Flow in the Entrance Region of a Porous Pipe*

(4th Report: Oscillatory Flow)

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This study deals with an axisymmetric oscillatory flow of an incompressible viscous liquid in a circular porous pipe, which starts from rest. Change of the axial velocity profile over an arbitrary cross-section of the pipe due to a step change and/or an oscillatory one of the flow rate is analytically obtained including a transient component of the velocity in the inlet region. The present analysis is based on Oseen's approximation of the momentum equation and the validity of this method is confirmed by the modified momentum integral method. Dynamic distributions of wall friction and pressure coefficients are precisely discussed on the boundary conditions for oscillatory injection and suction from the pipe wall. Furthermore the following two facts are made clear: one is that slip velocity on the wall has an immediate connection with wall friction, the other is that both the alternating component of the axial velocity and the slip velocity on the wall make the inlet short.

Key Words: Oscillatory Flow, Porous Pipe, Entrance Region, Slip Velocity, Oseen's Approximation, Green's Function, Unsteady Flow

1. Introduction

Flow analysis in a circular porous pipe is a significant contribution to knowledge of chemical plants and of capillary vessels. We have analyzed some flows in the entrance region of a porous pipe with a parabolic profile of inlet velocity in the 1st paper, with a uniform one in the 2nd, and a transient flow in the 3rd by using a modified momentum integral method. In this modified momentum integral method, we can strictly take the convective term of the equation of motion into consideration. However, this method has a demerit that the velocity profile in the boundary layer is approximated by a polynomial.

In this paper, we deal with an oscillatory flow which is expressed as a superposition of a pulsatile flow on an average one. Skalak-Wang(1977) investigated the pulsatile flow by the numerical method with the boundary condition that the velocity of injection or suction is constant and the slip velocity is zero at the wall. But this

method is very complicated. Hence in this paper, we try to apply Oseen's approximation to the flow in a porous pipe, which Yokota-Nakano(1979) and Atabek-Chang(1961) used in order to analyze a pulsatile flow in a rigid pipe. Oseen's approximation method has some merits of linearizing the equation of motion and of expressing the solution explicitly in spite of a bold treatment of the convective term of the equation of motion. In this paper we also approximate the axial component of the Navier-Stokes equation by Oseen's method and will search a new method for the strict solution. By using pulsatile injection and slip velocities as a boundary condition at the wall, we investigate pulsatile characteristics of the entrance region.

Nomenclature

- a: radius of the pipe
- B, B_1, B_2 : amplitudes of the oscillatory components
- C_p : pressure coefficient
 $= (p_e - p) / (1/2 \rho u_0^2)$
- J_n : Bessel function of the first kind, of order n
- i: imaginary unit
- $\bar{p}$: dimensionless pressure $= p / \rho u_0^2$
- p : pressure at the inlet
- $\bar{r}$: dimensionless radial coordinate
 $= r/a$
- Re : axial Reynolds number $= 2u_0 a / \nu$
- Re_w : wall Reynolds number $= 2v_{w0} a / \nu$
- $\bar{s}$: dimensionless Laplace parameter
 $= a^2 s / \nu$
- $\bar{t}$: dimensionless time $= t \nu / a^2$
- $\bar{u}$: dimensionless axial velocity
 $= u / u_0$
- u_0 : mean velocity at the inlet
- $\bar{u}_c$: dimensionless centerline velocity
in the pipe $= u_c / u_0$

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- $\bar{u}_e$: dimensionless velocity at the inlet $=u_e/u_0$
 $\bar{u}_w$: dimensionless slip velocity at the wall $=u_w/u_0$
 $\bar{u}_{w0}$: dimensionless mean velocity at the wall $=u_{w0}/u_0$
 $\bar{v}$: dimensionless radial velocity $=v/u_0$
 $\bar{v}_w$: dimensionless injection velocity at the wall $=v_w/u_0$
 $\bar{v}_{w0}$: dimensionless mean injection velocity at the wall $=v_{w0}/u_0$
 $\bar{x}$: dimensionless axial coordinate $=vx/(a^2u_0)$
 α_n : zero point of Bessel function of order 2, $J_2(\alpha_n)=0$
 δ : boundary layer thickness
 ν : kinematic viscosity
 ρ : density
 $\bar{\tau}_1, \bar{\tau}_2$: phase differences against the inlet velocity $=\tau_1\nu/a^2, \tau_2\nu/a^2$
 τ_w : dimensionless shearing stress at the wall $=-(\partial u/\partial r)_{r=1}$
 ω : dimensionless circular frequency $=a^2\omega/\nu$
 Suffixes
 -: dimensionless
 1: injection
 2: slip

2. Basic Equations

2.1 Basic equations

Here we will consider a porous pipe as shown in Fig.1. The radius of the porous pipe is a , the form of intake is a bell-mouth, and we suppose a pulsatile flow at the inlet which is expressed by Eq.(3). The pipe is porous and the fluid in the pipe is viscous and incompressible, and the flow has the following characteristics: When the suction velocity is very small in comparison with the main velocity, we may consider the pressure in the pipe almost constant over the cross-section. The radial velocity is very small compared with the axial velocity, so the radial equation of motion may be neglected. And $\partial^2 u/\partial x^2$ in the viscous term is also very small in comparison with $(1/r)\partial(r\partial u/\partial r)/\partial r$. Oseen's approximation that the convective term in the equation of motion is approximated by $u_0 \partial u/\partial x$ is widely applied to such an entrance flow. Hence the equation of motion which controls the flow in a porous pipe and the equation of continuity are described as follows:

$$\frac{\partial u}{\partial t} + u_0 \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{dp}{dx} + \nu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \quad (1)$$

Fig.1 Schematic illustration of the entrance flow in a circular porous pipe

$$\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} r v = 0 \quad (2)$$

2.2 Boundary conditions

The inlet velocity at the entrance is expressed by the following equation:

$$u_e(t) = u_0(1 + B \sin \omega t) \quad (3)$$

The injection velocity " v_w " and the slip velocity " u_w " at the wall vary according to the inlet velocity " $u_e(t)$ ". Therefore the injection velocity may be described as follows:

$$v_w(t) = v_{w0}\{1 + B_1 \sin \omega(t - \tau_1)\} \quad (4)$$

The average injection velocity is negative for suction and positive for injection. And the slip velocity at the wall is written in the same way:

$$u_w(t) = u_{w0}\{1 + B_2 \sin \omega(t - \tau_2)\} \quad (5)$$

Here τ_1 and τ_2 indicate phase lags of the injection and slip velocities against the inlet velocity, respectively.

3. Theoretical Analysis

3.1 Transformation of the basic equation

At first let us eliminate the pressure gradient term of Eq.(1). For this purpose, we integrate the equation of motion over the cross-section taking the equation of continuity into consideration. By virtue of the constant pressure over the cross-section, the pressure term can be expressed by the viscous term and the inertial term. Substituting this pressure term into Eq.(1) leads to the elimination of the pressure gradient in Eq.(1). Therefore the equation of motion is transformed as:

$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}}{\partial \bar{x}} = \frac{\partial \bar{u}_e(t)}{\partial t} - R_w \left\{ \bar{x} \frac{d\bar{k}(\bar{t})}{d\bar{t}} + \bar{k}(\bar{t}) \right\} - 2 \left(\frac{\partial \bar{u}}{\partial \bar{r}} \right)_{\bar{r}=1} + \frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}} \quad (6)$$

Here $\bar{k}(\bar{t}) = 1 + B_1 \sin \bar{\omega}(\bar{t} - \bar{\tau}_1)$, and bar "-" stands for dimensionless quantities according to Nomenclature. Hereafter, we will omit the bar for dimensionless quantities for the sake of brevity. The second term on the right hand side of Eq.(6) arises from the inertial term of the equation of motion. When the wall Reynolds number is zero, Eq.(6) coincides with the well-known basic equation for a rigid pipe.

3.2 Solution in the Laplace space

Let us solve Eq.(6) by the method of Laplace transform. Supposing that the initial value is zero, we take the Laplace transform of both sides of Eq.(6) with respect to the dimensionless time t . The variables after the Laplace transform are expressed in capital letters. Then Eq.(6) is rewritten as the following equation:

$$L(U) - \frac{\partial U}{\partial x} = -s U_e(s) + R_w(sx + 1)K(s) \quad (7)$$

where L is a differential operator expressed as

$$L = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - s - 2 \left(\frac{\partial}{\partial r} \right)_{r=1}$$

This curious differential operator has some interesting characters. The boundary conditions for the partial differential equation (7) are as follows;

$$\left. \begin{aligned} U &= U_w \quad (r=1) \\ \text{where, } U_w(s) &= u_{w0} \left(\frac{1}{s} + \frac{B_2 \omega}{s^2 + \omega^2} \right) e^{-r_2 s} \\ \frac{\partial U}{\partial r} &= 0 \quad (r=0) \\ U &= U_e \quad (x=0) \\ \text{where, } U_e(s) &= \frac{1}{s} + \frac{B\omega}{s^2 + \omega^2} \end{aligned} \right\} \quad (8)$$

A general solution of Eq.(7) is given as the sum of a particular solution U_p and a homogeneous solution U_h . The particular solution U_p is a solution of the following inhomogeneous differential equation (9) with an inhomogeneous boundary condition (10).

$$L(U_p) - \frac{\partial U_p}{\partial x} = -s U_e(s) + R_w(sx+1)K(s) \quad \dots\dots\dots (9)$$

$$\left. \begin{aligned} U_p &= U_w \quad (r=1) \\ \frac{\partial U_p}{\partial r} &= 0 \quad (r=0) \end{aligned} \right\} \quad \dots\dots\dots (10)$$

The particular solution can be expressed by virtue of a solution of impulse response in the Laplace space, that is to say, as the following Green function:

$$G(r, s) = \{J_0(i\sqrt{s}r) - J_0(i\sqrt{s})\} / J_2(i\sqrt{s}) \quad \dots\dots\dots (11)$$

As a result we obtain the following particular solution:

$$U_p = U_w - (U_w - U_e)G - R_w K \left(xG + \frac{\partial G}{\partial s} \right) \quad \dots\dots\dots (12)$$

where the Green function g is a solution of the differential equation $L(G) = -s$. It is easily verified that the particular solution Eq.(12) satisfies the differential equation (9) and the boundary condition (10) by using $L(\partial G/\partial s) = G - 1$. On the other hand the homogeneous solution U_h is a solution of the following homogeneous differential equation:

$$L(U_h) - \frac{\partial U_h}{\partial x} = 0 \quad \dots\dots\dots (13)$$

with homogeneous boundary conditions

$$\left. \begin{aligned} U_h &= 0 \quad (r=1) \\ \frac{\partial U_h}{\partial r} &= 0 \quad (r=0) \end{aligned} \right\} \quad \dots\dots\dots (14)$$

We can obtain a homogeneous solution by the method of separation of variables. After some calculations, the homogeneous solution can be expressed as

$$U_h = \phi_0 \phi_0 + \sum \phi_n \phi_n(r) e^{-(s + \alpha_n^2)x} \quad \dots\dots\dots (15)$$

where $\sum$ indicates a sum from $n=1$ to ∞ , α_n is the n -th positive root of $J_2(\alpha) = 0$, and $\phi_0, \phi_n (n=1, 2, \dots)$ are a set of the non-normalized, but perfect orthogonal functions expressed by the following equations:

$$\left. \begin{aligned} \phi_0 &= 1 \\ \phi_n &= \frac{J_0(\alpha_n r)}{J_0(\alpha_n)} - 1 \end{aligned} \right\} \quad \dots\dots\dots (16)$$

Here we do not normalize ϕ_n in order to simplify calculations. Though ϕ_0 is orthogonal to the other ϕ_n , it does not satisfy the boundary condition at $r=1$. All of the others $\phi_n (n=1, 2, \dots)$ satisfy the boundary condition at $r=1$. Therefore $U_h = 0$ at $r=1$ leads to $\phi_0 = 0$. The others ϕ_n are determined by the inlet boundary condition and then the total velocity U satisfies the boundary condition (8) at $x=0$. Therefore we get the following equation:

$$U_e = U_p + U_h \quad (x=0) \quad \dots\dots\dots (17)$$

The right hand side of this equation is a sum of Eqs.(12) and (15) where $x=0$. We multiply both sides of Eq.(17) by $r\phi_n$ and integrate them over the cross-section. Let us put $\alpha_0 = 0$ and δ_{nm} Kronecker's delta. The orthogonal condition of ϕ_n for all n except $n=0$:

$$\int_0^1 r \phi_n \phi_m dr = \delta_{nm} \frac{\alpha_n^2}{8} \quad (n=0, m \text{ includes } 0) \quad \dots\dots\dots (18)$$

leads to the following coefficients:

$$\phi_n = 4 \left\{ \frac{U_e - U_w}{s + \alpha_n^2} + \frac{R_w K}{(s + \alpha_n^2)^2} \right\} \quad \dots\dots\dots (19)$$

As we can see, the solution in the Laplace space is completely determined.

3.3 Solution in the time domain

For the purpose of the inverse Laplace transform, we make a rational partially-fractional decomposition of the Green function. The result is as follows:

$$G(r, s) = 2(1-r^2) + \sum \frac{4s}{\alpha_n^2(s + \alpha_n^2)} \left\{ \frac{J_0(\alpha_n r)}{J_0(\alpha_n)} - 1 \right\} \quad \dots\dots\dots (20)$$

Then the solution in the Laplace space is expressed as the sum of a homogeneous solution and a particular solution.

$$U = U_p + U_h \quad \dots\dots\dots (21)$$

But from Eqs.(12) and (20) the particular solution is described by the characteristic function ϕ_n as follows:

$$\begin{aligned} U_p &= U_w - (U_w - U_e + R_w K x) \{ 2(1-r^2) \\ &+ \sum \frac{4s}{\alpha_n^2(s + \alpha_n^2)} \phi_n \} - R_w K \sum \frac{4}{(s + \alpha_n^2)^2} \phi_n \quad \dots\dots\dots (22) \end{aligned}$$

On the other hand, by substituting ψ_n expressed by Eq.(19) into Eq.(15), the homogeneous solution is rewritten as

$$\begin{aligned} U_h &= 4 \sum \left\{ (U_e - U_w) \frac{1}{s + \alpha_n^2} \right. \\ &+ \left. R_w K \frac{1}{(s + \alpha_n^2)^2} \right\} \phi_n e^{-(s + \alpha_n^2)x} \quad \dots\dots\dots (23) \end{aligned}$$

We can easily take the inverse Laplace transform of Eq.(21) and then obtain the solution of time domain as follows:

(1) Inverse Laplace transform of U_p

The particular solution is composed of three solutions, i.e., a solution for a rigid pipe, a solution influenced by the injection velocity, and a solution influenced by the slip velocity. Hence,

$$\begin{aligned} u_p(x, r, t) &= [2(1-r^2)u_e(t; B) + u_{pr}(r, t)] \\ &+ [-2(1-r^2)R_w x u_e(t - \tau_1; B_1) \\ &+ u_{pi}(x, r, t)] \\ &+ [-2(1-r^2)u_{w0} u_e(t - \tau_2; B_2) + u_{ps}(r, t)] \quad \dots\dots\dots (24) \end{aligned}$$

u_{pr} , u_{pi} and u_{ps} are expressed

respectively as the sum of a pulsatile term (0) and a transient term (t), and they are described as follows:

$$u_{pr} = u_{pro} + u_{prt}$$

$$u_{pi} = u_{pio} + u_{pit}$$

$$u_{ps} = u_{pro} + u_{prt}$$

Here, the solution for a rigid pipe does not depend on x , whose components are described as follows:

$$u_{pro}(r, t; B) = 4 \sum \frac{B\omega}{\alpha_n^4 + \omega^2} \phi_n \left(\frac{\omega}{\alpha_n^2} \sin \omega t + \cos \omega t \right) \dots (25)$$

$$u_{prt}(r, t; B) = 4 \sum \phi_n \left(\frac{1}{\alpha_n^2} - \frac{B\omega}{\alpha_n^4 + \omega^2} \right) e^{-\alpha_n^2 t} \dots (26)$$

Components of the solution representing the influence of the injection velocity are

$$u_{pio}(x, r, t; B_1) = -R_w x u_{pro}(r, t - \tau_1; B_1) - R_w f(r, t - \tau_1; B_1) \dots (27)$$

$$u_{pit}(x, r, t; B_1) = -R_w x u_{prt}(r, t - \tau_1; B_1) + R_w g(r, t - \tau_1; B_1) \dots (28)$$

Components of the solution representing the influence of the slip velocity are

$$u_{pro}(x, r, t; B_2) = u_{w0} \{1 - B_2 \sin \omega(t - \tau_2)\} - u_{w0} u_{pro}(r, t - \tau_2; B_2) \dots (29)$$

$$u_{prt}(x, r, t; B_2) = -u_{w0} u_{prt}(r, t - \tau_2; B_2) \dots (30)$$

and they do not depend on x either, where

$$f(r, t; B) = 4 \sum \phi_n \left[\frac{1}{\alpha_n^4} + \frac{B}{(\alpha_n^4 + \omega^2)^2} \{ (\alpha_n^2 - \omega^2) \sin \omega t - 2\alpha_n^2 \omega \cos \omega t \} \right] \dots (31)$$

$$g(r, t; B) = 4 \sum \phi_n \left\{ \frac{t}{\alpha_n^2} + \frac{1}{\alpha_n^4} - \frac{B\omega}{\alpha_n^4 + \omega^2} t - \frac{2B\alpha_n^2 \omega}{(\alpha_n^4 + \omega^2)^2} \right\} e^{-\alpha_n^2 t} \dots (32)$$

(2) Inverse Laplace transform of U_h

We take the inverse Laplace transform of the homogeneous solution just like the particular solution. That is

$$u_h(x, r, t) = [h(x, r, t - x; B) + u_{hr}(x, r, t)] + [0 + u_{hi}(x, r, t)] + [-u_{w0} h(x, r, t - \tau_2 - x; B_2) + u_{hs}(x, r, t)] \dots (33)$$

where,

$$u_{hr}(x, r, t) = u_{hro}(x, r, t; B) - u_{prt}(r, t; B)$$

for $t > x$;

$$u_{hi}(x, r, t) = u_{hio}(x, r, t; B_1) - u_{pit}(x, r, t; B_1)$$

for $t > x + \tau_1$;

$$u_{hs}(x, r, t) = u_{hso}(x, r, t; B_2) - u_{prt}(r, t; B_2)$$

for $t > x + \tau_2$.

For the time except the above, u_{hr} , u_{hi} and u_{hs} are zero. Where, the pulsatile component of the homogeneous solution and a function h in Eq.(33) are as follows:

$$u_{hro}(x, r, t; B) = -u_{pro}(r, t - x; B) e^{-\alpha_n^2 x}$$

$$u_{hio}(x, r, t; B_1) = R_w f(r, t - \tau_1 - x; B_1) e^{-\alpha_n^2 x}$$

$$u_{hso}(x, r, t; B_2) = u_{w0} u_{pro}(r, t - \tau_2 - x; B_2) e^{-\alpha_n^2 x}$$

$$h(x, r, t; B) = 4 \sum \frac{1}{\alpha_n^2} \phi_n e^{-\alpha_n^2 x} (1 + B \sin \omega t)$$

It is very easy for us to give a compact Green's function expression of solution in the integral form. But here, we will show a result of integration in order to calculate it concretely. These solutions are apparently intricate, but we should pay attention to the fact that they are made of some fundamental and elementary functions and that all the transient terms are offset by $u_p + u_h$ for $t > \max(x + \tau_1, x + \tau_2)$. In particular, on the assumption that there is no injection velocity and the wall Reynolds number is zero, this velocity does not depend on x and it is uniform everywhere in the axial direction at a great distance from the inlet. Furthermore, concerning the dependence of the solution on the axial direction, we can recognize that, due to the injection at the wall, only the particular solution which expresses the influence of injection velocity is linear with respect to x , and we can also recognize that the pulsatile component of the homogeneous solution decays exponentially in the x direction.

The analytical solution mentioned above for $u_{w0} = 0$ and $R_w = 0$ which is a result of the pulsatile flow in a rigid pipe

(a) $\bar{x} = 0.04$

(b) $\bar{x} = 0.08$

Fig.2 Comparison of axial velocity Profiles ($B=0.5$, $R_w=1$, $\bar{u}_{w0}=0.01$, $\bar{\omega}=33.3$, $\bar{t}=10$, —:Oseen's approximation, - - -:Momentum integral method)

Fig.3 Axial velocity profiles (Injection, $B=0.25$, $R_w=1$, $\bar{u}_{w0}=0.01$, $\bar{\omega}=42$) Fig.4 Axial velocity profiles (Suction, $B=0.25$, $R_w=-1$, $\bar{u}_{w0}=0.01$, $\bar{\omega}=42$) Fig.5 Pulsatile component of velocity ($B=0.5$, $\bar{u}_{w0}=0.01$, $\bar{\omega}=42$) Fig.6 Centerline velocities ($B=0.25$, $\bar{u}_{w0}=0.01$, $\bar{\omega}=42$)

coincides with Yokota-Nakano's approximate solution. And its flow properties are confirmed by experiments using a Laser Doppler flow velocity meter.

4. Calculating Results and Discussion

4.1 Available range of each parameter

In this numerical study we will consider the range of each parameter to be about $0 \leq \bar{u}_{w0} \leq 0.2$, $|R_w| \leq 3$, $B < 0.5$ and $x < 0.3$ from two viewpoints of the degree of approximation of the basic differential equation and of the characteristics of a porous pipe. As the phase delay is considered to be very small in the case of an incompressible fluid, we may set $\tau_1 = \tau_2 = 0$. Furthermore we will have specified parameters $B_1 = B_2 = B$ and $\omega = 42$ as a representative dimensionless frequency. As for time, we will only investigate a pulsatile stationary state on the condition that $t \gg x$.

4.2 Velocity profiles

Let us apply the momentum integral method introduced in the last paper to a pulsatile flow. The comparison of the

momentum integral method with Oseen's approximate method adopted in this analysis is made in Fig.2. Though a little difference appears between the two methods in the neighborhood of the joint of boundary layer and potential core, two results by these different methods agree well on the whole. Next, change of axial velocity profiles over one period is shown in Figs.3 and 4. Fig.3 is for the case of injection and Fig.3-(a) and (b) for the cases of $x=0.08$ and $x=0.12$, respectively. In these cases, velocity profiles scarcely change in the axial direction, but in the case of suction (Fig.4-(a),(b)) the velocity considerably increases. Suction and injection have remarkable effects on the results in the range from $x=0.12$ to $x=0.20$. We can easily recognize these phenomena from the observation of change of the centerline velocity in the axial direction, which will be mentioned in the section 4.3. The definition of $\theta = \omega t$ in the curves (a,b,...,h) over a period of time is common to all of the following figures. Profiles of the pulsatile component of the velocity in the cases of suction and injection are shown in Fig.5. A concave

which is called an annular effect can be observed at the center of some velocity profiles.

4.3 Centerline velocity

We can easily grasp influence of porosity of a pipe on the flow according to the time change of the centerline velocity. The result is shown in Fig.6. In the case (a) of injection, the increase of velocity is suppressed at $x=0.1$ and

after this it slows down. On the other hand, in the case (b) of suction the velocity is always increases. But in the case of a rigid pipe, the centerline

(a) Injection ($R_w=1$)

(b) Suction ($R_w=-1$)

Fig.7 Wall shearing stresses
($B=0.25$, $\bar{u}_{w0}=0.01$, $\bar{\omega}=42$)

(a) Injection ($R_w=1$)

(b) Suction ($R_w=-1$)

Fig.8 Pressure coefficients
($B=0.25$, $\bar{u}_{w0}=0.01$, $\bar{\omega}=42$)

Fig.9 Axial velocity profiles ($B=0.25$, $\bar{\omega}=42$, $\bar{x}=0.08$)

Fig.10 Axial velocity profiles ($B=0.25$, $\bar{\omega}=42$, $\bar{x}=0.08$)

velocity gradually approaches a fixed value in each cycle as x increases. The value is 2 in the steady state. If the entrance region in a porous pipe is defined as a region from the inlet to the point where the centerline velocity begins to change in proportion to x , it is fairly short as compared with that in a rigid pipe in a steady flow ($x=0.26$) which was confirmed by Nikuradse's experiment.

4.4 Wall shearing stress

The dimensionless wall shearing stress is defined by $\tau_w = -(\partial u / \partial r)_{r=1}$. The calculating results are shown in Fig. 7. Figure 7-(a) is injection and (b) is suction. In the case of a steady flow in a rigid pipe for $R_w=0$, $u_{w0}=0$ and $B=0$, it gradually approaches a value 4 as $x \rightarrow \infty$. On the other hand, in the case of a porous pipe we recognize that the wall shearing stress in injection decreases as the velocity gradient decreases, but it increases in suction. The reason why the wall shearing stress of the entrance region decreases immediately is that the uniform velocity profile becomes round quickly due to the viscosity of the fluid.

4.5 Pressure coefficient

Pressure in the fluid is normalized by ρu_0^2 . Using the solution for velocity, we can obtain a velocity gradient from the following equation;

$$-\frac{dp}{dx} = \frac{\partial u_z}{\partial t} - R_w x \frac{\partial k(t)}{\partial t} - R_w k(t) - 2 \left(\frac{\partial u}{\partial r} \right)_{r=1} \quad \dots (34)$$

Integrating Eq.(34) with respect to x and rearranging it, leads to a pressure coefficient C_p . The result is shown in Fig. 8. As x increases, the time change of pressure in suction becomes larger than in injection. And the pressure drop in suction is larger than in injection or in the case of a rigid pipe. This fact is clearly explained by Eq.(34) on the supposition that R_w is negative.

4.6 Influence of slip velocity

When we compare Fig. 3-(a) with Fig. 9-(a), the change of the slip velocity from 0.01 to 0.02 has little influence on the velocity gradient. But as shown in Figs. 3-(a) and 9-(c), it has a great influence on the velocity gradient in the case of a large slip velocity ($u_{w0}=0.04$) due to porosity of the pipe wall. The centerline velocity of $u_{w0}=0.04$ is fairly small as compared with that of $u_{w0}=0.02$. We can explain this fact as follows: that is to say, the growth of velocity on the centerline is prevented by the slip flow on the wall. Owing to this, the ratio of the boundary layer to the potential core gets larger and consequently the entrance region becomes smaller.

4.7 Influence of wall Reynolds number

In Fig. 9-(a),(b) and Fig. 10-(a),(b) we show the result in the case of the duplicated wall Reynolds number. In comparison with the slip velocity discussed in the last section, wall Reynolds number has a strong effect on the velocity gradient. We examine precisely

(a) Injection ($R_w=2$)

(b) Suction ($R_w=-2$)

Fig. 11 Wall shearing stresses ($B=0.25$, $u_{w0}=0.01$, $\bar{\omega}=42$)

(a) Injection ($R_w=1$)

(b) Suction ($R_w=-1$)

Fig. 12 Axial velocity profiles ($B=1$, $u_{w0}=0.01$, $\bar{\omega}=42$, $\bar{x}=0.12$)

(a) Injection ($R_w=1$)

(b) Suction ($R_w=-1$)

Fig. 13 Pressure coefficients ($B=1$, $u_{w0}=0.01$, $\bar{\omega}=42$)

the influence of the wall Reynolds number on the wall shearing stress and show it in Figs.7 and 11. The amplitude of vibration decreases very much as x increases in the case of double injection volume. However, in the case of double suction volume, the amplitude becomes large. The inertia of mass on the centerline in injection decreases owing to the loss of the flowing-out mass volume. As a result, the entrance region gets shorter in the same way as in the case of existence of the slip velocity. Hence the boundary layer develops more quickly. From the above-mentioned fact, we can control the slip velocity and the wall Reynolds number changing the porosity of the pipe wall. Further we can control the pulsatile shearing stress and the entrance region.

4.8 Influence of amplitude

The available range of B in a rigid pipe is defined as $B < 0.5$ at $x < 0.3$ by Yokota-Nakano(1979). In a porous pipe it is further limited owing to the influence of the radial velocity. But here we also try to calculate for $B=1.0$ beyond the range in order to grasp the flow behavior from a viewpoint of qualitative analysis. As an example, we will show velocity profiles in Fig.12 and the pressure coefficient in Fig.13. As to velocity profiles, we can find a partial back flow near the wall in both suction and injection. And as to pressure coefficient, the time when $C_p < 0$ is longer than in the case of $B=0.25$. The pressure coefficient takes a minimum value at $t=e$ in the figure, but in the velocity profile at the corresponding time, the back flow does not appear yet in Fig.12 and it appears later at $t=f$ or g . That is, we can qualitatively find a phase difference between velocity profile and pressure. Strictly speaking, we need a more precise analysis on characteristics of frequency responses.

5. Conclusions

In this paper, we approximated the convective term of the equation of motion by Oseen's method and examined characteristics of oscillatory flows over the entrance region in a porous pipe. Oseen's approximation has been found to be a good approximation for this case. We have also obtained the centerline velocity profile, the shearing stress at the wall and the pressure coefficient in the stationary oscillatory flow. We have further examined how the flow was influenced as the flow rate of injection,

the slip velocity and the amplitude of vibration were changed. As a result it has become clear that the wall Reynolds number has a great effect on the flow, so the inertial force for injection becomes weak suddenly and the velocity decreases rapidly before the velocity leads to develop in the entrance region. Hence, when the ratio of the effect of viscosity to the effect of inertia becomes relatively large, the boundary layer develops faster. On the contrary, the velocity increases faster in suction than in injection because of a sudden increase of the inertia force. And the larger the wall Reynolds number is, the smaller the amplitudes of centerline velocity, wall shearing stress and pressure coefficient in injection become. But these amplitudes become very large in suction. This can be explained by the fact that the pulsatile component is damped by viscosity as x increases, while it is amplified by inertia. The analytical expression of velocity which consists of particular and homogeneous solutions give us various informations on the flow by changing the parameters in it. In this paper we have examined some characteristics of the analytical solution in the time domain and further from the solution in the Laplace space we will be able to examine other characteristics of frequency response and the frequency dependence of the drag coefficient.

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