Oscillations of Laval Nozzle Flow with Condensation*

(Part 2, On the Mechanism of Oscillations and their Amplitudes)

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In the case of moist air or steam rapidly expanding in Laval nozzles and condensation occurring in the supersonic section of the flow, if the heat released by condensation exceeds a certain quantity, the flow becomes unstable and a periodic flow oscillation occurs. In the first report, an empirical relation for the frequency of the periodic flow was presented and the range of oscillations was clarified.

In the present paper, the flow oscillation has been examined experimentally for the case of rapid expansion of moist air, using a Ludwig tube and a supersonic indraft wind tunnel. As the result, the effects of the initial relative humidity of the moist air and the nozzle geometry on the amplitude of pressure oscillation have been clarified. Furthermore, based on the experimental results, the mechanism of the flow oscillation has been discussed.

Key words : Unsteady Flow, Compressible Flow, Phase Change, Condensation, Laval Nozzle, Fluid Vibration, Mechanism, Amplitude

1. Introduction

In the case of moist air or steam rapidly expanding in a Laval nozzle and condensation occurring in it, the flow will be affected by the latent heat released by condensation of water vapour. In such a flow, it is known that the heat exceeding a certain quantity can lead to an unstable and periodic character of the flow.

In the first report⁽¹⁾, this oscillation phenomenon was experimentally investigated using a Ludwig tube with a downstream diaphragm and ten kinds of two-dimensional circular nozzles. As the result, the effects of the shape of the nozzle and the relative humidity of moist air on the frequency of the periodic flow were made clear, and the conditions where the flow oscillation occurred were presented.

This report describes the amplitude of the oscillation of the pressure and discusses the mechanism of the oscillation. As for the amplitude, Saltanov et al.⁽²⁾ showed that the axial distribution of amplitudes is affected by the back

pressure of the channel. However, the effects of other factors such as the nozzle geometry and the relative humidity of supplied moist air on the amplitude remain unsolved. A numerical solution of a system of equations describing an unsteady one-dimensional flow with condensation in the supersonic part of a Laval nozzle is also reported⁽³⁾, but the mechanism of generation of the flow oscillation is not yet made clear.

In the present paper, a supersonic indraft tunnel is employed, as well as the Ludwig tube used in the first report, and the oscillation phenomenon which is caused when moist air is accelerated in a two-dimensional circular nozzle installed in the test section is experimentally investigated. The effects of the supply humidity and the nozzle configuration on the amplitude of the periodic flow are clarified. Further, the mechanism of the flow oscillation is discussed.

2. Experimental Apparatus and Method

Fig.1 Configuration of nozzle

* Received 18th March, 1983.

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The Ludwig tube and its measuring system used in the present experiment are the same as those in the previous paper⁽¹⁾. Its duration time of uniform and steady flow is only about 20ms, which is insufficient for exact measurement. For this reason, a supersonic indraft wind tunnel where moist air at atmospheric pressure is drawn into a vacuum tank is also employed. Its duration is about 5s, and the relative humidity of moist air at the stagnation state can be varied over the range of zero to 100 percent.

The test section of the Ludwig tube used in the present work is shown schematically in Fig.1. A convergent-divergent circular nozzle is mounted in the test section which is located upstream of the diaphragm, and pressure transducers (Kistler, Model 202A5, resonant frequency 250kHz) are mounted flush to the side wall along the center line of the nozzle.

The height of throat h^* and the radius of curvature R^* of the nozzle used in the present work are tabulated in Table 1, where Nozzles N1~N8 are used in the Ludwig tube experiment, and Nozzles D1~

Table 1 Dimensions of nozzles

Nozzle	h^* mm	R^* mm	$l(=\sqrt{R^*h^*})$ mm
N1	6	80	21.9
N2	6	160	31.0
N3	6	400	49.0
N4	6	1600	98.0
N5	12	155	43.1
N6	12	400	69.3
N7	18	80	38.0
N8	18	155	52.8
D1	26	160	64.5
D2	28	400	105.8
D3	48	400	138.6

Fig.2 Static pressure change with time at nozzle throat (Nozzle N3)

D3 are used in the wind tunnel experiment. The width of these nozzles is 38mm in all nozzles. The value of the characteristic length $l = \sqrt{h^*R^*}$ of the nozzle is also shown in Table 1, which relates the nozzle geometry to the temperature gradient along the nozzle axis⁽⁴⁾, and the cooling rate of the flow increases in a nozzle with a smaller characteristic length.

In either case of both Ludwig tube and wind tunnel experiments, the ratio of the stagnation pressure upstream of the nozzle to the back pressure of the channel is set so large that the flow becomes supersonic from the throat to a section far downstream of the nozzle. Therefore, the effect of the back pressure referred to in Ref.(2) cannot be included in the present experimental results.

3. Experimental Results and Discussion

3.1 Amplitude of pressure oscillation

An example of the history with time of the static pressure p_t at the nozzle throat in the case of Nozzle N3 in the Ludwig tube experiment is shown in Fig.2, where t is the time measured from the moment of rupture of the diaphragm. The curve AB shows the case where the relative humidity ϕ_0 at the stagnation state of the steady flow induced by an unsteady expansion wave generated by the diaphragm rupture is zero, that is, air is dry. In this case, the relative humidity ϕ_0 at the initial state of the driver section is necessarily zero, and there is no oscillation. On the other hand, the curve AC shows the case of $\phi_0 = 164\%$ ($\phi_0 = 100\%$). The pressure level p_{ti} indicated by a dashed line is a value calculated by assuming that the flow is choked at the nozzle throat and the flow expands isentropically. In the case of dry air (curve AB), p_t decreases at first and after becoming minimum, it increases gradually with time. The minimum value p_{td} is larger than the calculated value p_{ti} , as is evident from Fig.2. This may be due to the fact that the opening time of the diaphragm is not zero but finite, and that

Fig. 3 Relation between pressure ratio $(p_{tm} - p_{td})/p_{td}$ and relative humidity ϕ_0

the expansion wave attenuates with time. In the case of $\phi_0=164\%$ (curve AC), although the pressure oscillates due to the condensation, the mean value of the pressure changes with time as in the case of the dry air, and it is evident that the increase in pressure after the pressure becomes minimum does not have any effect on the amplitude Δp_t in Fig.2.

As seen from Fig.2, the minimum pressure p_{tm} in the case of moist air is larger than p_{td} in the case of the dry air. The ratio of the pressure difference $(p_{tm}-p_{td})$ to the pressure p_{td} is plotted against the relative humidity ϕ_0 in Fig.3. In any nozzle, the value of $(p_{tm}-p_{td})/p_{td}$ increases linearly with ϕ_0 , if ϕ_0 exceeds a certain value. The value of ϕ_0 where $(p_{tm}-p_{td})/p_{td}$ begins to increase represents a minimum relative humidity of the initiation of flow oscillation at the nozzle throat, and this will be discussed later.

As described in the previous paper⁽¹⁾, when $\phi_0 > 200$ percent, condensation occurs in the upstream region of the nozzle throat in the Ludwig tube experiment, and so only the results in the case of $\phi_0 < 200$ percent are shown in Fig.3. On the other hand, the maximum value of ϕ_0 in the wind tunnel experiment is 100 percent, of course, and the experimental values agree well with those in the Ludwig tube experiment.

The relation between the amplitude of the pressure oscillation Δp_t at the nozzle throat and the relative humidity ϕ_0 is shown in Fig.4, where Δp_t is non-dimensionalized by the minimum value p_{tm} described above. In any nozzle when ϕ_0 exceeds a certain value, a periodic flow occurs and the amplitude $\Delta p_t/p_{tm}$ increases with an increase in ϕ_0 , and approaches a certain value gradually. Comparing the results of Nozzles N2 ~ N4 which have the same throat height and so the same stagnation state upstream of the nozzle in the Ludwig tube experiment, the amplitude increases in the case of the nozzle with longer characteristic length l in Table 1. This means that the amplitude becomes larger in the case of a nozzle with a smaller cooling rate of the flow. This phenomenon will be discussed in detail later.

Fig.4 Amplitude of pressure oscillation at nozzle throat

The axial distribution of amplitudes in the case of Nozzle D3 is shown in Fig.5, taking the relative humidity ϕ_0 as a parameter. On the ordinate, the amplitude Δp is represented in non-dimensional form divided by the stagnation pressure p_0 at the nozzle inlet, and the abscissa is the ratio of the distance x from the nozzle throat to the characteristic length l of the nozzle. When $\phi_0=64\%$, $\Delta p/p_0=0$ at any position of x , and oscillation does not occur in the case of ϕ_0 being lower than this value. When $\phi_0=67\%$, the flow oscillation occurs only in the region downstream of the nozzle throat, and the amplitude increases with an increase in x/l at first, and after becoming maximum, it decreases. In the case of $\phi_0=70\%$, the maximum value of the amplitude is larger than that in the case of $\phi_0=67\%$. When ϕ_0 increases further, the region of the flow oscillation extends to the upstream part of the nozzle throat, and the maximum value of the amplitude decreases. As shown in Fig.4, the amplitude at the

Fig. 5 Effect of relative humidity on axial distribution of amplitudes of pressure oscillation

Fig. 6 Effect of configuration of nozzle on axial distribution of amplitudes of pressure oscillation

nozzle throat becomes larger as ϕ_0 increases. This can also be said from the experimental points at $x/l=0$ in Fig.5.

The effect of the configuration of the nozzle on the axial distribution of amplitudes is shown in Fig.6, where the relative humidity ϕ_0 is constant at 100%. As is evident from this figure, the amplitude becomes larger in the order of Nozzles D1, D2, and D3. This means that the amplitude becomes larger in lower cooling rate nozzles. The reason for this phenomenon will be described later.

3.2 Observation of initiation process of oscillation

An example of a series of schlieren photographs showing the flow near the nozzle throat in the initiation process of the oscillation is shown in Fig.7, which was obtained in the Ludwig tube experiment and the time shown in each photograph represents the time from the moment of rupture of the diaphragm. In Fig.(a), no disturbance is observed, but in Fig.(b) a weak disturbance normal to the flow appears just downstream of the nozzle throat. As is observed in Figs.(c) and (d), this disturbance grows with time and forms a clear normal shock (labeled A). The shock A propagates upstream. The shock S in Fig.(e) is a starting shock wave of the nozzle, and it moves downstream in the starting process. In Fig.(f), a new shock wave labeled B is observed just downstream of the nozzle throat. As shown in Fig.(g), these shocks A and B propagate upstream and attenuate owing to

Fig. 7 Schlieren photographs of periodic flow caused by condensation (Nozzle N5, $\phi_0=42\%$, $\phi_0=124\%$, $T_0=302\text{K}$)

the nozzle geometry, because they move into an increasing area of the nozzle, becoming sound waves. In this way, shock waves are generated sequentially just downstream of the nozzle throat and they propagate upstream, and the flow near the nozzle throat begins an oscillation.

3.3 Mechanism of initiation of oscillation

The axial distribution of Mach numbers of the flow with condensation is schematically shown in Fig.8, where each solid line represents the case of a different relative humidity. Q and Q_{cr} are latent heat of the phase change and heat leading to thermal choking per unit volume of the flow, respectively. The curve aef represents the Mach number distribution in the case of an isentropic expansion ($Q=0$), and the points b , c , d , and e on the curve indicate the onset of condensation when condensation occurs in the flow. As was explained previously, the position of the onset of condensation shifts towards the nozzle throat as the relative humidity ϕ_0 increases. For this reason, Q_{cr} in each curve in Fig.8 is not constant, but it becomes smaller when ϕ_0 becomes larger.

When the relative humidity of the flow increases from zero, condensation occurs downstream of the nozzle throat at a certain value of it. In the case of $Q < Q_{cr}$, the flow is not choked thermally, and the Mach number changes along the distance x as the curve $aepg$ in Fig.8. With an increase in the relative humidity, thermal choking of the flow occurs. The distribution of Mach numbers in this case is shown as the curve $adoh$ in Fig.8. When $Q > Q_{cr}$, a shock wave, mn in Fig.8 for example, is formed to adjust the flow condition. The flow just downstream of this shock is subsonic. However, as the effect of heat addition by the condensation which increases the Mach number of the flow overcomes the effect of enlargement of the cross-sectional area which decrease the Mach number, the flow is accelerated and becomes supersonic again.

In the case of the curve $acmni$ in Fig.8, the gradient of the Mach number at the position of the shock (dM/dx)_m is negative. Then the shock is stable and the oscillation does not occur even if

Fig. 8 Axial distribution of flow Mach numbers

more heat is released in the flow, because a stronger shock wave can stand at a further upstream position. However, if a shock is formed at the point k where $dM/dx=0$ and more heat is added in the flow, the shock becomes unstable, because there is no position where the shock can stand, and it propagates upstream. This is the initiation of the flow oscillation. Let ϕ_{inc} be the relative humidity in the case that the shock is formed at the position where $dM/dx=0$. Then, the criterion for the initiation of flow oscillation is expressed as $\phi_0 > \phi_{inc}$. The value of ϕ_{inc} is larger for a nozzle with a larger cooling rate of the flow, as was described in the previous paper ¹¹.

3.4 Mechanism of oscillation

Based on the experimental results described above, a schematic wave diagram in the case of the flow oscillation occurring in the nozzle is shown in Fig.9. This represents the case of the Ludwig tube experiment, and the abscissa ξ is the distance from the position of the diaphragm, and the ordinate t is the time from the moment of diaphragm rupture. The dotted line, dash-dotted line, and dashed line represent a particle path, the locus of the onset of condensation, and the locus of thermal choking of the flow, respectively. After an unsteady expansion wave generated by the diaphragm rupture passes through the nozzle, a gas particle labeled a in Fig.9, for example, is accelerated to the right, and it condenses at the point of the onset of condensation, and the flow falls into a thermal choking condition at the point A . The shock wave generated by the thermal choking ⁵ is accelerated to the left by the interaction with the compression waves between the points A and B . As the gas particles between the points b and c are compressed by this shock wave, the heat released in the condensation zone decreases, and the heat

Fig. 9 Schematic wave diagram representing flow oscillation

rate dq/dt between the points B and C becomes negative. The decrease in the rate of heat addition causes the generation of expansion waves ⁶ between the points B and C . The shock wave described above is attenuated by the interaction with these expansion waves. The gas particles between the points c and d which pass through the weakened shock wave and expansion waves are cooled and they condense easily, causing the formation of new compression waves between the points C and D , because $dq/dt > 0$ in this region ⁶. A new shock wave is generated by the coalescence of these compression waves and it propagates upstream.

In this way, in the oscillating flow, shock and expansion waves are formed and they propagate upstream alternately, and the period of the cycle is composed of the following four processes, as shown in Fig.10:

- (1) Increase in the rate of heat addition by condensation
- (2) Generation and upstream motion of a shock wave
- (3) Decrease in the rate of heat addition by condensation
- (4) Generation and upstream motion of expansion waves

The reason why there exists a maximum in the axial distribution of amplitudes in Fig.5 is explained as follows. In Fig.9, the shock wave generated by the thermal choking of the flow is at first strengthened by the compression waves generated in the increasing heat addition zone, and then it is weakened by the expansion waves generated in the decreasing heat addition zone. Therefore the amplitude becomes maximum at the position where the shock is most strengthened.

As described previously, the maximum value of the amplitude increases at first with an increase in the relative humidity ϕ_0 , and then decreases, for a fixed nozzle configuration. The increase at first may be due to a stronger interaction of the shock wave generated by the thermal choking with the compression waves, and the decrease after that may be due to the fact that the interaction is weakened, because the position of the shock wave shifts towards the nozzle throat as ϕ_0 increases.

As mentioned in Fig.6, when ϕ_0 is kept constant and the nozzle geometry is

Fig.10 One cycle of flow oscillation

changed, the maximum value of the amplitude along the nozzle axis becomes larger in a lower cooling rate nozzle, and the value of x/l where the amplitude is maximum becomes smaller. The reason is explained as follows. The supercooling at the point of the onset of condensation is smaller in a lower cooling rate nozzle, and so the heat required for thermal choking of the flow is smaller. If ϕ_0 is kept constant, however, the total heat released by condensation is the same, therefore the shock generated by the thermal choking is more strengthened by the interaction with the compression waves and a stronger shock wave moves upstream. For this reason, the amplitude increases and the point where the amplitude becomes maximum approaches the nozzle throat in lower cooling rate nozzles.

The flow oscillation occurs only in the region downstream of the nozzle throat in the case of $\phi_0=67\%$ in Fig.5. This may be due to the fact that the shock wave moving upstream in Fig.9 is so weakened by the interaction with the expansion waves that it cannot pass through the nozzle throat. It was also pointed out that $(p_{tm} - p_{td})/p_{td}$ increases with an increasing ϕ_0 . This increase may be caused by the fact that the shock passing through the nozzle throat becomes stronger as ϕ_0 increases.

4. Conclusions

The flow oscillation phenomenon which is caused by rapid expansion of moist air through a two-dimensional converging-diverging circular nozzle is experimentally investigated using a Ludwig tube with a downstream diaphragm and a supersonic indraft wind tunnel, and the amplitude of the pressure and the mechanism of oscillation are examined. The results obtained are summarized as follows:

(1) The axial distribution of amplitudes has a maximum downstream of the nozzle throat. The maximum value increases at first and then decreases with an increase in the relative humidity ϕ_0 at the

stagnation state of the nozzle inlet. For a larger ϕ_0 , the position where the amplitude becomes maximum approaches the nozzle throat, and the region of the oscillating flow extends to the upstream part of the nozzle throat.

(2) In the case of ϕ_0 being constant, the maximum value of the amplitude becomes larger in lower cooling rate nozzles.

(3) Let Q and Q_{cr} be the latent heat released by condensation and the heat leading to thermal choking per unit volume of the flow, respectively, then the thermal choking does not occur if $Q < Q_{cr}$. In the case of $Q > Q_{cr}$, the flow is thermally choked, and a normal shock is formed to adjust the flow condition. When the gradient of Mach number where the shock is formed, $(dM/dx)_{shock}$, is negative, the shock is stable at that position and no oscillation occurs. If a shock is formed where $(dM/dx)_{shock}=0$ and more heat is added in the flow, the shock becomes unstable and a flow oscillation occurs. Therefore, the criterion for the initiation of flow oscillation is expressed as $\phi_0 > \phi_{0,inc}$, where $\phi_{0,inc}$ represents the relative humidity in the case of $(dM/dx)_{shock}=0$.

(4) In the oscillating flow, shock waves generated by the thermal choking of the flow and expansion waves generated by the decrease in the rate of heat addition with condensation are formed and they propagate upstream alternately.

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