On the Positioning by a Linear Induction Motor*

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After an experimental discussion on the characteristics of the generated thrust and friction of a linear induction motor with a phase-controlled power source, the positioning characteristics of a servomechanism having a linear motor as its actuator have been analyzed by simulations and experiments.

The mechanism has been shown to be a stable positioning mechanism as second order system, though there may occur minute inching motions due to force variations in the vicinities of set points. As the inching motions cause deterioration of the response characteristics for small input values, an improvement of response is also discussed.

Key Words: Control Devices, Linear Motor, Induction Motor, Phase Control, Positioning Mechanism, Servomechanism

1. Introduction

Linear induction motors enable large-stroke motions and they can drive any electric conductive materials as their second conductors. They are almost free from maintenance because of their simple structures which are similar to squirrel-cage motors. However, it is generally said to be difficult to control precisely at low speeds the positions and speeds of induction motors unlike D.C. motors and pulse motors. Therefore, there are, if anything, few applications of the linear induction motors to the positioning servomechanisms, and their characteristics have not been made clear in detail. Thus, positioning characteristics of the linear induction motor with a phase-controlled power source are studied in the present paper.

2. Constitution of Servomechanism

The constitution of the positioning servomechanism for the present study is shown in Fig.1. The linear motor is a cylindrical single-phase induction motor, whose second conductor is a compound steel pipe covered with copper and finished by chrome-plating. The output displacement x_o and velocity $\dot{x}_o$ of the motor running on the horizontally fixed second conductor are converted into the feedback voltages e_o and e_t respectively by a potentiometer and a tachometer generator, which are rotated in accordance with the motion of motor by means of a wire and pulley device. The desired position x_i of the motor is also converted into a voltage e_i by a potentiometer and the controlled deviation e_c is obtained as the difference between e_i and e_o .

In the phase-angle controller, the

Fig.1 Block diagram of servomechanism

Fig.2 Firing angle and output voltage of thyristor

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firing angle α of the thyristors is determined in response to the deviation e . The absolute value circuit, comparator and two thyristors are included in the servomechanism in order to change the direction of thrust of the motor.

The phase-angle controller consists of a WJT(Uni-Junction-Transistor) phase-shifting circuit. The firing angle α and the output voltage E versus the input voltage e are shown in Fig.2, where a dead zone and saturation are observed. The dead zone is removed by adding a bias e_0 as is known from Fig.1. The output voltage E shown in Fig.2 is a reading of the voltmeter and it is proportional to the average voltage;

$$E_{ave} = \frac{1}{\pi} \int_{\alpha}^{\pi} E_0 \sqrt{2} \sin \theta d\theta$$

$$= \frac{\sqrt{2}}{\pi} E_0 (1 + \cos \alpha), \dots\dots\dots (1)$$

Where E_0 is the RMS voltage of the power source. The firing angle α in the figure is the result obtained from the wave of the output by using a synchroscope.

In the present paper, the voltage E is adopted as a simple expression of the input voltage of the motor, and the firing angle is added to it when necessary.

Fig.3 Guide of motor and detection of position and velocity

(a) Mean clearance in diameter

(b) Coefficient of static friction

Fig.4 Mean clearance and coefficient of static friction

3. Characteristics of Linear Motor

3.1 Friction

The main parameters of the linear motor used in the experiment are as follows.

Power source : Single phase 100V, 50Hz
No. of poles : 8
Pole pitch : 28 mm
Synchronous speed : 2.8 m/s
Rated current : 3.5 A
Rated thrust : 19.6 N
Mass : 2.01 Kg

The motor is supported on its second conductor rod with two teflon sliding bearings whose widths are 9.5 mm. The outer distance between the two bearings is 262 mm as shown in Fig.3. The average diameter, out of roundness (radius method) and surface roughness (maximum height) of the rod are 20.052 mm, 2.9 μm and 0.7 μm respectively. The average bearing clearance in diameter c is shown in Fig.4(a) with respect to the position of the motor x_0 .

The friction of the bearings changes its magnitude depending on the position of motor and the lubrication. In the case where the turbine oil #90 is used, the coefficients of static and kinetic frictions μ_s and μ_k become respectively as shown in Fig.4(b) and Fig.5 when the power supply is cut off. In Fig.4(b), marks o and I express respectively the mean value and the range of dispersion. The abscissa v in Fig.5 expresses the velocity of motor. From these figures, the friction of the linear motor is known to be fairly larger than the ones of rotary motors.

Because the motors may keep still under load in the positioning mechanisms, the virtual coefficient of static friction μ'_s

Fig.5 Coefficient of kinetic friction

Fig.6 Virtual coefficient of static friction

when the motor is supplied with a phase-controlled power is experimentally searched for, resulting in Fig.6. The coefficient is defined as $\mu_s'=(F_1-F_2)/(2Mg)$, where F_1 and F_2 are the maximum and minimum forces which keep the motor still, M is the mass of motor and g is the gravitational acceleration.

3.2 Thrust

The characteristics of starting thrust F_s versus supplied voltage E and virtual thrust F' versus velocity v are shown in Figs.7 and 8 respectively. The virtual thrust F' is obtained by measuring the acceleration of the motor which is supplied with a phase-controlled power of constant voltage E . Therefore, F' includes friction besides the generated thrust. From Figs.5 and 8, the true thrust F may be considered to decrease linearly with respect to the velocity v . Regarding the true thrust as a straight line passing two points, $(v, F)=(0, F_s)$ and $(v_s, 0)$, in the $F-v$ diagram, the true thrust F and the virtual coefficient of kinetic friction μ_k when the motor is supplied with power are obtained as shown in Fig.9. v_s is the

Fig.7 Starting thrust versus input voltage

Fig.8 Virtual thrust versus velocity

synchronous speed of the motor.

3.3 Vibration

From Figs.6 and 9, the coefficients of static and kinetic frictions seem to vary with respect to the input voltage of motor. This is because the motor vibrates due to the phase-controlled power source.

Thus, the vibration of motor is investigated in the case where the motor is fixed to the frame with a coil spring whose stiffness is 196 N/m. Some examples of variation of displacement x_0 with respect to time t are shown in Fig.10. The total amplitude A_x in the running direction becomes as shown in Fig.11. The frequency of vibration is 100 Hz, which is equal to twice the power source frequency. When the motor

Fig.9 Thrust and virtual coefficient of kinetic friction

Fig.10 Vibration of motor supplied with phase-controlled power

Fig.11 Total amplitude of vibration

is supplied with power of various sinusoidal voltages by a transformer, it hardly vibrates. That is, the amplitude of vibration increases as the input voltage increases, but its value is only about $2\mu\text{m}$ at $E=100\text{ V}$.

In Fig.11, the amplitude becomes maximum at about $E=60\text{ V}$, which is almost coincident with the voltage where the current is observed to rise most sharply and take the maximum peak value. On the contrary, the virtual coefficient of static friction of Fig.6 takes its minimum value at about $E=60\text{ V}$. Thus, considering the virtual coefficient to decrease proportionally to the amplitude of Fig.11, the coefficient becomes a full line shown in Fig.6.

3.4 Inching motion

For a constant input voltage of motor, the velocity of motor takes a certain constant value when the friction becomes equal to the generated thrust. This steady-state velocity seems to rise at $E=32\sim34\text{ V}$ as shown in Fig.12. However, the motor moves at very low speeds even when $E=20\sim33\text{ V}$, and the mean velocity is such as shown in Fig.13. The displacement of motor at $E=20\sim33\text{ V}$ is shown in Fig.14, where minute inching motions with a period of 10 ms are observed. The inching motions appear when the maximum thrust during a half cycle of the alternating input voltage exceeds slightly the static friction.

Making the thyristor fire only once, the firing angle at which the motor stops just 10 ms after the firing is experimentally investigated. The result is $\alpha=0.60\pi\text{ rad}$. This angle corresponds to a reading of the voltmeter $E=32.8\text{ V}$, at which the velocity of motor rises as is known from Fig.12.

Fig.12 Steady-state velocity versus input voltage

Fig.13 Mean velocity of inching motion

Strictly speaking, the friction is different a little between the cases where the thyristor is fired only once and where it is fired every 10 ms . In fact, the motor comes to make a uniform motion with a speed of $\dot{x}_0 \approx 3.5 \times 10^{-3}\text{ m/s}$ at $E=32.8\text{ V}$ as shown in Fig.14. The input voltage E and velocity $\dot{x}_0$ when the motion of motor changes from inchings to a uniform motion are denoted by E_c and v_c respectively.

4. Characteristics of Servomechanism

The characteristics of the servomechanism shown in Fig.1 are analyzed in the case where the linear induction motor mentioned just above is adopted.

4.1 Equations

Denoting the coefficients of potentiometers and tachometer generator in Fig.1 by k_p and k_t respectively, the input voltage e of the phase-angle controller is given by the next equations.

$$\left. \begin{aligned} e_i &= k_p x_i \\ e_o &= k_p x_o \\ e_t &= k_t \dot{x}_o \\ e_e &= e_i - e_o - e_t \\ e &= |e_e| + e_o \end{aligned} \right\} \dots\dots\dots (2)$$

As mentioned above, the output of thyristor is expressed by a reading E of the voltmeter. Eliminating the dead zone $0 \leq e < 6.9\text{ V}$, E may be expressed as follows from Fig.2.

$$E = 94 - 105 \exp\{-4.39 \times (e - 6.9)\} \text{ (V)} \dots\dots (3)$$

The full line shown in Fig.2 indicates the above equation.

The thrust of motor F shown in Fig.9 is for a slip factor of $0 \sim 1$, that is for $e_e v \geq 0$. Assuming the thrust to lie on the extension to the $F-v$ lines when $e_e v < 0$, the next equation may be obtained from the fact mentioned in section 3.2.

$$F = F_s \left\{ 1 - \frac{|v|}{v_s} \operatorname{sgn}(e_e v) \right\} \operatorname{sgn}(e_e) \text{ (N)} \dots\dots (4)$$

Where the starting thrust F_s is given by a curve shown in Fig.7, whose equation is as follows.

$$F_s = 2.18 \times 10^{-2} E^{1.54} \text{ (N)} \dots\dots\dots (5)$$

Fig.14 Inching motion

Because the virtual coefficient of static friction μ'_s varies with respect to the input voltage of motor $E(V)$, the full line shown in Fig.6 is adopted, the equation of which becomes approximately as follows.

$$\mu'_s = a_1 + a_2 E^2 + a_3 E^3 + a_4 E^4 + a_5 E^5 \dots (6)$$

Where,

$$\left. \begin{aligned} a_1 &= 2.24 \times 10^{-1} & a_2 &= 7.87 \times 10^{-5} \\ a_3 &= -7.26 \times 10^{-8} & a_4 &= 1.13 \times 10^{-7} \\ a_5 &= -4.73 \times 10^{-10} \end{aligned} \right\} \dots (7)$$

The virtual coefficient of kinetic friction μ'_k takes almost the same value of 0.35 at $v \approx 1.2$ m/s without regard to the supplied voltage E as is observed in Fig.9. Thus, putting $\mu'_k = \mu'_s$ at $v=0$, μ'_k is expressed as follows.

$$\mu'_k = \frac{0.35 - \mu'_s}{1.2} |v| + \mu'_s \dots (8)$$

As mentioned in section 3.4, the variation of velocity of the motor due to that of thrust during a half cycle of alternating voltage of the power source is negligibly small in the case of $E \geq E_c$ or $|v| \geq v_c$. Thus, substituting $\dot{x}_0 = v$ into Eq. (2) and calculating Eqs. (4) and (8) for F and μ'_k , the motion of motor may be obtained from the next equation.

$$M\dot{v} = F - \mu'_k Mg \operatorname{sgn}(v)$$

$$\therefore \dot{v} = \frac{F}{M} - \mu'_k g \operatorname{sgn}(v) \dots (9)$$

In the case of $E < E_c$ besides $|v| < v_c$, the motor makes inching motions. However the inching motions themselves are neglected and the servomechanism is analyzed here by use of their mean velocities, because the motion of motor cannot be controlled within a half cycle of voltage alternation of power source. The mean velocity v becomes nearly zero under $E \approx 21$ V as shown in Fig.13, and the velocity is expressed as follows.

$$\left. \begin{aligned} \text{When } 21(V) \leq E < E_c \\ v &= 4.2 \times 10^{-5} (E - 21)^{1.8} \operatorname{sgn}(e_e) \text{ (m/s)} \\ \text{When } E < 21(V) \\ v &= 0. \end{aligned} \right\} \dots (10)$$

The curve shown in Fig.13 indicates the above equation.

4.2 Simulation

When the displacement x_0 and velocity

Fig.15 Step response

$v(=x_0)$ of the motor at a certain time $t=t_0$ are given, the thrust F and the virtual coefficient of friction μ'_k can be obtained from Eqs. (2) ~ (8). Because the thyristor is fired once in a half period $T=0.01$ s of the voltage alternation of power source, the displacement x_r and velocity v_r at $t=t_0+T$ can be obtained by solving Eq. (9) or (10) assuming F and μ'_k to be constant for the interval T .

When $E \geq E_c$ or $|v| \geq v_c$,

$$\left. \begin{aligned} x_r &= x_0 + vT \\ &+ \frac{1}{2} \left\{ \frac{F}{M} - \mu'_k g \operatorname{sgn}(v) \right\} T^2 \end{aligned} \right\} \dots (11)$$

$$v_r = v + \left\{ \frac{F}{M} - \mu'_k g \operatorname{sgn}(v) \right\} T$$

When $21(V) \leq E < E_c$ and $|v| < v_c$ (m/s),

$$\left. \begin{aligned} x_r &= x_0 + vT \\ v_r &= 4.2 \times 10^{-5} (E - 21)^{1.8} \operatorname{sgn}(e_e) \end{aligned} \right\} \dots (12)$$

When $E < 21(V)$ and $|v| < v_c$,

$$\left. \begin{aligned} x_r &= x_0 + vT \\ v_r &= 0. \end{aligned} \right\} \dots (13)$$

Replacing x_0 and v in Eqs. (11) ~ (13) by x_r and v_r respectively, the displacement and velocity of the motor at $t=t_0+2T$ are obtained.

4.3 Results and discussions

The results of experiments and simulations about a servomechanism manufactured according to Fig.1 shall be described here with discussions.

Some responses to a step input of magnitude $x_s = 0.3$ m are shown in Fig.15. The full lines and broken lines represent respectively the results of experiments and simulations. Though cyclings occur when the velocity feedback is little, the output is observed to converge stably to the set point even when $k_t = 0$. The value of k_t at the critical damping state is $k_{tc} \approx 0.7$ V·s/m.

Step responses for $k_t = 0.7 k_{tc} = 0.47$ V·s/m become as shown in Fig.16. The displacement curves at early responses are the same without regard to the magnitude of input x_s , which implies that the thrust of motor is in the state of saturation.

Examples of the settling time t_s , which is required for the controlled deviation to become less than $\pm 5\%$ of the set magnitude are shown in Fig.17. The settling time is known to become quite long as the input step becomes small. This is because the

Fig.16 Step response

motor makes inching motions with very small mean speeds in the neighborhood of the set point. When the input step is large, differences of the settling times due to the values of k_p are negligibly small. This is because the motor runs with its saturated thrust when the controlled deviation is large. Assuming that a Bang-Bang control is possible, and giving the motor the maximum acceleration and deceleration with its saturated thrust, the simulation gives the settling time shown by the chain line in Fig.17. When $x_s > 0.1$ m, the settling time of the servomechanism of Fig.1 is almost equal to the minimum value realized by the Bang-Bang control.

As an example of frequency responses, responses to sinusoidal inputs with amplitudes of $A=0.05$ and 0.10 m are shown in Fig. 18. The results of simulation are indicated by full and broken lines, which are obtained by connecting smoothly the calculated points for various input frequencies. Approximating the gain by two straight lines, both slopes of the lines over break frequencies for $A=0.05$ and 0.10 m take almost the same value of about -40 dB/dc. The phases at break frequencies for both input amplitudes take

Fig.17 Settling time

Fig.18 Frequency response

also the same value of about $-\pi/2$ rad.

From the results of step and frequency responses mentioned just above, the servomechanism of Fig.1 is known to be a stable system similar to the second order one though it is a nonlinear and discontinuous system. Good coincidence of the simulations with the experiments is also verified.

4.4 Improvement of response

The positioning mechanism with a linear induction motor which is controlled by the phase-control method is very slow in response to small inputs as shown in Fig.17 due to the inching motion of the motor in the neighborhoods of set points. Thus, the coefficient about the velocity feedback k_t which makes the settling times minimum for small step inputs has been searched for by simulations, resulting in Fig.19(a). When the coefficient about the position feedback k_p is kept constant, the coefficient k_t is known to decrease considerably in its magnitude as the input decreases. This is because the damping effect of friction increases relatively to the thrust of motor. When the coefficient k_t of Fig.19(a) is adopted, the settling time becomes as shown in Fig.19(b), where the settling time is observed to increase suddenly as the input step decreases below the value corresponding to $k_t=0$. By the way, the control system of Fig.1 itself cannot regulate automatically the value of k_t depending on the input magnitude. A microcomputer, for example, may be required to realize the automatic regulation of k_t .

(a) Coefficient of velocity feedback

(b) Minimum settling time

Fig.19 Coefficient of velocity feedback and minimum settling time for small step input

It is thinkable that an increase of the bias voltage may eliminate the inching motions in the neighborhoods of set points and make the responses rapid. However, vibrations become apt to occur due to the elasticities and backlashes of the detecting means and the noises of the electric circuits. Increasing the bias of the experimental mechanism, the vibrations shown in Fig.20 are observed in the vicinity of set point. Though no big vibrations are observed in (a), vibrations mainly due to the backlash and the noise are observed respectively in (b) and (c).

Fig.20 Relation between vibration of motor and bias voltage

5. Conclusions

Positioning characteristics of a linear induction motor have been analyzed in the case where the motor is controlled by the phase-control method. The results are summarized as follows.

(1) The bearing friction of a linear motor is fairly greater than that of a rotary motor, and the motor makes minute inching motion in the neighborhood of set point due to the variation of generated thrust.

(2) The inching motion is directly affected by the static friction. The virtual coefficient of static friction when the motor is supplied with a phase-controlled power varies with the firing angle α of the thyristor, and it becomes minimum at about $\alpha = 0.4\pi$ rad. The variation of the friction is caused by the vibration of motor due to the variation of thrust, and it is hardly caused when sinusoidal powers less than the rated voltage are supplied.

(3) Characteristics of generated thrust and friction of a linear induction motor have been formulated on the basis of experimental results. Calculating the thrust and friction from the formulae at intervals of the half period $T = 0.01$ s of the alternation of power source, and assuming them constant for the interval T , a servomechanism which has a linear motor as its actuator has been analyzed. The results of analyses are in good coincidence with those of experiments.

(4) By the simulations and experiments, the servomechanism studied in the present paper has been confirmed to be capable of stable positioning. The settling times for large step inputs are nearly equal to those of Bang-Bang controls. However, the times for small step inputs become quite long due to the inching motions. Thus, the relation between the position and velocity feedbacks which give the minimum settling times has been studied for small step inputs.