Kinematic Analysis of Stephenson Six-Link Mechanisms*
(2nd Report, Index of Motion
-Transmission Characteristics)

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This paper deals with the motion-transmission characteristics of
Stephenson six-link mechanisms of three kinds which have two closed
five-link loops including the fixed and driving links. The positions
of mechanisms are analytically determined at which the deviations of
displacements of links due to small changes of kinematic constants and
the forces acting at pairs due to external forces on links become
extremely large.

Then, a standardized expression of the functionally variable part
of the denominator of the deviations of displacements of links which
coincides with one of the forces acting at pairs is adopted as an
index of the motion-transmission characteristics. Moreover, the geo-
metric property of the index is made clear and the relation between
the index and the mechanical error in the output is experimentally
examined.

Key Words: Mechanism, Kinematics, Six-Bar Linkage, Transmission
Index, Mechanical Error.

1. Introduction

In the cases of such mechanisms that
the domain of the angular displacement of
the driving link includes positions at
which the deviations of displacements of
links due to small changes of kinematic
constants and the forces acting at pairs
due to external forces on links become
extremely large, it is difficult to main-
tain a theoretical accuracy of motion in a
stage of practical use.

In the case of a four-link mechanism,
the above-mentioned positions coincide
with the limit positions of rotation of
the driving link, and the main factor on
which the deviations of displacements of
links and the forces acting at pairs de-
pend in the neighbourhoods of them is the
absolute value of the sine of the trans-
mision angle [1]. In the cases of plane
six-link mechanisms whose fixed and driv-
ing links are included in a closed four
-link loop, the same result is deduced
immediately. Therefore, the motion-trans-
mision characteristics is reflected by
means of Alt’s transmission angles.

However, in the cases of Stephenson
six-link mechanisms of three kinds which
have two closed five-link loops including
the fixed and driving link (hereinafter
referred to as the complex six-link mecha-
nism), Alt’s transmission angles [2], [3]
are insufficient to reflect the motion
-transmission characteristics. In order to
search for an index in place of Alt’s
transmission angles, H. Mathaesi studied
the steam of the power between two adja-
cent links of six-link mechanisms whose
driving link can make a complete rotation,
but he gave the final estimation of motion
-transmission characteristics on the basis of
the forces acting at pairs [4]. K. Oga-
wara extended the transmission index [5]
for spatial mechanisms developed by G.
Sutherland and B. Roth to a plane multi-
link mechanism whose fixed link is a bina-
ry link and which has more than two trans-
mision loops, but he did not discuss the
relation between the index and the charac-
teristics of motion-transmission [6].

Thus, in this paper, the positions of
mechanisms are analytically determined at
which the deviations of displacements of
links due to small changes of kinematic
constants and the forces acting at pairs
due to external forces on links become
extremely large. Then, a new index of
motion-transmission characteristics of
complex six-link mechanisms is presented
and its properties are examined theo-
ettically and experimentally.

2. Deviations of displacements of
links due to small changes of
kinematic constants

Let the names and the symbols of
pairing points, kinematic constants and
angular displacements of complex six-link
mechanisms be such as shown in Fig. 1. The
discussions are developed in detail as for
the S22F mechanism. As for S23F and S32F
mechanisms, only the results are stated.
2.1 First approximation

Equating the components of the x- and y-axes in both sides of two closed loop equations in a vector form passing through five pairing points A, B, E₁, C₁, D and A, B, E₂, C₂, D, respectively, of the S22F mechanism arranged in the rectangular coordinate system O-xy as shown in Fig. 1, we have the following basic equations of displacement analysis.

\[ x = x_0 + Z_1 \cos \gamma_1 + Z_2 \cos \gamma_2 + Z_1 \cos \gamma_3 = Z_1 \cos \gamma_1 \]  
\[ y = y_0 + Z_1 \sin \gamma_1 + Z_2 \sin \gamma_2 + Z_1 \sin \gamma_3 = Z_1 \sin \gamma_1 \]  
\[ x = x_0 + Z_1 \cos \gamma_1 + Z_2 \cos (\gamma_2 + \alpha_1) + Z_1 \cos \gamma_3 = Z_1 \cos (\gamma_1 + \alpha) \]  
\[ y = y_0 + Z_1 \sin \gamma_1 + Z_2 \sin (\gamma_2 + \alpha_1) + Z_1 \sin \gamma_3 = Z_1 \sin (\gamma_1 + \alpha) \]

Letting \( \Delta \gamma_1, \Delta \gamma_2, \Delta \gamma_3, \Delta \gamma_4 \) denote the deviations of angular displacements of links \( \gamma_1, \gamma_2, \gamma_3, \gamma_4 \) and \( \gamma_5 \) respectively when the values of kinematic constants \( Z_1, Z_2, \alpha_1, \alpha_2, x_0 \) and \( y_0 \) change by small quantities \( \Delta Z_1, \Delta Z_2, \Delta \alpha_1, \Delta \alpha_2, \Delta x_0 \) and \( \Delta y_0 \) respectively, we have the following linear equations with these deviations as first approximation.

\[ \Delta x = Z_1 \sin \gamma_1 \Delta \gamma_1 - Z_2 \sin \gamma_2 \Delta \gamma_2 + Z_1 \sin \gamma_3 \Delta \gamma_3 = -Z_1 \sin \gamma_1 \Delta \gamma_1 \]  
\[ \Delta y = Z_1 \cos \gamma_1 \Delta \gamma_1 + Z_2 \cos \gamma_2 \Delta \gamma_2 + Z_1 \cos \gamma_3 \Delta \gamma_3 = Z_1 \cos \gamma_1 \Delta \gamma_1 \]  
\[ \Delta x = Z_1 \sin \gamma_1 \Delta \gamma_1 - Z_2 \sin (\gamma_2 + \alpha_1) \Delta \gamma_2 - Z_1 \sin \gamma_3 \Delta \gamma_3 = -Z_1 \sin (\gamma_1 + \alpha) \Delta \gamma_1 \]  
\[ \Delta y = Z_1 \cos \gamma_1 \Delta \gamma_1 + Z_2 \cos (\gamma_2 + \alpha_1) \Delta \gamma_2 + Z_1 \cos \gamma_3 \Delta \gamma_3 = Z_1 \cos (\gamma_1 + \alpha) \Delta \gamma_1 \]  

where

\[ \Delta \gamma_1 = \Delta x_1 - \Delta x_2 + \cos \gamma_1 \Delta z_1 + \cos \gamma_2 \Delta z_2 + \cos \gamma_3 \Delta z_3 - \cos \gamma_1 \Delta z_1 \]  
\[ \Delta \gamma_2 = \Delta y_1 - \Delta y_2 + \sin \gamma_1 \Delta z_1 + \sin \gamma_2 \Delta z_2 + \sin \gamma_3 \Delta z_3 - \sin \gamma_1 \Delta z_1 \]  
\[ \Delta \gamma_3 = \Delta x_1 - \Delta x_2 + \cos \gamma_1 \Delta z_1 + \cos (\gamma_2 + \alpha_1) \Delta z_2 + \cos \gamma_3 \Delta z_3 - \cos (\gamma_1 + \alpha) \Delta z_1 - Z_1 \cos (\gamma_1 + \alpha) \Delta \alpha_1 \]  
\[ \Delta \gamma_4 = \Delta y_1 - \Delta y_2 + \sin \gamma_1 \Delta z_1 + \sin (\gamma_2 + \alpha_1) \Delta z_2 + \sin \gamma_3 \Delta z_3 - \sin (\gamma_1 + \alpha) \Delta z_1 - Z_1 \sin (\gamma_1 + \alpha) \Delta \alpha_1 \]  

When the values of \( \Delta \gamma_1, \Delta \gamma_2, \Delta \gamma_3, \Delta \gamma_4 \) are known, Eqs. (5) ~ (8) are the simultaneous linear equations with four unknowns \( \Delta \gamma_2, \Delta \gamma_3, \Delta \gamma_4 \) and \( \Delta \gamma_4 \). Then, if the determinant composed of the coefficients of unknowns (hereinafter referred to as the coefficient determinant)

\[ U(\gamma_1) = \begin{vmatrix} -Z_1 \sin \gamma_1 & -Z_1 \sin \gamma_1 & Z_1 \sin \gamma_1 & 0 \\ Z_1 \cos \gamma_1 & Z_1 \cos \gamma_1 & -Z_1 \cos \gamma_1 & 0 \\ Z_1 \sin (\gamma_1 + \alpha_1) & Z_1 \sin (\gamma_1 + \alpha_1) & -Z_1 \cos (\gamma_1 + \alpha_1) & Z_1 \cos \gamma_1 \end{vmatrix} \]  

is not zero, the deviations of angular displacements \( \Delta \gamma_2, \Delta \gamma_3, \Delta \gamma_4, \Delta \gamma_4 \) are determined as finite values. Here, the accuracy of a solution is assured only if the absolute value of the coefficient determinant \( U(\gamma_1) \) is greater than an order of small changes of kinematic constants \( \Delta \).

On the other hand, if \( U(\gamma_1) \) is less than or equal to \( \Delta \), we must deduce the functional relations of a quadratic form with the deviations of angular displacements of links as second approximation.

At any rate, generally, the deviations of angular displacements of links due to small changes of kinematic constants become extremely large in those cases, so that the motion-transmission characteristics become poor. Hence, in this paper, we adopt \( U(\gamma_1) \) as a quantity for estimating the motion-transmission characteristics of the S22F mechanism, and clarify its kinematic properties.

2.2 Geometrical conditions for \( U(\gamma_1) \) to become zero

Equation (9) is expanded as follows.

\[ U(\gamma_1) = Z_1 Z_2 \sin (\gamma_1 - \alpha) \sin (\gamma_1 + \alpha_1) - Z_1 Z_2 \sin (\gamma_1 - \gamma_2) \sin (\gamma_1 - \gamma_2 - \alpha) \]

\[ = -\Delta \gamma_1 \gamma_2 \gamma_1 \gamma_2 \]
Fig. 2 Configurations of pairing points when \( U(y_1) \) becomes zero.

Hence, it is found that the coefficient determinant \( U(y_1) \) becomes zero in the following five cases:

(a) Two equalities \( \sin(y_1-y_3)=0 \) and \( \sin(y_3+y_4)=0 \) hold. That is, three straight lines \( \text{BE}_1, \text{E}_1\text{C}_1 \) and \( \text{C}_1\text{D} \) lie on a straight line.

(b) Two equalities \( \sin(y_1-y_3)=0 \) and \( \sin(y_3-y_4)=0 \) hold. That is, two straight lines \( \text{E}_1\text{C}_1, \text{C}_1\text{D} \) and \( \text{E}_2\text{C}_2, \text{C}_2\text{D} \) lie on a straight line respectively.

(c) Two equalities \( \sin(y_3-y_4)=0 \) and \( \sin(y_3+y_4)=0 \) hold. That is, two straight lines \( \text{E}_2\text{C}_2, \text{E}_1\text{C}_1 \) and \( \text{E}_1\text{C}_1, \text{E}_2\text{C}_2 \) lie on a straight line respectively.

(d) Two equalities \( \sin(y_3-y_4)=0 \) and \( \sin(y_3+y_4)=0 \) hold. That is, three straight lines \( \text{BE}_2, \text{E}_2\text{C}_2 \) and \( \text{C}_2\text{D} \) lie on a straight line.

(e) The proportional expression

\[
\frac{Z_1 \sin(y_1-y_3)}{Z_3 \sin(y_1-y_3)} = \frac{Z_1 \sin(y_1-y_3)}{Z_3 \sin(y_1-y_3)}
\]

holds. That is, the triangle whose vertices are the pairing point \( B \) and two feet of perpendiculars from \( B \) to straight lines \( \text{C}_1\text{E}_1 \) and \( \text{C}_2\text{E}_2 \), is similar to the triangle whose vertices are the pairing point \( D \) and two feet of perpendiculars from \( D \) to them.

The seven pairing points of the six-link mechanism in the above-mentioned cases form the configurations shown in (a)~(e) of Fig. 2 respectively. Hence, except that the pairing point \( B \) coincides with \( D \) in such a mechanism that the lengths of the fixed and driving links are equal to each other, four cases (a)~(d) are unified into the case (e), where three straight lines passing through two pairing points \( \text{BE}_1, \text{E}_1\text{C}_1 \) and \( \text{E}_2\text{C}_2 \) intersect at a point.

Hereupon, it is found that the value of \( U(y_1) \) generally becomes zero at only the limit positions of motion.

2.3 \( U(y_1) \)'s of S23F and S32F mechanisms

Using the same symbols of kinematic constants and angular displacements shown in Figs. 1~(b) and (c) as for the S23F and S32F mechanisms respectively, the coefficient determinants of them are expressed as follows.

[In the case of the S23F mechanism]

\[
U(y_1) = Z_1 Z_2 Z_3 (Z_1 \sin(y_1-y_3) \sin(y_3+y_4) - Z_2 \sin(y_1-y_3) \sin(y_3+y_4))
\]

[In the case of the S32F mechanism]

\[
U(y_1) = Z_1 Z_2 Z_3 (Z_1 \sin(y_1-y_3) \sin(y_3+y_4) - Z_2 \sin(y_1-y_3) \sin(y_3+y_4))
\]

Moreover, it is found in the same manner as in the case of the S22F mechanism that the values of \( U(y_1) \)'s in the cases of S23F and S32F mechanisms also become zero when three straight lines \( \text{CD} \), \( \text{BE}_1 \), \( \text{BE}_2 \) and \( \text{BE}_3 \) intersect at a point respectively.

3. Forces acting at pairs due to external forces on links

we develop the discussions on the S22F mechanism.

Let \( F_2, F_3, F_4 \) and \( F_6 \) denote the external forces on the links \( \text{BE}_1\text{E}_2, \text{E}_1\text{C}_1, \text{C}_1\text{D}, \text{E}_2\text{C}_2 \) which act along the lines passing through the points \( H_2, H_3, H_4, H_6 \) and make the angles of \( \varphi_2, \varphi_3, \varphi_4, \varphi_6 \) with respect to the line \( \text{BE}_1, \text{E}_1\text{C}_1, \text{C}_1\text{D}, \text{E}_2\text{C}_2 \) respectively. Moreover, let the input torque, the components along the \( x \)- and \( y \)-axes of forces acting at fixed pairs \( A \) and \( D \), and the components along binary links and perpendiculars to them of forces acting at moving pairs of ternary links \( B \), \( E_1 \), \( E_1 \), \( E_2 \) and \( C_2 \) be denoted by the symbols shown in Fig. 3.

Then, from the equilibrium conditions of forces and moment acting on the links \( \text{AB}, \text{BE}_1\text{E}_2, \text{E}_1\text{C}_1, \text{C}_1\text{D}, \text{E}_2\text{C}_2 \), the components along perpendiculars to binary links \( \text{C}_1\text{E}_1 \) and \( \text{C}_2\text{E}_2 \), namely, \( F_{C_1}, F_{C_2} \).
$F_{Cn}$ and $F_{Dn}$ are yielded directly as follows, where $h_2=\frac{B_2}{h_2}$, $h_3=\frac{E_2}{h_2}$, $h_4=\frac{D_4}{h_4}$, $h_4=\frac{E_4}{h_4}$.

\[
F_{Cn} = (h_n/2)F_2 \sin \alpha,
F_{Dn} = (1 - h_n/2)F_2 \sin \alpha
\]

The remaining components of forces at pairs: $F_{Cn}$, $F_{Dn}$, $F_{Cn}$, $F_{Dn}$, $F_{A1}$, $F_{A2}$, $F_{A3}$, $F_{A4}$, $F_{A5}$, $F_{A6}$, $F_{A7}$, $F_{A8}$, and the input torque $T_n$ are determined as the solution of the following simultaneous linear equations composed of equilibrium equations.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & \cos \gamma_1 & \sin \gamma_1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & \cos \gamma_1 & \sin \gamma_1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cos \gamma_2 & \sin \gamma_2 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cos \gamma_3 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \cos \gamma_4 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cos \gamma_5 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \cos \gamma_6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \cos \gamma_7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cos \gamma_8 \\
\end{bmatrix}
\begin{bmatrix}
F_{Cn} \\
F_{Dn} \\
F_{Cn} \\
F_{Dn} \\
F_{A1} \\
F_{A2} \\
F_{A3} \\
F_{A4} \\
F_{A5} \\
F_{A6} \\
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
T_n \\
0 \\
F_{A1} \\
F_{A2} \\
F_{A3} \\
F_{A4} \\
F_{A5} \\
F_{A6} \end{bmatrix}
\]

where

\[f_i = -F_i \cos(\gamma_i + \alpha_i) + (1 - h_i/2)F_2 \sin \alpha_i \sin \gamma_i - (1 - h_i/2)F_2 \sin \alpha_i \sin \gamma_i
\]

The coefficient determinant of Eq. (15) is expanded as follows.

\[u(\gamma) = Z_2Z_4 \sin(\gamma_7 - \gamma_1) \sin(\gamma_7 - \gamma_1 - \alpha_1) - Z_2Z_4 \sin(\gamma_7 - \gamma_1) \sin(\gamma_7 - \gamma_1 - \alpha_1)
\]

This equation coincides with the resultant equation obtained by dividing both sides of Eq. (10) by $Z_2Z_4$. Hence, it is found that the position in which the forces acting at pairs due to external forces on links become extremely large coincides with the position in which the deviations of displacements of links due to small changes of kinematic constants become extremely large. Besides, the component forces $F_{Cn}$, $F_{Dn}$, $F_{A1}$, $F_{A2}$, $F_{A3}$, $F_{A4}$ of Eq. (14) and the nonzero elements $f_i$ (i = 4-9) of the column matrix in the right hand side of Eq. (15) contain terms which are in inverse proportion to $Z_1$ or $Z_2$, so that their lower limits need to be determined in consideration of the maximum of forces acting at pairs in synthesis of mechanisms.

Hereupon, it is concluded that the factor on which the forces acting at pairs depend is $Z_2Z_4u(\gamma_1)$, namely, $U(\gamma_1)$.

Developing the same discussions on the S23F and S32F mechanisms as on the S22F mechanism, we have $u(\gamma_1)$'s of them as follows.

In the case of the S23F mechanism

\[u(\gamma) = Z_2Z_4 \sin(\gamma_7 - \gamma_1) \sin(\gamma_7 - \gamma_1) - Z_2Z_4 \sin(\gamma_7 - \gamma_1) \sin(\gamma_7 - \gamma_1 - \alpha_1)
\]

In the case of the S32F mechanism

\[u(\gamma) = Z_2Z_4 \sin(\gamma_7 - \gamma_1) \sin(\gamma_7 - \gamma_1) - Z_2Z_4 \sin(\gamma_7 - \gamma_1) \sin(\gamma_7 - \gamma_1 - \alpha_1)
\]

Moreover, it is found that the factors on which the forces at pairs depend in the cases of S23F and S32F mechanisms are $Z_2Z_4u(\gamma_1)$ and $Z_2Z_4u(\gamma_1)$, namely, $U(\gamma_1)$'s.

4. Index of motion-transmission characteristics

At limit positions of motion, the deviations of displacements of links due to small changes of kinematic constants and the forces acting at pairs due to external forces on links become so extremely large that it is difficult to maintain a theoretical accuracy of motion. The value of $U(\gamma_1)$ becomes zero at limit positions of motion and its absolute value becomes large as the driving link of the six-link mechanism departs from these positions.

The functional part of $U(\gamma_1)$ of a four-link mechanism derived in the same manner as in the chapter 3 coincides with the sine of the transmission angle. Moreover, the one of a six-link mechanism whose fixed and driving links are included in the closed four-link loop coincides with the product of two sines of angles between the coupler and driven links or two coupler links, namely, $\sin \gamma_1 \sin \gamma_2$.

The angles \( \psi_1 \) and \( \psi_2 \) of mechanisms of three kinds W22, W23, W32, which are obtained from the chain of Watt-type, and these of mechanisms of two kinds S23, S32, which are obtained from the chain of Stephenson-type, are shown in (a)~(e) of Fig. 4. They may be regarded as expanded forms of the transmission angle.

4.1 Definition of indices

The lower and upper limits of kinematic constants are specified in consideration of constraints for the purpose of
practical use. Besides, the mechanism must be synthesized such that the function \( U(\gamma_1) \) which affects the deviations of displacements of links and the forces acting at pairs may take as large values as possible.

Therefore, in this paper, we standardize the expressions in right-hand sides of Eqs. (10), (12) and (13) such as to take values between 0 and 1, and adopt them as indices of motion-transmission characteristics. Hereinafter, they are denoted by \( T^\prime s \). The indices \( T^\prime s \) of complex six-link mechanisms S22F, S23F and S32F, which have two closed five-link loops including the fixed and driving links, are given by the following equations.

[In the case of the S22F mechanism]

\[
r = \frac{Z_2 Z_3 \sin(\gamma_1 - \gamma_2 - \alpha_1) - Z_2 Z_3 \sin(\gamma_1 - \gamma_2 - \alpha_1)}{Z_2 Z_3 + Z_2 Z_1}
\]

[In the case of the S23F mechanism]

\[
r = \frac{Z_1 Z_3 \sin(\gamma_1 - \gamma_3 - \alpha_1) \cdot \sin(\gamma_1 + \alpha_1 - \gamma_3) - Z_1 Z_3}{Z_1 Z_3 + Z_2 Z_1}
\]

[In the case of the S32F mechanism]

\[
r = \frac{Z_1 Z_3 \sin(\gamma_1 - \gamma_3 - \alpha_1 - \gamma_3) - Z_1 Z_3 \sin(\gamma_1 - \gamma_3 - \alpha_1)}{Z_1 Z_3 + Z_2 Z_1}
\]

4.2 Geometrical properties of indices

we investigate the geometrical property of the numerator of Eq. (19) by using Fig. 5-(a).

Let \( Q \) and \( R \) denote points of intersection of two straight lines \( BD \), \( C_1 E_1 \) and \( BD \), \( C_1 E_1 \) respectively. Moreover, let \( \xi \) and \( \eta \) denote angles between them, so we have following equalities.

\[
\frac{BD}{RQ + QB} = \frac{Z_1 \sin(\gamma_1 - \gamma_2 - \alpha_1)}{Z_1 \sin(\gamma_1 - \gamma_3 - \alpha_1)}
\]

\[
\frac{BD}{QB} = \frac{Z_1 \sin(\gamma_1 - \gamma_3 - \alpha_1)}{Z_1 \sin(\gamma_1 - \gamma_3 - \alpha_1)}
\]

\[
BD \sin \xi = Z_3 \sin(\gamma_1 - \gamma_3) - Z_3 \sin(\gamma_1 - \gamma_3)
\]

Substituting Eqs. (22) and (23) into the resultant equation obtained by elimination of \( QR \) from Eqs. (22) and (23), we have

\[
BD \cdot RQ \sin \xi \sin \eta = \frac{Z_1 \sin(\gamma_1 - \gamma_3) Z_3 \sin(\gamma_1 - \gamma_3 - \alpha_1)}{Z_1 \sin(\gamma_1 - \gamma_3) Z_3 \sin(\gamma_1 - \gamma_3 - \alpha_1)}
\]

Here, letting \( T \) denote the root of the perpendicular to the straight line \( BD \) from the point of intersection \( S \) of the circle of center \( Q \), which is in contact with the straight line \( C_2 E_2 \), and the straight line \( C_1 E_1 \), the value of \( RQ \sin \xi \sin \eta \) becomes equal to the length of the segment \( ST \). Therefore, it is found that numerator of Eq. (19) is equal to the product of two segments \( BD \) and \( ST \).

Fig. 4 Transmission angles of simple six-link mechanisms.

Fig. 5 Geometrical analysis of indices of motion-transmission characteristics of complex six-link mechanisms.
In the cases of S23F and S32F mechanisms, by constructing points Q, R, S, T and angles $\xi$, $\eta$ as shown in Fig. 5-(b) and (c) respectively, it is found that the segments ST's are equal to the numerators of Eqs. (20) and (21) respectively.

Then, the indices $\tau$'s are evaluated easily by the following equations.

- [In the case of the S22F mechanism] 
  \[ r = \frac{BD \cdot ST}{(Z_1 + Z_2)} \] 

- [In the case of the S23F mechanism] 
  \[ r = \frac{ST}{(Z_1 + Z_2)} \]

- [In the case of the S32F mechanism] 
  \[ r = \frac{ST}{(Z_1 + Z_2)} \]

Besides, the value of $\tau$ becomes zero at only the limit positions of motion and its absolute value becomes large as the driving link departs from these positions. Specially, if the kinematic constants of S22F, S23F and S32F mechanisms satisfy the configurations shown in Figs. 6-(a), (b) and (c) respectively, the values of $\tau$'s become unity at these positions.

4.3 Comparison of $\tau$'s with Alt's transmission angles

The transmission angle defined as an angle between the direction of absolute motion of the output link and the direction of relative motion of the transmission link with respect to the driving link is sometimes unfit to estimate the motion-transmission characteristics of complex six-link mechanisms.

In the case of the S22F mechanism shown in Fig. 7-(a), pairing points $E_1$ and $E_2$ of the ternary coupler link $BE_1E_2$ lie on the driving link $AB$, so that these pairing points and the instantaneous center $I$ and $B$ of $BE_1E_2$ with respect to the fixed link and the driving link respectively lie on a straight line. Then the values of both transmission angles $\mu_1$ and $\mu_2$ become zero.

However, letting $T$ denote the foot of the perpendicular to the straight line BD from the point of intersection $S$ of the straight line $C_1E_1$ and the circle to be contact with the straight line $C_2E_2$ whose center is the point of intersection $Q$ of BD and $C_1E_1$, the ratio of the product of segments $BD$ and $ST$ to the sum of products of two lengths of links $Z_1Z_2+Z_2Z_3$, namely, the value of $\tau$ is greater than 0.5. Hence, the motion-transmission characteristics are acceptable.

In the case of the S23F mechanism shown in Fig. 7-(b), binary coupler links $B_1E_1$ and $B_2E_2$ lie on a straight line, so that two points of intersection $Q$ and $R$ of these links and the driven link CD coincide. That is, this mechanism has at a limit position. Hence, the motion-transmission characteristics are extremely poor.

However, the instantaneous center $I$ of the ternary coupler link $E_1E_2C$ with respect to the fixed link coincides with the point of intersection of the straight line, which passes through the fixed pairing point $A$ and is parallel to $B_1E_1$ or $B_2E_2$, and the driven link CD, so that both transmission angles $\mu_1$ and $\mu_2$, which are equal to the angles between two straight lines $E_2I$, $B_1E_1$ and $E_2C$, $B_2E_2$ respectively, take values of about 90°.

In the case of the S32F mechanism in Fig. 7-(c), two points of intersection $Q$ and $R$ of the binary coupler link $BE$ and driven links $D_1C_1$ and $D_2C_2$ respectively coincide. That is, this mechanism is at a limit position. Hence, the motion-trans-
mission characteristics is extremely poor.

However, the coupler link BC lies on the pole path normal of the coupler link C1C2E with respect to the fixed link. Then, the transmission angle \( \mu \), which evaluated as the angle between the direction of absolute motion of the pairing point E and the direction of its relative motion with respect to the driving link, takes a value of 90°.

It seems that the above-mentioned failures to estimate the motion-transmission characteristics have been caused by a naive extension of the conception of the transmission angle of the plane four-link mechanism to the complex six-link mechanisms, in disregard of such a fact that the position of their pairing points are determined as the point of intersection of the circle and the tricircular curve of sixth order, namely, the coupler curve of the four-link mechanism.

5. Examples of characteristic analysis and experiments

5.1 Examples of characteristic analysis

Figure 8 is the angular displacement curve of the driven link of the S22F mechanism whose kinematic constants are as follows.

\[
\begin{align*}
Z_1 &= Z_2 = 50, \\
Z_3 &= Z_4 = 70.710 678, \\
\alpha_1 &= 315°, \\
\alpha_2 &= 0, \\
(x_0, y_0) &= (0, 0), \\
(x_0, y_0) &= (-50, 0)
\end{align*}
\]

The curves of solid and broken lines in Fig. 9 are the trajectories of the pairing points \( E_1 \) and \( E_2 \) of coupler links. The index of motion-transmission characteristics \( \tau \) varies as shown in Fig. 10, when the driving link moves in domains of motion, and becomes zero at limit positions. Then, such a curve of \( \tau \) that the expression in the sign of the absolute value of \( \text{Eq.}(19) \) takes a minus value is drawn in the minus region of \( \tau \) for the purpose of one to one correspondence to the curve of Fig. 8.

5.2 Experiments

The experimental equipment of the above-mentioned S22F mechanism has been designed with an accuracy of 0.02 mm, where the unit length and the diameters of turning-pair elements have been selected as 2.4 mm and 10 mm respectively. Moreover, two sets of compatible coupler links the diameters of whose cylinder elements \( E_1, E_2 \) are 10.5 mm and 11.0 mm have been prepared.

The mechanical errors shown in Fig. 11 have been resulted from subtractions of the upper and lower limits of \( \gamma_4 \) measured applying the counterclockwise and clockwise torques of 2.35 Nm on the driven link when the driving link is fixed at the positions of values of \( \gamma_4 \) with step-size of 1° (0.5° in neighbourhoods of limit posi-
5.3 Discussions

The mechanical errors become extremely large in neighbourhoods of limit positions of motion and decrease monotonously as the value of $\gamma_1$ increases or decreases from its value at these positions. Their tendencies to decrease are analogous regardless of the extent of the clearance of turning-pair elements and their magnitudes are in inverse proportion to the index of motion-transmission characteristics.

Strictly speaking, the mechanical errors in the neighbourhood of the limit position of motion such that $\gamma_1=90^\circ$ holds become extremely large, when $\gamma_1$ takes a slightly greater value than $90^\circ$. It seems that these phenomena have been yielded as a result of the torque applied on the driven link having caused the angular displacement to jump as shown in Fig. 12.

6. Conclusions

The motion-transmission characteristics of Stephenson six-link mechanisms which have two closed five-link loops including the fixed and driving links have been investigated. The results are summarized as follows.

1. The deviations of displacements of links due to small changes of kinematic constants generally become extremely large at only the limit positions of motion. The forces acting at pairs due to external forces on links become also extremely large at these position.
2. The indices $\tau$'s obtained by standardizing the coefficient determinants, which are the denominators of above-mentioned quantities, generally become zero at only the limit positions of motion, and its absolute values become large as the driving link departs from these position.
3. The mechanical errors tend to increase in inverse proportion as $\tau$ decreases. Moreover, the value of $\tau$ may be evaluated by simpler construction. Therefore, the indices $\tau$'s are useful to estimate the motion-transmission characteristics of complex six-link mechanisms.

References

(3) Alt, H., Z. VDI, Bd.96, Nr.8 (1954-3), 238.
(4) Matthes, H., Konstruktion, Bd.18, Nr.2 (1966), 45.