and a cantilever, and we can not find any other theoretical study on damped wheels.

In this paper, we carried out a theoretical analysis of the flexural vibration of a damped wheel which is composed of a main wheel, a thin annular rubber plate (hereafter named rubber) and a thin annular steel plate (hereafter named plate). By making use of a small model wheel which is about 1/6 of a real wheel used in the

Shinkansen, the vibration and noise are investigated experimentally. Then, the theoretical analysis is verified, and the relation between the noise reduction and the damped wheel's structure is also investigated.

2. Theoretical Analysis

Figure 1 shows a damped wheel which has a rubber and a steel thin annular plate on its web. The theoretical analysis of vibration of the main wheel and the plate are carried out individually by making use of Mindlin theory⁶⁾ as in the previous paper⁷⁾. A loss factor is introduced to discuss quantitatively the vibration of various damped wheels.

In the beginning, the vibration of the main wheel is analyzed. Let the complex Young's modulus E^* and the shear modulus G^* be given as,

$$E^* = E(1 + i\zeta), \quad G^* = G(1 + i\zeta) \dots \dots \dots (1)$$

where E : Young's modulus, G : shear modulus, and ζ : loss factor. The moments and shear forces in the Mindlin plate are given by

$$\left. \begin{aligned} M_r^* &= M_r(1 + i\zeta), & M_s^* &= M_s(1 + i\zeta) \\ M_{ro}^* &= M_{ro}(1 + i\zeta), & Q_r^* &= Q_r(1 + i\zeta) \\ Q_s^* &= Q_s(1 + i\zeta) \end{aligned} \right\} \dots (2)$$

Equations of motions are

Fig. 1 Damped wheel.

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$$\left. \begin{aligned} \frac{\partial M_r^*}{\partial r} + \frac{1}{r} \frac{\partial M_\theta^*}{\partial \theta} + \frac{M_r^* - M_\theta^*}{r} - Q_r^* &= \frac{\rho h^3}{12} \frac{\partial^2 \psi_r}{\partial t^2} \\ \frac{\partial M_\theta^*}{\partial r} + \frac{1}{r} \frac{\partial M_r^*}{\partial \theta} + \frac{2M_\theta^*}{r} - Q_\theta^* &= \frac{\rho h^3}{12} \frac{\partial^2 \psi_\theta}{\partial t^2} \\ \frac{\partial Q_r^*}{\partial r} + \frac{1}{r} \frac{\partial Q_\theta^*}{\partial \theta} + \frac{Q_r^*}{r} + P e^{i\omega t} &= \rho h \frac{\partial^2 w}{\partial t^2} \end{aligned} \right\} \dots\dots\dots (3)$$

where ψ_r : slope in r direction, ψ_θ : slope in θ direction, w : displacement in z direction, and P : external force. The deflection w and others are assumed to be given in the series of their eigenfunctions w_{ms} and others as,

$$w = \sum_{ms} C_{ms} w_{ms} e^{i\omega t}, \quad \psi_r = \sum_{ms} C_{ms} \psi_{r,ms} e^{i\omega t}, \text{ etc.} \dots\dots\dots (4)$$

where m : number of nodal diameters, s : number of nodal circles, C : unknown coefficients, Substituting these into Eq. (3), and by making use of orthogonality^{ms} of the eigenfunctions⁸, the following equation is obtained:

$$(\omega_{ms}^{*2} - \omega^2) C_{ms} = \iint P w_{ms} r dr d\theta, \dots\dots\dots (5)$$

where, $\omega_{ms}^{*2} = \omega_{ms}^2 (1 + i\zeta)$.

We can also have the following equation for the plate by the same procedure.

$$(\omega_{nt}^{*2} - \omega^2) C'_{nt} = \iint P' w'_{nt} r dr d\theta, \dots\dots\dots (6)$$

where, suffix ' refers to the plate.

The external forces P and P' are the reacting forces by the rubber, so they are given as,

$$P = -P' = k^* (w' - w) = k^* \left(\sum_{nt} C'_{nt} w'_{nt} - \sum_{ms} C_{ms} w_{ms} \right), \dots\dots\dots (7)$$

where $k^* = k(1 + i\eta)$, k : spring constant of the rubber, and η : loss factor of the rubber. Since the integration area of Eqs. (5) and (6) is the surface of the plate, substitution of Eq. (7) into these equations gives

$$(\omega_{ms}^{*2} - \omega^2) C_{ms} = k^* \int_0^{2\pi} \int_{r_1}^{r_2} \left(\sum_{pg} C'_{pg} w'_{pg} - \sum_{ij} C_{ij} w_{ij} \right) w_{ms} r dr d\theta, \dots\dots\dots (8)$$

$$(\omega_{nt}^{*2} - \omega^2) C'_{nt} = k^* \int_0^{2\pi} \int_{r_1}^{r_2} \left(\sum_{ij} C_{ij} w_{ij} - \sum_{pg} C'_{pg} w'_{pg} \right) w'_{nt} r dr d\theta, \dots\dots\dots (9)$$

The eigenfunctions can be shown in a form in which variables r and θ are separated.

$$w_{ms}(r, \theta) = W_{ms}(r) \cos m\theta \dots\dots\dots (10)$$

By making use of orthogonality of trigonometrical function, Eqs. (8), (9) can be rewritten as

$$(\omega_{ms}^{*2} - \omega^2) C_{ms} = k^* \Pi_m \left(\sum_g C'_{mg} \int_{r_1}^{r_2} W'_{mg} W_{ms} r dr - \sum_j C_{mj} \int_{r_1}^{r_2} W_{mj} W_{ms} r dr \right), \dots\dots\dots (11)$$

$$(\omega_{nt}^{*2} - \omega^2) C'_{nt} = k^* \Pi_n \left(\sum_j C_{nj} \int_{r_1}^{r_2} W_{nj} W'_{nt} r dr - \sum_g C'_{ng} \int_{r_1}^{r_2} W'_{ng} W'_{nt} r dr \right), \dots\dots\dots (12)$$

$$\text{where } \Pi_m = \int_0^{2\pi} \cos^2 m\theta d\theta = \begin{cases} 2\pi & (m=0) \\ \pi & (m \geq 1) \end{cases} \dots\dots\dots (13)$$

$$\text{Let } I_m(i, j) = \Pi_m \int_{r_1}^{r_2} W_{mi} W_{mj} r dr, \dots\dots\dots (14)$$

$$J_m(i, j) = \Pi_m \int_{r_1}^{r_2} W'_{mi} W'_{mj} r dr, \dots\dots\dots (15)$$

$$L_m(i, j) = \Pi_m \int_{r_1}^{r_2} W_{mi} W'_{mj} r dr, \dots\dots\dots (16)$$

Then, Eqs. (11) and (12) are rewritten in a matrix form as,

$$AC = \omega^2 C \dots\dots\dots (17)$$

where

$$A = \begin{bmatrix} k^* I_m(1, 1) + \omega_{m1}^{*2} & k^* I_m(1, 2) & \dots & -k^* L_m(1, 1) & -k^* L_m(1, 2) & \dots \\ k^* I_m(2, 1) & k^* I_m(2, 2) + \omega_{m2}^{*2} & \dots & -k^* L_m(2, 1) & -k^* L_m(2, 2) & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ -k^* L_m(1, 1) & -k^* L_m(1, 2) & \dots & k^* J_m(1, 1) + \omega_{m1}^{*2} & k^* J_m(1, 2) & \dots \\ -k^* L_m(2, 1) & -k^* L_m(2, 2) & \dots & k^* J_m(2, 1) & k^* J_m(2, 2) + \omega_{m2}^{*2} & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$C = (C_{m1}, C_{m2}, \dots, C'_{m1}, C'_{m2}, \dots)^T$$

The complex natural frequency $\omega = \Omega + i\beta$ is obtained by solving the above set of equations.

Now, we consider the displacement under the external force $P_0 e^{i\omega t}$ which acts on the rim (r_0, θ_0) . In the following equations, the time term $e^{i\omega t}$ will be omitted. The external force is given by Dirac's delta function as,

$$P = k^*(w' - w) + P_0 \delta(r - r_0) \delta(\theta - \theta_0) \quad (18)$$

In this case, Eq. (11) becomes

$$(\omega_{ms}^2 - \Omega^2) C_{ms} = k^* \Pi_m \left(\sum_j C_{mj} \int_{r_1}^{r_2} W'_m W_{ms} r dr - \sum_j C_{mj} \int_{r_1}^{r_2} W_m W_{ms} r dr \right) + P_0 W_{ms}(r_0, \theta_0) \quad (19)$$

The equation for the plate under the external force is the same as Eq. (12), in which ω is replaced by Ω . These two equations can be rewritten in the following matrix form.

$$A' C = B \quad (20)$$

where

$$A' = A - \Omega^2 E \quad (E: \text{unit matrix})$$

$$B = P_0 \{w_{m1}(r_0, \theta_0), w_{m2}(r_0, \theta_0), \dots, 0, 0, \dots\}^T$$

From this simultaneous equation, the coefficients C and C' are determined. Substituting these coefficients into Eq. (4), the deflections w_{ms} and w_{pt} under the external force are obtained.

3. Results of Numerical Calculations

The numerical calculation for the damped wheel shown in Fig. 1 was carried out. The following figures are used in the calculation;

- R_1 (inner radius of web) = 25.0 mm
- R_2 (outer radius of the web) = 62.5 mm
- R_3 (outer radius of the rim) = 75.0 mm
- H_1 (thickness of the web) = 5.0 mm
- H_2 (thickness of the rim) = 20.6 mm
- $E = 2.06 \times 10^{11}$ N/m²
- $G = 8.13 \times 10^{10}$ N/m²
- $\nu = 0.3$
- $\rho = 7.86 \times 10^3$ kg/m³
- $\zeta = 3.0 \times 10^{-3}$
- $P_0 = 9.8$ N

in which ζ is obtained from experiments. Table 1 shows the dominant natural frequencies f_{ms} of the main wheel. The natural frequencies of the damped wheel are very close to the natural frequencies of the main wheel, and depend slightly on the structural shape of the damper, i.e. thickness of the rubber h_r , thickness of the plate h_p , inner and outer radii of the absorber r_1 and r_2 . Only when the fundamental frequency f_{pr} of the damper

$$f_{pr} = \frac{1}{2\pi} \sqrt{\frac{E_r}{h_r h_p \rho_p}} \quad (21)$$

E_r : rubber's Young's modulus,
 ρ_p : plate's density

is in the vicinity of f_{ms} , the natural frequency of the damped wheel splits into two.

Figure 2 shows the relation between the acceleration levels of each mode and f_{pr} , in which the external force acts on the rim and the acceleration is measured at the same point, and $\eta = 0.5$, $r_1 = 31.0$ mm, and $r_2 = 61.0$ mm. In this figure, 0 dB represents the acceleration of the wheel without a damper, and the symbols A, B, C, D, E, and F indicate the modes shown in Table 1.

Here, we compare the vibration amplitudes and the phases of the main wheel and the plate. When $f_{pr} = f_{ms}$, the plate makes a larger vibration than the main wheel. In the other region, the main wheel makes a larger vibration. Concerning the phase, at $f_{pr} < f_{ms}$, both are in the same phase, at $f_{pr} = f_{ms}$, the phase lag begins to appear and it increases with f_{pr} . This tendency depends on the loss factor η . When η increases, the phase lag curve has a gentle slope.

Table 1 Natural frequencies of the damped wheel.

	frequency (Hz)	mode (m,s)
A	1349	(0,0)
B	954	(1,0)
C	2305	(2,0)
D	6208	(3,0)
E	11300	(4,0)
F	17114	(5,0)

Fig. 2 Results of calculations on the relations between the acceleration levels and f_{pr} .

When η increases, the vibration becomes small. One example of the relation between the acceleration level and loss factor is shown in Fig. 3, in which $E_r = 4.9 \times 10^8 \text{ N/m}^2$, $r_1 = 31.0 \text{ mm}$, $r_2 = 61.0 \text{ mm}$, $h_r = 4.0 \text{ mm}$, and $h_p = 5.0 \text{ mm}$. In this case, $f_{pr} = 8900 \text{ Hz}$. The r mode whose natural frequency is close to f_{pr} remarkably decreases in the acceleration level.

4. Experimental Results and Discussions

Table 2 shows the shapes of the plate and the rubber which are used in the experiment. Two kinds of experiments are carried out: non-running and running experiments. In the non-running experiment, the rim is sinusoidally excited by an electromagnetic exciter. Since the exciter contacts with the wheel, the frequency spectrum slightly differs from that of the wheel alone. For example, a peak at 3000 Hz appears only in the coupled vibration of the wheel and the exciter. Since the D, E, and F modes are dominant in the radiated noise, mainly these modes are discussed here.

4.1 Non-running experiment

4.1.1 Effect of the damper's structure

The acceleration levels of the main wheel and one with a rubber are shown in Fig. 4. In this experiment, natural and nitrile rubbers are used, and the wheel of this type is named a rubber coated wheel. In this figure and the following figures, the ordinate is set at $1g = 0 \text{ dB}$, and W_{st} represents a wheel without a damper. Figure 5 shows the acceleration levels of the wheel with the damper (PA, RA). As shown in Fig. 4, the vibration of the rubber coated wheel is slightly smaller than that of the standard web wheel. However, the damper reduces the vibration of the high order modes D, E, and F by more than 10 dB, as shown in Fig. 5.

In Fig. 5, we can see that the nitrile rubber has larger damping effect than the natural rubber, except in the case of the D mode. This is explained by the loss factor of the nitrile rubber being greater than that of the natural rubber. This corresponds to the theoretical prediction that the vibration decreases with a loss factor shown in Fig. 3. In the D mode, f_{pr} of the natural rubber is in the vicinity of f_{30} , and its vibration remarkably decreases.

Figure 6 shows the difference in the acceleration level between the dampers (PA, RA) and (PA, RB) to investigate the effect of the natural rubber's thickness. The wheel with rubber RA (5 mm thickness) makes smaller vibration than the wheel with RB (2 mm thickness) in the low order modes, and it is opposite with the high order modes. It is because f_{pr} of the wheel with RA is in the low f_{pr} frequency region, and f_{pr} of the wheel with PB is in the higher frequency region.

The same experiment with the nitrile rubber is carried out. The results show that the vibration decreases with the rubber's thickness, but f_{pr} does not have so clear effect as the natural rubber.

Table 2 Shapes of the rubber and the plate.

Rubber (r_1, r_2, h)	Plate (r_1, r_2, h)
RA (31, 61, 5)	PA (31, 61, 4)
RB (31, 61, 2)	PB (31, 61, 2)
RC (46, 61, 5)	PC (46, 61, 4)
RD (31, 46, 5)	PD (31, 46, 4)

Fig. 3 Results of calculations on the relations between the acceleration levels and η .

Fig. 4 Acceleration levels of the rubber coated wheels.

Fig. 5 Acceleration levels of the damped wheels with the natural and the nitrile rubbers.

From the above experiments, we find that the nitrile rubber is more effective for the vibration reduction than the natural rubber. Therefore, in the following experiment, only the nitrile rubber is used.

We investigate the effects of the inner and the outer radii of the damper. Three kinds of the damped wheel are built: W_1 (PA, RA) whose damper is attached to the whole area of the web, W_2 (PC, RC) whose damper is attached to the outer half area of the web, and W_3 (PD, RD) whose damper is attached to the inner half area of the web. Figure 7 shows the experimental results about these wheels. As shown in this figure, W_1 and W_2 make smaller vibration than W_3 , especially in the high order modes.

Table 3 shows the theoretically calculated vibration levels of these three wheels. This corresponds to the experimental results. From this we can conclude that the damper should be located as far outside of the web as possible.

4.1.2 Vibration amplitudes and phases of main wheel and plate

The vibration amplitudes and the phases of the main wheel and the plate of three damped wheels W_1 (PA, RA), W_4 (PB, RA), and W_5 (PB, RB) are investigated. In general, the main wheels make a larger vibration than the plates. However, the plate makes a larger vibration at 7000 - 8500 Hz for W_1 , 8000 - 10000 Hz for W_4 , and 8000 - 14000 Hz for W_5 . The main wheel and the plate have the same phase angle in the low frequency range. For W_1 and W_4 , the phase lag begins to appear at about 10000 Hz, and it increases with the frequency. The phase lag of W_4 is slightly larger than that of W_1 . However, W_5 does not have a phase lag in the measuring frequency range (0-20 kHz).

Regarding the theoretical prediction, the plate makes a larger vibration than the main wheel only at around f_{pr} . Also, the phase lag begins to appear at about f_{pr} and it increases with the frequency. When E_r is supposed to be 4.9×10^8 N/m², f_{pr} 's of W_1 , W_4 , and W_5 are 8900, 12600, and 20000 Hz, respectively. This theoretical prediction considerably agrees with the experimental results.

4.2 Rolling experiment

The amplitude of the rolling vibration strongly depends on the corrugation and the roughness of the contact surface of the wheels. Since the corrugation and the roughness increase with operation, it is difficult to keep the surface conditions constant. In this experiment, we use a grooved wheel which has grooves of 0.2mm depth, 0.2mm width, and 1mm interval on the periphery. The grooves are so large that the corrugation and roughness can be neglected, and we can compare the vibration amplitudes of various wheels. The exciting force has a dominant component whose frequency is $f_z = NZ$ (N: rotational speed, Z: number of grooves). Then we can compare the vibration amplitude of arbitrary mode by setting $f_z = f_{ms}$.

Figures 8 and 9 show the vibration acceleration and the noise, respectively. Comparing W_{st} and W_1 in Fig. 8, the damper reduces the vibration by about 20 dB at the D, E, and F modes, and the total level is decreased by 16.5 dB. The theoretical prediction of the vibration reduction shown in Fig. 2 is 8 dB at the D mode, 13 dB at the E mode, and 8 dB at the F mode. These values are smaller than the experimental results, but this difference can be revised by choosing appropriate loss factor and other constants. The decreasing tendencies of both results correspond to each other.

As shown in Fig. 9, the noise is considerably decreased in the natural modes as well as the vibration, and the total level is decreased by 16.7 dB. Then, we can conclude that the damped wheel has enough effect of noise reduction.

Concerning the damped wheels W_4 (PB, RA) and W_5 (PB, RA), the same experiments as W_1 are carried out. The results are similar to those of W_1 .

Table 3 Result of calculations on the acceleration reduction level of the damped wheel W_1 , W_2 , and W_3 .

Mode	W_1	W_2	W_3
A	3.63	2.39	2.95
B	4.90	2.58	5.03
C	3.27	2.33	1.27
D	7.66	7.87	0.61
E	12.79	12.49	3.28
F	8.05	7.43	4.39

Fig. 6 Acceleration levels of the damped wheels with two values of rubber thickness.

Fig. 7 Acceleration levels of the damped wheels with three values of damper's diameter.

Fig. 8 Acceleration levels of the rolling damped wheel.

Fig. 9 Sound pressure levels of the rolling damped wheel.

5. Conclusions

In this paper, the vibration of a damped wheel is theoretically analyzed under the assumption that the rubber is a spring with viscous damping which works only in the axial direction. To be precise, the force in the circumferential and the radial directions should be taken into consideration. We also have to consider the frequency dependence of the rubber's Young's modulus and loss factor. However, in order to have an outline of the vibration and the noise abatements, the theoretical analysis mentioned in this paper is useful. In this study, the damper is attached on one side of the web. However, it is possible to attach the dampers on both sides. In that case, we can expect further abatement.

The following conclusions with the effective damped wheel are obtained.

- 1) The rubber coated wheel does not reduce the vibration, but the damped wheel composed of the main wheel, a thin annular plate and a rubber plate can reduce the vibration by about 20 dB.
- 2) The rubber should have a large loss factor.
- 3) The damper should be attached to the outer side of the wheel.
- 4) The vibration is remarkably decreased at the fundamental frequency of the damper.

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