Effects of Variations of Maximum Stress and Stress Ratio on the Behaviors of Biaxial Mechanical Ratcheting*

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The effects of variations of maximum effective stress and stress ratio on the behaviors of biaxial strain accumulations were experimentally investigated using thin-walled tubular specimens of a carbon steel subjected to combined cyclic axial load and steady internal pressure. When the maximum effective stress was changed with number of cycles keeping the stress ratio constant, the strain accumulating rate under biaxial stress condition was equal to the rate under uniaxial stressing. The rate of strain accumulation in biaxial stressing was smaller compared with the rate in the uniaxial condition when the stress ratio was changed with number of cycles. A constitutive equation of biaxial mechanical ratcheting is proposed. The predictions by the present equation agree well with the experimental observations.

Key Words : Plasticity, Mechanical Ratcheting, Cyclic Loading, Biaxial Strain Accumulation, Variation of Stress, Constitutive Equation

1. Introduction

Many reports (1)-(7) on the biaxial mechanical ratcheting have been published for the case that the strain has accumulated only in a direction of steady load superposed upon large cyclic loading. The results in these studies have given valuable information on the prediction of the strength and of the deformation of structures subjected to static load superposed upon cyclic loading with constant strain amplitude. However, few reports on the biaxial strain accumulations in both directions of steady load and of cyclic loading have been published. The authors have experimentally examined the behaviors of biaxial strain accumulations using thin-walled tubular specimens of a carbon steel subjected to combined internal pressure and cyclic axial loading in the 1st report (8). On the basis of the experimental results, they have formulated the magnitude of biaxial strain accumulations as a function of maximum stress, stress ratio, the angle of bent stress path and number of cycles for the case that the stress conditions are not varied with number of

cycles. They have also proposed a constitutive equation of a uniaxial mechanical ratcheting (cyclic creep) under varied stress conditions in previous reports (9), (10).

This paper describes the effects of the variations of maximum stress and stress ratio on the behaviors of biaxial strain accumulations. Sudden variations of the maximum effective stress and stress ratio occur in structures subjected to random cyclic loading. Gradual changings of stress distribution in the structures are also induced by the mechanical ratcheting even under steady cyclic loading conditions (11). Then the behaviors of mechanical ratcheting under two types of varied stress conditions, i.e., stepwise and gradual changes of stressings, are investigated.

2. On the constitutive equation of mechanical ratcheting

Stresses and strains are expressed in the deviatoric stress plane (12) ($\bar{\sigma}_1$, $\bar{\sigma}_2$) and deviatoric plastic strain (12) ($\bar{\epsilon}_1$, $\bar{\epsilon}_2$) as

$$\bar{\sigma}_1 = (3/2)S_{11}, \quad \bar{\sigma}_2 = (\sqrt{3}/2)(S_{11} + 2S_{22}) \cdots (1)$$

$$\bar{\epsilon}_1 = \epsilon_{11}^p, \quad \bar{\epsilon}_2 = (1/\sqrt{3})(\epsilon_{11}^p + 2\epsilon_{22}^p), \quad \cdots \cdots (2)$$

where S_{ij} and ϵ_{ij}^p denote the deviatoric stress and deviatoric plastic strain in the Cartesian coordinates. The plastic strain incremental vector $d\epsilon^p$ and stress vector σ are given by the equations

$$d\epsilon^p = d\bar{\epsilon}_1 e_1 + d\bar{\epsilon}_2 e_2 \cdots \cdots (3)$$

$$\sigma = \bar{\sigma}_1 e_1 + \bar{\sigma}_2 e_2, \quad \cdots \cdots (4)$$

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where e_i ($i=1, 2$) indicates a unit vector in the direction of $\bar{\epsilon}_i$. The magnitudes of these vectors $|d\bar{\epsilon}^p|$, $|\bar{\sigma}|$ are the effective plastic strain increment and the effective stress of Mises-type, respectively, which, hereafter, will be expressed as $d\bar{\epsilon}$ and $\bar{\sigma}$. The maximum effective stress $\bar{\sigma}_{\max}$, stress ratio R and the angle θ_σ of bent stress path, as illustrated in Fig.1, are defined by the equations,

$$\left. \begin{aligned} \bar{\sigma}_{\max} &= |\bar{\sigma}_{\max}| = \sqrt{\bar{\sigma}_1^2 + \bar{\sigma}_2^2} \\ R &= \bar{\sigma}_{1\min} / \bar{\sigma}_{1\max} \\ \theta_\sigma &= \tan^{-1}(\bar{\sigma}_2 / \bar{\sigma}_{1\max}) \end{aligned} \right\} \dots\dots\dots (5)$$

where $\bar{\sigma}_1$ and $\bar{\sigma}_2$ represent the cyclic stress and steady stress, respectively. If the relation between stress $\bar{\sigma}$ and plastic strain incremental vector $d\bar{\epsilon}^m$ in the monotonic loading can be expressed by Prandtl-Reuss' equation

$$d\bar{\epsilon}^m = d\bar{\sigma} / H' \bar{\sigma}, \quad (6)$$

with the tangent modulus of plasticity H' , the rate $\dot{\bar{\epsilon}}^r = d\bar{\epsilon}^r / dN$ and the direction θ_ϵ of the ratcheting strain accumulations are then defined as follows,

$$\left. \begin{aligned} \dot{\bar{\epsilon}}^r &= \dot{\bar{\epsilon}}^p - \dot{\bar{\epsilon}}^m = \dot{\bar{\epsilon}}_1 e_1 + \dot{\bar{\epsilon}}_2 e_2, \quad \dot{\bar{\epsilon}}^r = |\dot{\bar{\epsilon}}^r| \\ \dot{\bar{\epsilon}}^m &= d\bar{\epsilon}^m / dN \\ &= \begin{cases} (1 - \bar{\sigma}_{\max}^* / \bar{\sigma}_{\max}) \bar{\sigma}_{\max} / H'; & \bar{\sigma}_{\max} \geq \bar{\sigma}_{\max}^* \\ 0; & \bar{\sigma}_{\max} < \bar{\sigma}_{\max}^* \end{cases} \\ \theta_\epsilon &= \tan^{-1}(\dot{\bar{\epsilon}}_2 / \dot{\bar{\epsilon}}_1) \end{aligned} \right\} \dots\dots\dots (7)$$

Here, $\dot{\bar{\epsilon}}^p = d\bar{\epsilon}^p / dN = \bar{\epsilon}_{(N)}^p - \bar{\epsilon}_{(N-1)}^p$ denotes the plastic strain rate at N -th cycle and $\bar{\sigma}_{\max}^*$ indicates the largest 'maximum effective stress' in the preceding stress cycles.

The ratcheting strain rate $\dot{\bar{\epsilon}}_u^r$ under varied uniaxial stress condition was given by the constitutive equation of strain hardening type (9), (10) as

$$\dot{\bar{\epsilon}}_u^r = \alpha C^{1/\alpha} \exp(-\gamma R / \alpha) \bar{\sigma}_{\max}^{\beta/\alpha} \bar{\epsilon}_u^{(1-1/\alpha)}, \dots\dots (8)$$

where C , α , β and γ are material constants. If the maximum effective stress $\bar{\sigma}_{\max}$ and stress ratio R do not change with number of cycles N , Eq.(8) yields

$$\bar{\epsilon}_u^r = C \exp(-\gamma R) \bar{\sigma}_{\max}^{\beta/\alpha} N^\alpha, \dots\dots\dots (9)$$

Fig. 1 Schematic representation of stress path in biaxial mechanical ratcheting

The biaxial strain accumulation can be expressed by the following constitutive equation (8) for the condition that the maximum effective stress and stress ratio do not change with number of cycles,

$$\left. \begin{aligned} \bar{\epsilon}^r &= \bar{\epsilon}^r (\cos \theta_\epsilon e_1 + \sin \theta_\epsilon e_2) \\ \theta_\epsilon &= \tan^{-1}(\bar{\epsilon}_2^r / \bar{\epsilon}_1^r) \end{aligned} \right\} \dots\dots\dots (10)$$

Here, the magnitude of effective ratcheting strain accumulation $\bar{\epsilon}^r$ in biaxial cyclic stress is found to be nearly equal to the uniaxial strain accumulation $\bar{\epsilon}_u^r$. The direction θ_ϵ of ratcheting strain is formulated as a function of stress ratio R and the angle θ_σ of bent stress path as

$$\theta_\epsilon = D \exp(-\xi R) \theta_\sigma^\eta, \dots\dots\dots (11)$$

where D , ξ and η are material constants.

Now, let us express the biaxial strain accumulation by the following rate-type equation in the case that the maximum effective stress $\bar{\sigma}_{\max}$ and stress ratio R are varied with number of cycles,

$$\left. \begin{aligned} \dot{\bar{\epsilon}}^r &= \dot{\bar{\epsilon}}^r (\cos \theta_\epsilon e_1 + \sin \theta_\epsilon e_2) \\ \theta_\epsilon &= \tan^{-1}(\dot{\bar{\epsilon}}_2^r / \dot{\bar{\epsilon}}_1^r) \end{aligned} \right\} \dots\dots\dots (12)$$

If the rate of biaxial strain accumulation $\dot{\bar{\epsilon}}^r$ is equal to the uniaxial ratcheting strain rate $\dot{\bar{\epsilon}}_u^r$ even under varied strain conditions, it can be given by Eq.(8). If the direction θ_ϵ of ratcheting strain accumulation is not affected by the change of maximum effective stress and of stress ratio, the direction θ_ϵ can be expressed by Eq.(11). Hereafter, we will consider the behaviors of biaxial mechanical ratcheting under varied stress conditions, comparing the strain accumulations predicted by Eqs. (8), (11) and (12) with experimental data.

3. Experimental procedures

Thin-walled tubular specimens of a carbon steel (S25C) were employed, which had been machined from a bar annealed at 870°C for an hour to the dimensions illustrated in Fig.2. The chemical

Table 1 Chemical composition of specimen

C	Si	Mn	P	S	sol. Al
0.27	0.29	0.38	0.023	0.017	0.033

Fig. 2 Specimen

composition of the specimen is shown in Table 1. The dimensions, the heat treatment and the chemical composition of the specimen were the same as those employed in the tests in the 1st report⁽³⁾.

The axial and circumferential strain accumulations were measured in the tests under combined cyclic axial load and internal pressure. The deviatoric stress $\bar{\sigma}_1$, $\bar{\sigma}_2$ and deviatoric plastic strains $\bar{\epsilon}_1$, $\bar{\epsilon}_2$ are expressed using the axial and circumferential true stresses σ_z , σ_θ and the axial and circumferential plastic strains ϵ_z^p , ϵ_θ^p by the equations

$$\bar{\sigma}_1 = \sigma_z - \sigma_\theta / 2, \quad \bar{\sigma}_2 = \sqrt{3} \sigma_\theta / 2, \quad \dots \dots \dots (13)$$

$$\bar{\epsilon}_1 = \epsilon_z^p, \quad \bar{\epsilon}_2 = (\epsilon_z^p + 2\epsilon_\theta^p) / \sqrt{3}, \quad \dots \dots \dots (14)$$

The true stresses σ_z and σ_θ were calculated using the analog computation unit during a test by the equations

$$\sigma_z = \sigma_z^{(n)}(1 + e_z), \quad \sigma_\theta = \sigma_\theta^{(n)}(1 + e_z + 2e_\theta), \quad \dots \dots (15)$$

where $\sigma_z^{(n)}$, $\sigma_\theta^{(n)}$ and e_z , e_θ denote the nominal stresses and nominal strains, respectively. Tests were carried out controlling the calculated true stresses σ_z and σ_θ .

The following three conditions of varied stressings were employed in the tests:

- (1) The maximum effective stress $\bar{\sigma}_{max}$ was varied with number of cycles N keeping the stress ratio R constant.
 - (2) The stress ratio R was varied keeping the maximum effective stress $\bar{\sigma}_{max}$ constant.
 - (3) Both maximum effective stress $\bar{\sigma}_{max}$ and stress ratio R were changed.
- The angle θ_σ of bent stress path was kept constant during a test. The cyclic axial loads were imposed sinusoidally at the rate of 0.1 to 0.2 Hz.

4. Experimental results and discussions

4.1 Variation of maximum effective stress with constant stress ratio

Figure 3(a) shows the relation between plastic strain accumulations $|\epsilon^p|$ and number of cycles N under the condition that the maximum effective stress $\bar{\sigma}_{max}$ are gradually increased or decreased with number of cycles keeping the stress ratio R constant. Here, the notation ϵ^p denotes a difference between total plastic strain accumulation ϵ^p and plastic prestrain ϵ_0^p which is expressed as

$$\epsilon^p = \epsilon^p - \epsilon_0^p = \bar{\epsilon}_1 e_1 + \bar{\epsilon}_2 e_2, \quad \dots \dots \dots (16)$$

The norm of the plastic strain vector ϵ^p defined by Eq.(16) is employed for evaluating the magnitude of strain accumulation instead of the length of the strain trajectory defined by the equation

$$\bar{\epsilon} = \int_0^{\epsilon^p} d\bar{\epsilon} = \int_0^{\epsilon^p} d\bar{\epsilon}, \quad \dots \dots \dots (17)$$

because it is difficult to determine $\bar{\epsilon}'$ accurately when the direction θ_ϵ of the ratcheting strain is changed with number of cycles. The plastic strain trajectories in the strain plane ($\bar{\epsilon}_1$, $\bar{\epsilon}_2$) are illustrated in Fig.3(b). Figures 4(a) and (b) show the magnitudes of strain accumulations and the plastic strain trajectories under the condition that the maximum effective stress is changed stepwise. The solid lines in Figs.3 and 4 represent the calculated results by the constitutive equation (8), (11) and (12). In Figs.4(a) and (b), close agreement between the predicted and experimental strain accumulations is obtained. It is found from Figs.3(b) and 4(b) that the direction of the strain accumulations is not affected by the variation of the maximum effective stress. The predicted directions θ_ϵ by Eq.(11) agree comparatively well with the experimental results.

(a) Relationship between strain accumulations and number of cycles

(b) Strain trajectories

Fig. 3 Ratcheting behaviors under the condition that the maximum effective stresses are gradually varied keeping the stress ratio constant

4.2 Variation of stress ratio with constant maximum effective stress

Figure 5(a) and (b) show the magnitudes of the strain accumulations and the plastic strain trajectories when the stress ratios are gradually increased or decreased with number of cycles keeping the maximum effective stress constant. Figure 6(a) and (b) represent the results of the tests under the condition that the stress ratios are varied stepwise with constant maximum effective stress. The broken lines in the figures exhibit the predicted results by Eqs.(8), (11) and (12). As shown in Fig.5(a), for the case of gradual change in stress ratio ($\dot{R} \neq 0$ at any stress cycles), the magnitudes of strain accumulations in biaxial stressing ($\theta_\sigma \approx 15^\circ$) are smaller than those in uniaxial stressings. Equations (8), (11) and (12) overestimate the magnitudes of biaxial strain accumulations, though the predicted results for uniaxial ratcheting agree comparatively well with the actual ones. On the contrary, for the case of stepwise change in stress ratio ($\dot{R} = 0$ at any cycles except for the stress cycle at which the stress ratio is just changed),

(a) Relationship between strain accumulations and number of cycles

(b) Strain trajectories

Fig. 4 Ratcheting behaviors under the condition that the maximum effective stress are varied stepwise keeping the stress ratio constant

(a) Relationship between strain accumulations and number of cycles

(b) Strain trajectories

Fig. 5 Ratcheting behaviors under the condition that the stress ratios are gradually varied keeping the maximum effective stress constant

(a) Relationship between strain accumulations and number of cycles

(b) Strain trajectory

Fig. 6 Ratcheting behaviors under the condition that the stress ratios are changed stepwise keeping the maximum effective stress constant

the magnitudes of strain accumulations in biaxial stressing are almost the same as those in uniaxial stressing. A change in the direction of the strain accumulation $|\dot{\theta}_e|$ for a constant θ_σ is given by differentiating Eq.(11) as

$$\dot{\theta}_e = -\xi D \exp(-\xi R) \theta_\sigma \dot{R} \dots (18)$$

From Eq.(18), the above mentioned phenomenon, i.e., a decrease of the rate of strain accumulation in biaxial stressing ($\theta_\sigma \neq 0$) with a gradually changed stress ratio ($\dot{R} \neq 0$), can be explained by the hypothesis that the rate of strain accumulation decreases with the change of the direction $|\dot{\theta}_e|$. Figure 7 shows the relationship between relative strain accumulations ($\bar{\epsilon}^r/\bar{\epsilon}_u^r$) and $|\dot{\theta}_e|$ calculated by Eq.(18). Here, median value of $|\dot{\theta}_e|$ in the test is used in Fig.7. It is evident from the figure that the rate of strain accumulation decreases with an increasing $|\dot{\theta}_e|$.

5. A constitutive equation of the mechanical ratcheting under biaxial stressing

To describe the decrease of the rate of strain accumulation with an increasing $|\dot{\theta}_e|$, now Eq.(8) is modified to

Fig. 7 Relationship between the change of the directions of strain accumulations $|\dot{\theta}_e|$ and the relative strain accumulations ($\bar{\epsilon}^r/\bar{\epsilon}_u^r$)

Fig. 8 Relationship between g and $|\dot{\theta}_e|$

$$\left. \begin{aligned} \bar{\epsilon}^r &= \bar{\epsilon}^r (\cos \theta_e e_1 + \sin \theta_e e_2) \\ \theta_e &= D \exp(-\xi R) \theta_\sigma^2 \\ \bar{\epsilon}^r &= g(|\dot{\theta}_e|) \\ &\quad \times \alpha C^{1/a} \exp(-\gamma R/a) \bar{\sigma}_{\max}^{1/a} \bar{\epsilon}^{r(1-1/a)} \end{aligned} \right\} \dots (19)$$

Here, the monotonically decreasing function $g(|\dot{\theta}_e|) \cdot (\leq 1)$ can be determined as follows. In the case that $|\dot{\theta}_e|$ does not change in a test, the function $g(|\dot{\theta}_e|)$ has a constant value and it is given by integrating Eq.(8) as

$$\left. \begin{aligned} g &= \bar{\epsilon}^{r(1/a)} / \int_0^N h(\bar{\sigma}_{\max}, R) dn \\ h(\bar{\sigma}_{\max}, R) &= C^{1/a} \exp(-\gamma R/a) \bar{\sigma}_{\max}^{1/a} \end{aligned} \right\} \dots (20)$$

In the case of uniaxial ratcheting, the relations $g=1$ and $\bar{\epsilon}^r = \bar{\epsilon}_u^r$ are given, so that the following equation can be derived from Eq.(20),

$$\int_0^N h(\bar{\sigma}_{\max}, R) dn = \bar{\epsilon}_u^{r(1/a)} \dots (21)$$

(a) Relationship between strain accumulations and number of cycles

(b) Strain trajectories

Fig. 9 Ratcheting behaviors under the condition that both maximum effective stress and stress ratios are varied

Substituting Eq.(21) into Eq.(20), the function g can be determined as

$$g = (\bar{\epsilon}^r / \bar{\epsilon}_L^r)^{1/\alpha} \quad (22)$$

Figure 8 shows the relation between the values of g and $|\theta_c|$. The experimental equation of g can be given by

$$g = 1 - 0.9(1 - e^{-500|\theta_c|}) \quad (23)$$

The magnitudes of strain accumulations and plastic strain trajectories under varied stress conditions calculated by Eqs.(19) and (23) are drawn by solid lines in Figs.3 to 6. Figures 9(a) and (b) show the comparisons between the experimental and the calculated strain accumulating behaviors in the case that both maximum effective stress and stress ratio are changed with number of cycles. The predicted strain accumulations agree comparatively well with the experimental ones.

6. Conclusions

The effects of variations of maximum stress and stress ratio on the behaviors of biaxial mechanical ratcheting were experimentally examined. The present findings are summarized as follows:

- (1) The rate of strain accumulation in biaxial stressing is smaller compared with that under the uniaxial condition when the stress ratio is changed with number of cycles.
- (2) The direction of strain accumulation is not affected by the variations of maximum effective stress and of stress ratio. The direction is formulated as a function of stress ratio and the angle of

bent stress path.

(3) A constitutive equation of the mechanical ratcheting is proposed, which can describe a decrease of the rate of strain accumulation due to a change in the direction of strain accumulation. The predicted strain accumulations agree comparatively well with the experimental ones.

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